

EFFECT OF COPULA AND FAULT TOLERANCE FACTOR ON AVAILABILITY OF THE SYSTEM

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Abstract:

In this paper we discuss the effect of Copula and Fault tolerance factor on Availability of the system consists of nine states and each state contains two subsystems A and B. Both subsystems contain two units which are similar. The units of the subsystem either in the condition of working or failed due to hardware failure, human error, Catastrophic failure, Common cause failure. Expressions for availability of the system is evaluated under different conditions. With the help of these numerical calculations and graphs are also developed.

Index terms: System, Availability, Copula, Fault tolerant factor, Failure, Gumbel-Hougaard family copula

I. Introduction:

In the dynamic landscape of modern technological systems, ensuring uninterrupted functionality in the face of potential failures is a critical challenge. Despite rigorous investigations and advancements in fault detection, the inevitability of system failures persists. To address this inherent vulnerability, fault tolerance emerges as a paramount concept, embodying the proactive measures taken to maintain system operation even when confronted with errors.

One compelling strategy for fortifying fault tolerance lies in the utilization of the Copula technique, a powerful statistical tool that describes the interdependence among variables. Copulas have garnered significant attention across diverse domains due to their versatility and efficacy in modelling complex relationships. This research explores the application of the Gumbel-Hougaard family copula to facilitate efficient and effective repairs in the wake of system failures.

II. Literature review:

The scientists conducted mostly their research activities on evaluation readiness in a variety of circumstances. Dr. Mas'abu Musa, Mr. Ibrahim Yusuf, Miss. Aisha Umar Dakingari [1] performed a technique for the most important system components by carrying out a dependability, availability, maintainability and reliability analysis on the solar water pumping system. Longxiang fang, Jinling lu, and Shuai Zhang [2] proved that the maximum claims of accidents would have uniform stochastic order when the matrix would change to another matrix in a certain mathematical reasoning. Praveen Kumar, Poornima [3] conducted a dependability study on a multi-station complex engineering system where three subsystems faced imminent disaster at time t . Ibrahim Yusuf, Abdullahi Sanusi, Mus'abu Musa, Surajo Sulaiman [4] focused on the performance of a chillers water system in multi- stations. Abdullahi Sanusi and Ibrahim Yusuf [5] are finding out how the fault-tolerant feature of the model increases its availability and profitability and also gives a plan for optimal maintenance.

III. Assumptions:

1. To start with, the state of the system is in a good condition.
2. As a matter of fact, all system failure rates remain unvarying and have the exponential distribution.
3. All types of failures occur at any time, regardless of whether one or two units from both sub systems are operational.
4. Two repairs' facilities work together to repair the system in its completely failed state.
5. The repairs of completely failed states or units are modelled using Gumbel-Hougaard Family Copula or arbitrary exponential distribution.
6. The system functions as if it were new after being repaired.

IV. Model description:

The model/system consists of nine states $S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8$. If a fault occurs in the system, it recovers immediately using the fault tolerance factor C. However, if the system is unable to recover, then it enters a complete failure state and must be repaired back to its original state using an exponential distribution or Copula repair.

V. Description of States:

States	Description
S_0	It is the ideal state in which the two subsystems and their respective units work perfectly.
S_1	In this state, the two units in subsystem 2 are working perfectly, but the first unit of subsystem 1 has failed.
S_2	In this state, one unit has previously failed in subsystem 1, and the first unit from subsystem 2 unexpectedly failed, but the other units from both subsystems 1 and 2 are fully operational. The system is now operational.
S_3	This state denotes total failure as a result of the failure of both units in subsystem 1.
S_4	This state represents total failure as a result of the failure of both subsystem 2 units.
S_5	This state denotes complete failure due to human operator error.
S_6	This state also represents complete failure due to hardware failure.
S_7	This state is a complete failure due to common cause failure.
S_8	This state is a complete failure due to catastrophic failure.

VI. Notations:

t	Time
s	Laplace Transform variable
S_i	Transition states of the system
λ_1	Failure rate of subsystem 1
λ_2	Failure rate of subsystem 2
λ_{hf}	Failure rate due to hardware failure
λ_h	Failure rate due to human error
λ_{cc}	Failure rate due to common cause failure
λ_{cf}	Failure rate due to catastrophic failure
C	Fault tolerance factor
$\mu(x)$	Rate of repair for the completely failed state
$G_i(t), i = 0, 1, 2, 3, 4, 5, 6, 7, 8$	The probability of the system being in state S_i at any given time t.
$\bar{G}_i(s)$	Laplace transform of $G_i(t)$
$G_i(x, t), i = 3, 4, 5, 6, 7, 8$	The probability density function of the failed states of the system at any given time t, multiplied by the elapsed repair time x.
$\phi(x) = \exp\left[x^\theta + \{\log \mu(x)\}^\theta\right]^{-\frac{1}{\theta}}$	Where $\phi(x)$ is the joint probability of repair rate from completely failed state to perfect state, from Gumbel-Hougaard family Copula.
$\bar{H}(s) = \int_0^\infty \phi(x) e^{-sx - \int_0^x \phi(x) dx} dx$	$\bar{H}(s)$ denotes the probability density function of the Laplace transformation of $\phi(x)$.

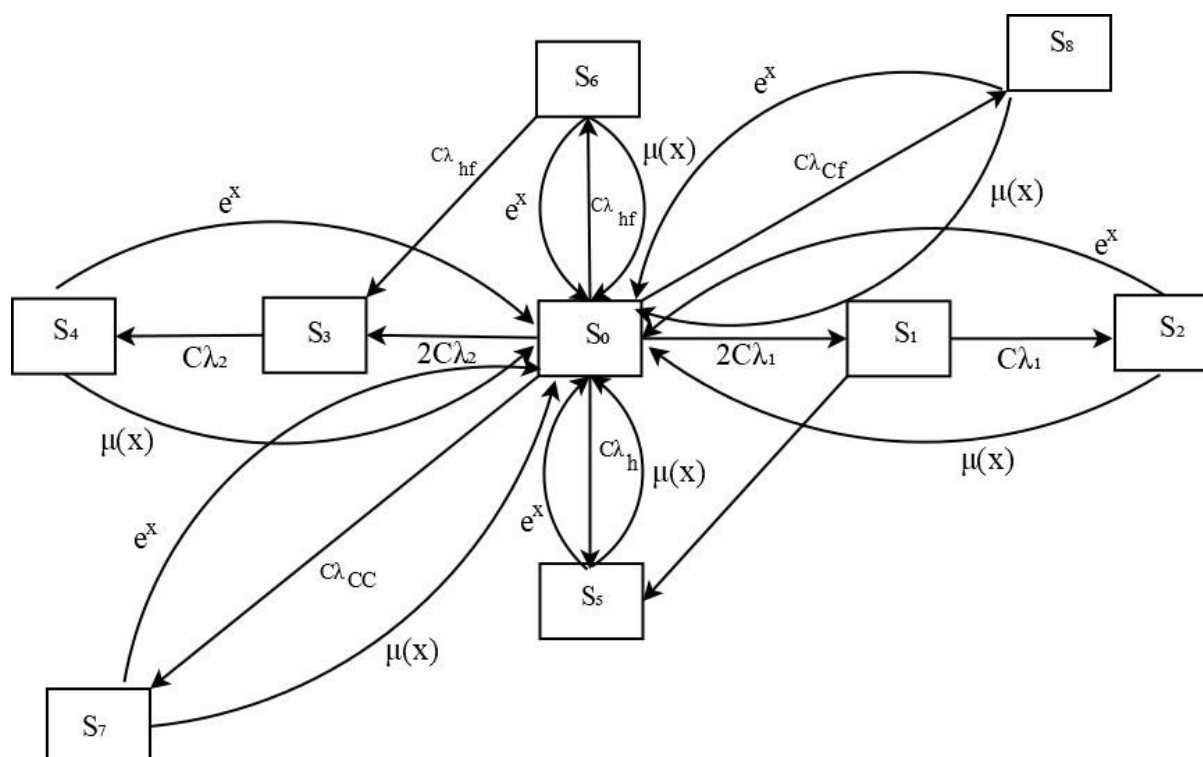


Fig 1: System Configuration

VII. Model formulation and solution:

For the model under consideration, one can derive the following set of difference differential equations using elementary probability and continuity arguments as:

$$\left[\frac{\partial}{\partial t} + 2\lambda_1 C + 2\lambda_2 C + \lambda_{hf} C + \lambda_h C + \lambda_{cc} C + \lambda_{cf} C \right] G_0(t) = \int_0^\infty [\mu(x) + e^x] G_3(x, t) dx + \int_0^\infty [\mu(x) + e^x] G_4(x, t) dx \\ + \int_0^\infty [\mu(y) + e^y] G_5(y, t) dy + \int_0^\infty [\mu(y) + e^y] G_6(y, t) dy \\ + \int_0^\infty [\mu(z) + e^z] G_7(z, t) dz + \int_0^\infty [\mu(z) + e^z] G_8(z, t) dz \quad (1)$$

$$\left[\frac{\partial}{\partial t} + \lambda_1 C + \lambda_h C \right] G_1(x, t) = 2\lambda_1 C G_0(t) \quad (2)$$

$$\left[\frac{\partial}{\partial t} + \lambda_2 C + \lambda_{hf} C \right] G_2(x, t) = 2\lambda_2 C G_0(t) \quad (3)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \{\mu(x) + e^x\} \right] G_3(x, t) = 0 \quad (4)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \{\mu(x) + e^x\} \right] G_4(x, t) = 0 \quad (5)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \{\mu(y) + e^y\} \right] G_5(y, t) = 0 \quad (6)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \{\mu(y) + e^y\} \right] G_6(y, t) = 0 \quad (7)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \{ \mu(z) + e^z \} \right] G_7(z, t) = 0 \quad (8)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \{ \mu(z) + e^z \} \right] G_8(z, t) = 0 \quad (9)$$

The Boundary conditions:

$$G_3(0, t) = 2\lambda_1^2 C G_0(t) \quad (10)$$

$$G_4(0, t) = 2\lambda_2^2 C G_0(t) \quad (11)$$

$$G_5(0, t) = \lambda_h C (1 + 2\lambda_1 C) G_0(t) \quad (12)$$

$$G_6(0, t) = \lambda_{hf} C (1 + 2\lambda_2 C) G_0(t) \quad (13)$$

$$G_7(0, t) = \lambda_{cc} C G_0(t) \quad (14)$$

$$G_8(0, t) = \lambda_{cf} C G_0(t) \quad (15)$$

The Initial Conditions are

$$G_0(0) = 1 \text{ and the remaining state probabilities are zero at } t = 0 \quad (16)$$

VIII. Solution of the model:

Taking Laplace transforms of equations 1-9 and (10-15) then we get

$$\begin{aligned} [s + 2\lambda_1 C + 2\lambda_2 C + \lambda_{hf} C + \lambda_h C + \lambda_{cc} C + \lambda_{cf} C] \bar{G}_0(s) = & \int_0^\infty [\mu(x) + e^x] \bar{G}_3(x, s) dx + \int_0^\infty [\mu(x) + e^x] \bar{G}_4(x, s) dx \\ & + \int_0^\infty [\mu(y) + e^y] \bar{G}_5(y, s) dy + \int_0^\infty [\mu(y) + e^y] \bar{G}_6(y, s) dy \\ & + \int_0^\infty [\mu(z) + e^z] \bar{G}_7(z, s) dz + \int_0^\infty [\mu(z) + e^z] \bar{G}_8(z, s) dz \end{aligned} \quad (17)$$

$$[s + \lambda_1 C + \lambda_h C] \bar{G}_1(x, s) = 2\lambda_1 C \bar{G}_0(s) \quad (18)$$

$$[s + \lambda_2 C + \lambda_{hf} C] \bar{G}_2(x, s) = 2\lambda_2 C \bar{G}_0(s) \quad (19)$$

$$\left[s + \frac{\partial}{\partial x} + \{ \mu(x) + e^x \} \right] \bar{G}_3(x, s) = 0 \quad (20)$$

$$\left[s + \frac{\partial}{\partial x} + \{ \mu(x) + e^x \} \right] \bar{G}_4(x, s) = 0 \quad (21)$$

$$\left[s + \frac{\partial}{\partial y} + \{ \mu(y) + e^y \} \right] \bar{G}_5(y, s) = 0 \quad (22)$$

$$\left[s + \frac{\partial}{\partial y} + \{ \mu(y) + e^y \} \right] \bar{G}_6(y, s) = 0 \quad (23)$$

$$\left[s + \frac{\partial}{\partial z} + \{ \mu(z) + e^z \} \right] \bar{G}_7(z, s) = 0 \quad (24)$$

$$\left[s + \frac{\partial}{\partial z} + \{ \mu(z) + e^z \} \right] \bar{G}_8(z, s) = 0 \quad (25)$$

Boundary conditions:

$$\bar{G}_3(0, s) = 2\lambda_1^2 C \bar{G}_0(s) \quad (26)$$

$$\bar{G}_4(0, s) = 2\lambda_2^2 C \bar{G}_0(s) \quad (27)$$

$$\bar{G}_5(0, s) = \lambda_h C (1 + 2\lambda_1 C) \bar{G}_0(s) \quad (28)$$

$$\bar{G}_6(0, s) = \lambda_{hf} C (1 + 2\lambda_2 C) \bar{G}_0(s) \quad (29)$$

$$\bar{G}_7(0, s) = \lambda_{cc} C \bar{G}_0(s) \quad (30)$$

$$\bar{G}_8(0, s) = \lambda_{cf} C \bar{G}_0(s)$$

We get the following equations by solving (17) to (25), using the equations (26) to (30)

$$\bar{G}_0(s) = \frac{1}{(s + 2\lambda_1 C + 2\lambda_2 C + \lambda_{hf} C + \lambda_h C + \lambda_{cc} C + \lambda_{cf} C) - \bar{H}(s) [2\lambda_1^2 C + 2\lambda_2^2 C + \lambda_h C (1 + 2\lambda_1 C) + \lambda_{hf} C (1 + 2\lambda_2 C) + \lambda_{cc} C + \lambda_{cf} C]} \quad (31)$$

$$\bar{G}_1(s) = \left[\frac{2\lambda_1 C}{s + \lambda_1 C + \lambda_h C} \right] \bar{G}_0(s) \quad (32)$$

$$\bar{G}_2(s) = \left[\frac{2\lambda_2 C}{s + \lambda_2 C + \lambda_{hf} C} \right] \bar{G}_0(s) \quad (33)$$

$$\bar{G}_3(s) = \left[\frac{1 - \bar{H}(s)}{s} \right] \bar{G}_3(0, s) \quad (34)$$

$$\bar{G}_4(s) = \left[\frac{1 - \bar{H}(s)}{s} \right] \bar{G}_4(0, s) \quad (35)$$

$$\bar{G}_5(s) = \left[\frac{1 - \bar{H}(s)}{s} \right] \bar{G}_5(0, s) \quad (36)$$

$$\bar{G}_6(s) = \left[\frac{1 - \bar{H}(s)}{s} \right] \bar{G}_6(0, s) \quad (37)$$

$$\bar{G}_7(s) = \left[\frac{1 - \bar{H}(s)}{s} \right] \bar{G}_7(0, s) \quad (38)$$

$$\bar{G}_8(s) = \left[\frac{1 - \bar{H}(s)}{s} \right] \bar{G}_8(0, s) \quad (39)$$

The system's operational state availability is given by:

$$\bar{G}_{up}(s) = \bar{G}_0(s) + \bar{G}_1(s) + \bar{G}_2(s) = \left[1 + \left[\frac{2\lambda_1 C}{s + \lambda_1 C + \lambda_h C} \right] + \left[\frac{2\lambda_2 C}{s + \lambda_2 C + \lambda_{hf} C} \right] \right] \bar{G}_0(s) \quad (40)$$

IX. Special Cases:

Availability Analysis:

i. System availability when neither the Copula nor the Fault Tolerance factor is present:

The expression for Laplace transforms of the system availability when neither the Copula nor the Fault Tolerance factor is present is given by

$$\bar{G}_{up}(s) = \left[1 + \left[\frac{2\lambda_1}{s + \lambda_1 + \lambda_h} \right] + \left[\frac{2\lambda_2}{s + \lambda_2 + \lambda_{hf}} \right] \right] \bar{G}_0(s) \quad (41)$$

Where

$$\bar{G}_0(s) = \frac{1}{(s + 2\lambda_1 + 2\lambda_2 + \lambda_{hf} + \lambda_h + \lambda_{cc} + \lambda_{cf}) - \frac{\mu}{s + \mu} [2\lambda_1^2 + 2\lambda_2^2 + \lambda_h(1 + 2\lambda_1) + \lambda_{hf}(1 + 2\lambda_2) + \lambda_{cc} + \lambda_{cf}]} \quad (42)$$

Take $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\lambda_h = 0.03$, $\lambda_{hf} = 0.04$, $\lambda_{cc} = 0.05$, $\lambda_{cf} = 0.06$ and $\mu = 1$, using the Laplace transform then we get

$$G_{up}(t) = 0.02000000000e^{-0.04000000000t} + 0.04000000000e^{0.06000000000t} + 0.03494282789e^{-0.6200000000t} (28.61817604 \cosh(0.5723635208t) + 19. \sinh(0.5723635208t)) \quad (43)$$

Table 1 and Fig.2 illustrate the system's availability when Copula and Fault Tolerance factor are both absent, taking $t = 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60$.

Time(t)	Availability $G_{up}(t)$
0	1.0000
4	0.859325438
8	0.8410499271
12	0.8020161869
16	0.7496171591
20	0.6900676888
24	0.6277384200
28	0.5656234888
32	0.5057038531
36	0.4492221622
40	0.3968897443
44	0.3490416113
48	0.3057518405
52	0.2669188353
56	0.2323277328
60	0.2016955939

Table 1

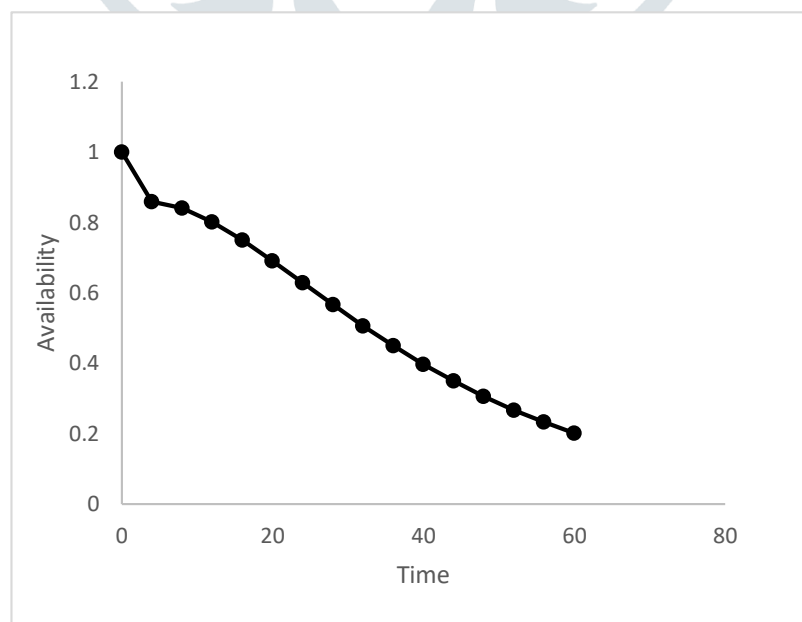


Fig 2

ii. System availability in the presence of Copula:

The system availability, when a Copula is considered, without taking into account the Fault Tolerance factor, is expressed as:

$$\bar{G}_{up}(s) = \left[1 + \left[\frac{2\lambda_1}{s + \lambda_1 + \lambda_h} \right] + \left[\frac{2\lambda_2}{s + \lambda_2 + \lambda_{hf}} \right] \right] \bar{G}_0(s) \quad (44)$$

Where,

$$\bar{G}_0(s) = \frac{1}{(s + 2\lambda_1 + 2\lambda_2 + \lambda_{hf} + \lambda_h + \lambda_{cc} + \lambda_{cf}) - \frac{e}{s+e} \left[2\lambda_1^2 + 2\lambda_2^2 + \lambda_h(1+2\lambda_1) + \lambda_{hf}(1+2\lambda_2) + \lambda_{cc} + \lambda_{cf} \right]} \quad (45)$$

Take $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\lambda_h = 0.03$, $\lambda_{hf} = 0.04$, $\lambda_{cc} = 0.05$, $\lambda_{cf} = 0.06$, $\mu = 1$, $x = 1$, $\theta = 1$ using the Laplace transform then we get

$$G_{up}(t) = 1.422076998e^{-0.04000000000t} - 5.453325784e^{-0.06000000000t} + 1.833209367 \cdot 10^{-15} e^{-1.479150000t} \left(\begin{array}{l} 2.744503097 \cdot 10^{15} \cosh(1.426003254t) \\ + 2.674529648 \cdot 10^{15} \sinh(1.426003254t) \end{array} \right) \quad (46)$$

We obtain Table 2 and Fig.3 for system availability in the presence of Copula taking $t = 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60$.

Time(t)	Availability $G_{up}(t)$
0	1.00000
4	0.937936005
8	0.904994483
12	0.850529803
16	0.784089547
20	0.712311364
24	0.6396992999
28	0.5692069398
32	0.5026730080
36	0.4411450580
40	0.3851188295
44	0.3347137792
48	0.2898009047
52	0.2500948137
56	0.2152191774
60	0.1847525780

Table 2

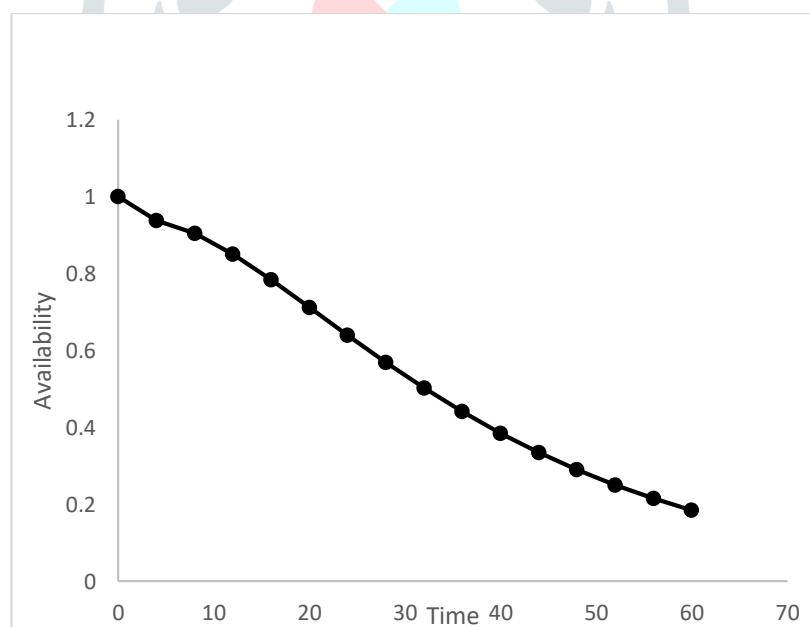


Fig 3

iii. System availability, excluding the Copula, considering the Fault Tolerance factor

The following equation illustrates the system availability when disregarding the Copula in the presence of the Fault Tolerance Factor.

$$\bar{G}_{up}(s) = \left[1 + \left[\frac{2\lambda_1 C}{s + \lambda_1 C + \lambda_h C} \right] + \left[\frac{2\lambda_2 C}{s + \lambda_2 C + \lambda_{hf} C} \right] \right] \bar{G}_0(s) \quad (47)$$

Where

$$\bar{G}_0(s) = \frac{1}{(s + 2\lambda_1 C + 2\lambda_2 C + \lambda_{hf} C + \lambda_h C + \lambda_{cc} C + \lambda_{cf} C)} - \frac{\mu}{s + \mu} \left[2\lambda_1^2 C + 2\lambda_2^2 C + \lambda_h C (1 + 2\lambda_1 C) + \lambda_{hf} C (1 + 2\lambda_2 C) + \lambda_{cc} C + \lambda_{cf} C \right] \quad (48)$$

By substituting $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\lambda_h = 0.03$, $\lambda_{hf} = 0.04$, $\lambda_{cc} = 0.05$, $\lambda_{cf} = 0.06$ and $\mu = 1$, using $t = 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60$, and the Laplace transform, we obtain various numerical results for different values of the Fault tolerance factor, which are shown in Table 3 and Fig.4 below.

When $c = 0.1$,

$$G_{up}(t) = 1.245000000e^{-0.004000000000t} - 9.289719626e^{-0.006000000000t} + 1.836624389 \cdot 10^{-7} \cdot e^{-0.5120000000t} \left(\begin{array}{l} 4.904953308 \cdot 10^7 \cdot \sinh(0.5064227483t) \\ + 4.9246431 \cdot 10^7 \cosh(0.5064227483t) \end{array} \right) \quad (49)$$

When $c = 0.3$

$$G_{up}(t) = 1.305263158e^{-0.008000000000t} - 7.373134328e^{-0.012000000000t} + 9.550130137 \cdot 10^{-9} e^{-0.5240000000t} \left(\begin{array}{l} 7.326086338 \cdot 10^8 \cdot \sinh(0.5130458068t) \\ + 7.40081137 \cdot 10^8 \cdot \cosh(0.5130458068t) \end{array} \right) \quad (50)$$

When $c = 0.5$

$$G_{up}(t) = 1.372222222e^{-0.012000000000t} - 6.099378882e^{-0.018000000000t} + 2.655057543 \cdot 10^{-7} e^{-0.5360000000t} \left(\begin{array}{l} 2.157074401 \cdot 10^7 \cdot \cosh(0.5198615200t) \\ + 2.1173033 \cdot 10^7 \cdot \sinh(0.5198615200t) \end{array} \right) \quad (51)$$

iv. System availability in the presence of both Copula and Fault tolerance factor:

The system availability incorporating both Copula and fault tolerance factor are used is given below

$$\bar{G}_{up}(s) = \left[1 + \left[\frac{2\lambda_1 C}{s + \lambda_1 C + \lambda_h C} \right] + \left[\frac{2\lambda_2 C}{s + \lambda_2 C + \lambda_{hf} C} \right] \right] \bar{G}_0(s) \quad (52)$$

Where

$$\bar{G}_0(s) = \frac{1}{(s + 2\lambda_1 C + 2\lambda_2 C + \lambda_{hf} C + \lambda_h C + \lambda_{cc} C + \lambda_{cf} C)} - \frac{e}{s + e} \left[2\lambda_1^2 C + 2\lambda_2^2 C + \lambda_h C (1 + 2\lambda_1 C) + \lambda_{hf} C (1 + 2\lambda_2 C) + \lambda_{cc} C + \lambda_{cf} C \right] \quad (53)$$

Take $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\lambda_h = 0.03$, $\lambda_{hf} = 0.04$, $\lambda_{cc} = 0.05$, $\lambda_{cf} = 0.06$, $\mu = 1$, $x = 1$, $\theta = 1$ using the Laplace transform then we get various outcomes for different values of Fault tolerance factor, which are shown in Table 4 and Fig.5 below.

When $c = 0.1$

$$G_{up}(t) = -11.09488514e^{-0.006000000000t} + 1.209919710e^{-0.004000000000t} + 6.409597338 \cdot 10^{-16} e^{-1.371150000t} \left(\begin{array}{l} 1.698229210 \cdot 10^{16} \cosh(1.365508102t) \\ + 1.696136314 \cdot 10^{16} \sinh(1.365508102t) \end{array} \right) \quad (54)$$

When $c = 0.3$

$$G_{up}(t) = -9.047322290e^{-0.01800000000t} + 1.250963773e^{-0.01200000000t} + 1.456099007 \cdot 10^{-14} e^{-1.395150000t} \left(\begin{matrix} 6.041044239 \cdot 10^{14} \cosh(1.378449742t) \\ + 6.013672831 \cdot 10^{14} \sinh(1.378449742t) \end{matrix} \right) \quad (55)$$

When $c = 0.5$

$$G_{up}(t) = -7.6277161923e^{-0.03000000000t} + 1.295159962e^{-0.02000000000t} + 5.906304530 \cdot 10^{-19} e^{-1.419150000t} \left(\begin{matrix} 1.230249317 \cdot 10^{19} \sinh(1.391684951t) \\ + 1.241385696 \cdot 10^{19} \cosh(1.391684951t) \end{matrix} \right) \quad (56)$$

Availability $G_{up}(t)$			
Time (t)	C=0.1	C=0.3	C=0.5
0	1.00000	1.000000000	1.000000000
4	0.983618181	0.952654246	0.923818895
	0.984190672	0.952324725	0.920317972
12	0.984565400	0.949017755	0.909203794
16	0.984471395	0.942461970	0.891216018
20	0.983924291	0.933070425	0.867844449
24	0.982943803	0.921226026	0.840356468
28	0.981549200	0.907274526	0.809816912
32	0.979759142	0.891527565	0.7771162572
36	0.977591448	0.874265656	0.7429953190
40	0.975063555	0.855740875	0.7080665363
44	0.972192305	0.836179374	0.6728326195
48	0.968993777	0.815783659	0.6377026157
52	0.965483700	0.794734720	0.6030058357
56	0.961677209	0.773193923	0.5690039418
60	0.957588985	0.751304818	0.5359012998

Table 3

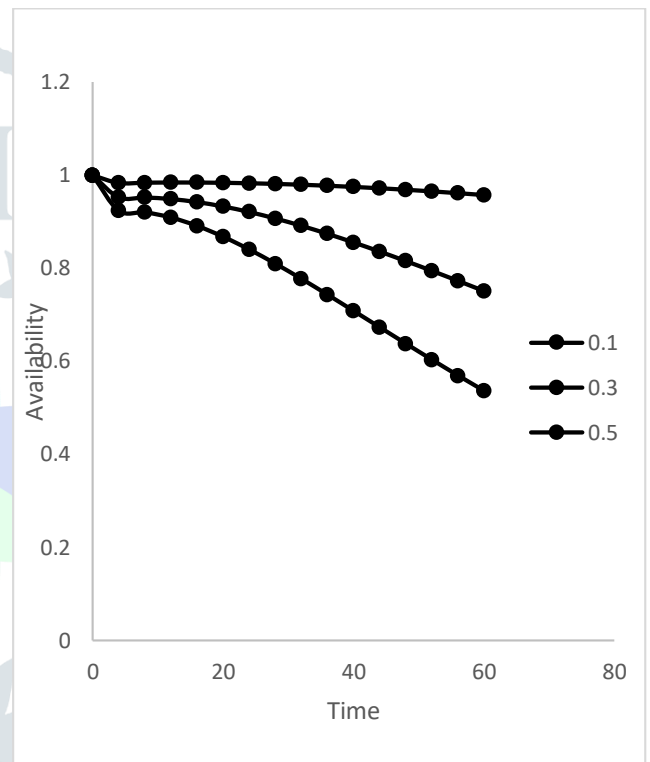


Fig 4

Availability $G_{up}(t)$			
Time (t)	C=0.1	C=0.3	C=0.5
0	1.000000000	1.000000000	1.000000000
4	0.994448772	0.982832399	0.970608150
8	0.995086953	0.981324320	0.963239163
12	0.995229655	0.976085122	0.947226596
16	0.994898503	0.967599352	0.924443206
20	0.994114193	0.956304407	0.896463976
24	0.992896843	0.942594688	0.864608115
28	0.991265880	0.926825280	0.829975444
32	0.989240003	0.909315343	0.793477969
36	0.986837544	0.890351515	0.755867520
40	0.984075966	0.870190482	0.717759170
44	0.980972202	0.849061884	0.679651914
48	0.977542639	0.827170687	0.641946235
52	0.973803003	0.804699405	0.604959250
56	0.969768718	0.781810278	0.568937951
60	0.965454460	0.758646959	0.534070310

Table 4

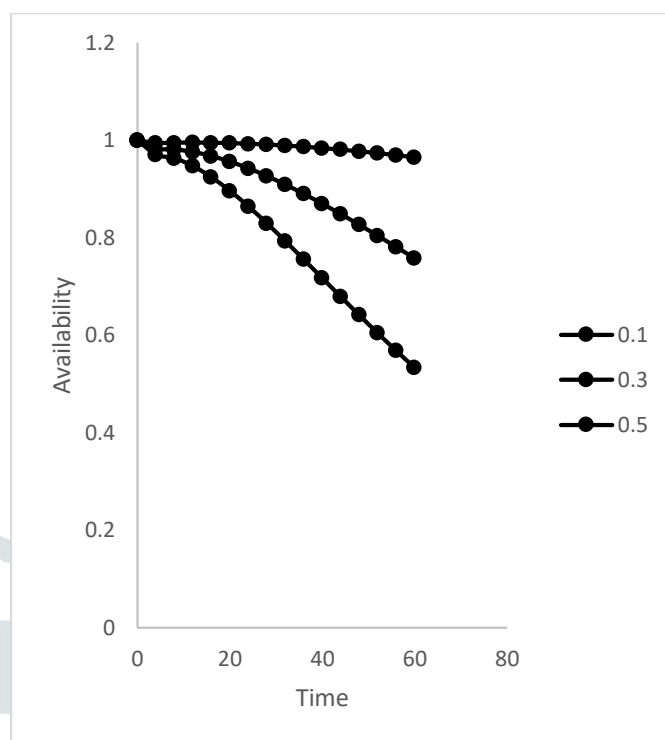


Fig 5

X. Result Analysis:

It appears that the author of this study focused on analysing the availability of a series-parallel system consisting of two subsystems. The analysis involved utilizing the Gumbel-Hougaard family Copula in conjunction with a fault tolerance factor. The main goal was to investigate and improve the system's availability through Four different approaches. Copula, Fault tolerance factor, and a combination of Copula and fault tolerance factor.

The study evaluated the system's performance under the following conditions:

1. **System availability when neither the Copula nor the Fault Tolerance factor is present:** This represents the baseline availability of the system without any special considerations.
2. **Availability in the presence of only Copula:** This assesses the impact of using the Copula alone on the system's availability.
3. **System availability, excluding the Copula, considering the Fault Tolerance factor:** This examines how the system's availability is affected when only the fault tolerance factor is considered.
4. **Availability in the presence of both Copula and Fault tolerance factor:** This examines how the system's availability is affected when both Copula and Fault tolerance are considered.

The expressions derived for these scenarios were validated numerically, and the results were presented in tables and figures. The specific case considered in the study revealed that the optimum system availability and benefit were achieved when the entire system underwent periodic repair facilitated by the Copula, and the Fault tolerance factor was invoked.

The numerical results, as presented in tables and figures, likely demonstrated the effectiveness of combining Copula and Fault tolerance factor in enhancing the overall availability of the series-parallel system. This information can be valuable for practitioners and researchers working on reliability and availability analysis in complex system, providing insights into optimal strategies for improving system performance. In all the four cases, the availability of the system decreases as the increase of time.

XI. References:

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