EVALUATION OF RELIABILITY OF BY USING BLOCK DIAGRAM APPROACH (Parallel-Series System)

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ABSTRACT:
This paper presents an approach for performing Reliability analysis of Bridge network with critical and non-critical human errors. In this paper we consider both the time dependent and time independent cases to evaluate the reliability of the parallel series system by using block diagram approach. The reliability parameters are evaluated.

Key words: Reliability, parallel series system, critical and non-critical human errors, Hazard rate.

1 INTRODUCTION

Humans play a pivotal role in the design, development and operational phases of engineering systems. Reliability evaluation of systems without taking into consideration the human element does not provide a realistic picture. Hence, there is a definite need for incorporating the occurrence of human errors in system reliability evaluation.

A human error is defined as a failure to perform a prescribed task (or the performance of a prohibited action) which could lead to disruption of scheduled operations or result in damage to property and equipment. Furthermore, depending upon the severity of human error consequences, human errors can be classified into two categories, namely, critical and non-critical. For our purpose the occurrence of a critical human error causes the entire system to fail, whereas the occurrence of a non-critical human error results in a single unit failure only.

In this paper we discuss the reliability analyses of bridge and parallel-series networks with critical and non-critical human errors [1-4]. A newly developed approach is used to perform system reliability analysis. This approach is a modified version of the block diagram approach and is demonstrated in this model. In this model we concerned with a parallel-series system. The system is composed of ‘m’ subsystems with ‘n’ units each.

2. ASSUMPTIONS:

The following assumptions are associated with analyses given below:

1. A unit can fail either due to a hardware failure or due to a non-critical human error.
2. The occurrence of a critical human error can result in total system failure but the occurrence of a non-critical human error can cause the failure of a single unit only.
3. Each unit failure is independent of others.
4.

3. PARALLEL – SERIES SYSTEM

This model is concerned with a parallel- series system shown in the following figure 1. The system is composed of *m* subsystems arranged in parallel. In turn, each subsystem is consisted of *n* units connected in series. As in the case of bridge network, a hypothetical unit representing critical human errors is connected in series with the parallel-series network. The failure of this hypothetical unit can result in system failure. As shown in figure 1, each unit is represented by a rectangle. The hardware failure and non-critical human error occurrence probabilities associated with each unit are represented by blocks in series.

The following symbols are associated with this model:

i	denotes i th unit, for $i = 1, 2, 3, \ldots\ldots\ldots\ldots, n$
j	denotes j th subsystem, for $j = 1, 2, 3, \ldots\ldots\ldots, m$
F_{ji}	Hardware failure probability of i th unit of j th subsystem
f_{ji}	Non-critical human error occurrence probability of i th unit of j th subsystem
f_C	Critical human error occurrence probability associated with the parallel-series system

$R_{H\,ji}$ Hardware reliability of *i* th unit of *j* th subsystem

$R_{NC\,ji}$ Reliability of *i* th unit of *j* th subsystem with respect to non-critical human errors

R_{ji} Reliability of *i* th unit of *j* th subsystem with respect to hardware failures and non-critical human errors

$R_{S\,j}$ Reliability of j th subsystem with respect to hardware failures and non-critical human errors

$R_{H,NC}$ Reliability of the parallel-series system with respect to hardware failures and non-critical human errors

s Laplace transform variable

R_C Reliability of the parallel-series system with respect to critical human errors

t Time

R_{PS} Reliability of the parallel-series system with respect to hardware failures, critical and non-critical human errors

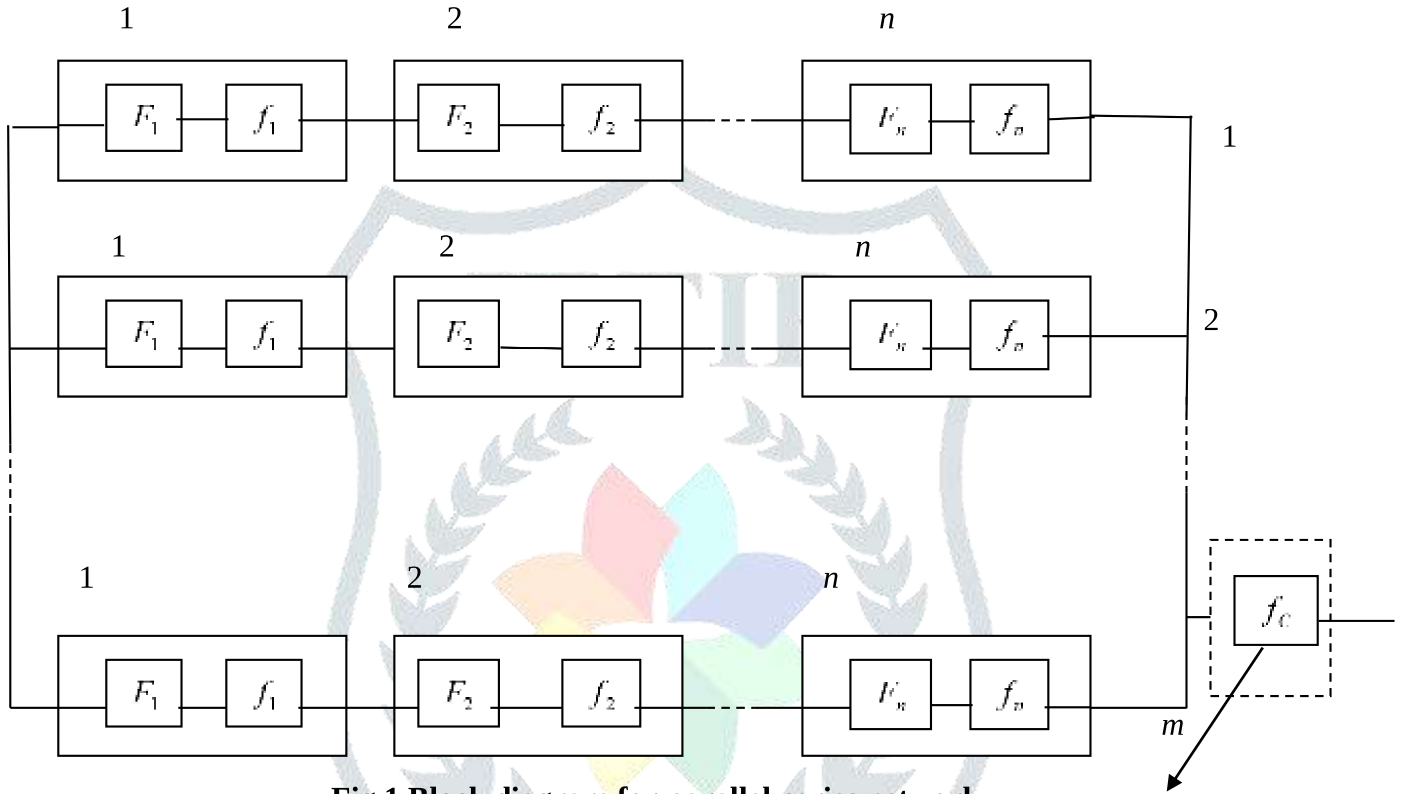

Fig.1 Block diagram for parallel-series network

4. The time independent reliability expressions for the following two cases are given below.

Case I: Non-identical units

Equations for $R_{H\,ji}, R_{NC\,ji}, R_{ji}, R_{S\,j}, R_{H,NC}$ and R_{ps} are as follows:

$$R_{H\,ji} = 1 - F_{ji} \text{ , for } i = 1,2,\ldots..,n \text{ and } j = 1,2,\ldots\ldots..,m \tag{1}$$

$$R_{NC\,ji} = 1 - f_{ji} \tag{2}$$

$$R_{ji} = R_{H\,ji} . R_{NC\,ji} \tag{3}$$

$$R_{S\,j} = \prod_{i=1}^{n} R_{ji} \tag{4}$$

$$R_{H,NC} = 1-\prod_{j=1}^{m}\left(1-R_{S\,j}\right) \tag{5}$$

$$R_{ps} = R_C \,.\, R_{H,NC} = \left(1-f_c\right)\left[1-\prod_{j=1}^{m}\left(1-\prod_{i=1}^{n} R_{H\,ji}.R_{NC\,ji}\right)\right] \tag{6}$$

Case II: Identical units:

By setting $R_{H\,ji} = R_H$ (i.e. $F_{ji} = F$) and $R_{NC\,ji} = R_{NC}$ (i.e. $f_{ji} = f$),

for i = 1,2,…..,n and j = 1,2,……..,m then the following expressions for reliability of the parallel-series system:

$$R_{PS} = R_C \left[1-\left(1-R^n\right)^m\right], \text{ where } R = R_H.R_{NC},\ R_H = 1-F \text{ and } R_{NC} = 1-f \tag{7}$$

The plots of Equation (7) are shown in figure 2 for the specified hypothetical of

F, f, m and n. Take R_H = 0.9, R_{NC} = 0.95 then R = 0.885.

f_C	R_C	R_{PS}
0.0	1	0.8593
0.03	0.97	0.8336
0.05	0.95	0.8164
0.07	0.93	0.7992
0.09	0.91	0.7820

m = 2, n = 3

f_C	R_C	R_{PS}
0.0	1	0.9159
0.03	0.97	0.8885
0.05	0.95	0.8701
0.07	0.93	0.8518
0.09	0.91	0.8335

m = 5, n = 6

f_C	R_C	R_{PS}
0.0	1	0.8784
0.03	0.97	0.8521
0.05	0.95	0.8345
0.07	0.93	0.8169
0.09	0.91	0.7993

m = 9, n = 10

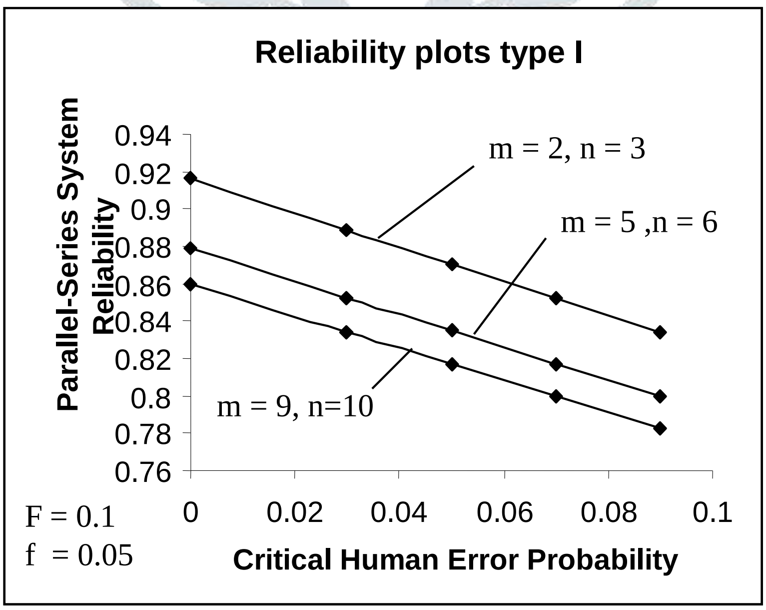

Fig. 2

5. The time dependent reliability analyses are developed for the following two cases:

Case A: Exponentially distributed failure times:

For exponentially distributed failure times, equations for R_H, R_{NC}, R and R_C are given below:

$$R_H(t) = e^{-\lambda_H t}, \tag{8}$$

Where λ_H is the constant hardware failure rate of a unit.

$$R_{NC}(t) = e^{-\lambda_{NC} \cdot t}, \tag{9}$$

Where λ_{NC} is the constant non-critical human error rate of a unit.

$$R(t) = e^{-zt}, \tag{10}$$

Where $z = \lambda_H + \lambda_{NC}$

$$R_C(t) = e^{-\lambda_C t}, \tag{11}$$

Where λ_C is the constant critical human error rate associated with the system.

The reliability of a *n*-identical unit series subsystem is given by

$$R_S(t) = e^{-nzt} \tag{12}$$

The reliability of the parallel-series system with respect to hardware failures and non-critical human errors is

$$R_{H,NC}(t) = 1 - \left(1 - e^{-nzt}\right)^m \tag{13}$$

The total reliability of the parallel-series system with human errors is

$$R_{ps} = e^{-\lambda_c t}\left[1 - \left(1 - e^{-nzt}\right)^m\right] = e^{-\lambda_c t} - \sum (-1)^k \binom{m}{k} e^{-(\lambda_c + knz)t} \tag{14}$$

The mean time to failure of the system is given by

$$\text{M.T.T.F}_{\text{ps}} = \int_0^\infty R_{ps}(t)\, dt = \frac{1}{\lambda_c} - \sum (-1)^k \binom{m}{k} \frac{1}{\lambda_c + knz} \tag{15}$$

The plots of equation (15) are shown in figure 3 for the assumed values of the model

parameters. Let $\lambda_H = 0.09$, $\lambda_{NC} = 0.05$ then $z = \lambda_H + \lambda_{NC} = 0.14$

λ_C	M.T.T.F
0.01	3.47469
0.03	3.29502
0.05	3.13172
0.07	2.98273
0.09	2.8463

m = 2, n = 3

λ_C	M.T.T.F
0.01	3.20001
0.03	3.06085
0.05	2.93196
0.07	2.81232
0.09	2.70102

m = 3, n = 4

λ_C	M.T.T.F
0.01	2.91835
0.03	2.80823
0.05	2.70499
0.07	2.60806
0.09	2.51691

m = 4, n = 5

λ_C	M.T.T.F
0.01	2.67161
0.03	2.58222
0.05	2.4977
0.07	2.41769
0.09	2.34187

m = 5, n = 6

λ_C	M.T.T.F
0.01	2.46148
0.03	2.38728
0.05	2.31667
0.07	2.24942
0.09	2.18532

m = 6, n = 7

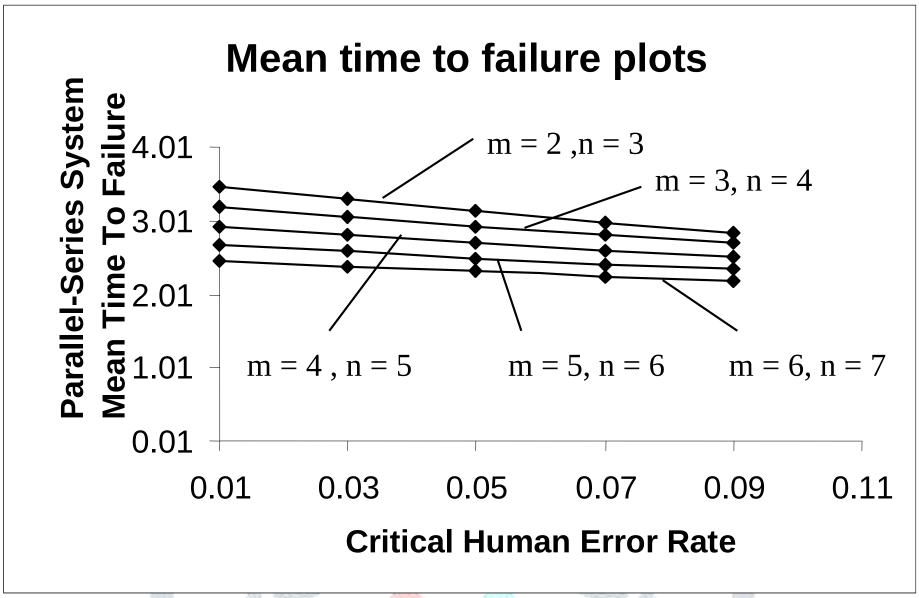

Fig .3

These plots show the effect of increasing critical human error rate λ_C on the system mean time to failure for different values of m and n .

The variance of time to failure of the parallel-series system is

$$\sigma^2_{ps} = -2\,\underset{s\to 0}{Lt}\, R'_{ps}(s) - \left(\text{M.T.T.F}_{\text{ps}}\right)^2 \tag{16}$$

$$= \frac{2}{\lambda_c^2} - 2\sum_{k=0}^{m}(-1)^k\binom{m}{k}(\lambda_c + k\,n\,z)^{-2} - \left[\frac{1}{\lambda_c} - \sum_{k=0}^{m}(-1)^k\binom{m}{k}(\lambda_c + k\,n\,z)^{-1}\right]^2 \tag{17}$$

Where $R'_{ps}(s)$ denotes the derivative of the Laplace transform of the system reliability function.

The failure density function is expressed as

$$f_{ps}(t) = -\frac{dR_{ps}(t)}{dt} = \lambda_c\, e^{-\lambda_c t} - \sum(-1)^k\binom{m}{k}(\lambda_c + k\,n\,z)e^{-(\lambda_c + k\,n\,z)\,t} \tag{18}$$

The hazard rate function of the parallel-series system is given by

$$h_{p\,s}(t) = \frac{f_{ps}(t)}{R_{ps}(t)} \tag{19}$$

Case B: Rayleigh Distributed Failure times:

For Rayleigh distributed failure times, the time dependent equations for R_H, R_{NC}, R and R_C are as follows:

$$R_H(t) = e^{-\beta_H t^2}, \tag{20}$$

Where $\beta_H = \frac{1}{\alpha_H}$; α_H is the scale parameter associated with the Rayleigh distribution representing hardware failure times of a unit.

$$R_{NC}(t) = e^{-\beta_{NC} \cdot t^2}, \tag{21}$$

Where $\beta_{NC} = \frac{1}{\alpha_{NC}}$; α_{NC} is the scale parameter associated with the Rayleigh distribution representing failure times of a unit due to non-critical human errors.

$$R(t) = R_H(t).R_{NC}(t) = e^{-wt^2}, \tag{22}$$

Where $w = \beta_H + \beta_{NC}$

$$R_C(t) = e^{-\beta_C t^2}, \tag{23}$$

Where $\beta_C = \frac{1}{\alpha_C}$, α_C is the scale parameter associated with the Rayleigh distribution representing bridge system failure times with respect to critical human errors.

The reliability of a subsystem with respect to hard ware failures and non-critical human errors is

$$R_S(t) = e^{-nwt^2} \tag{24}$$

Similarly, equations for $R_{H,NC}(t)$ and $R_{ps}(t)$ are:

$$R_{H,NC}(t) = 1 - \left(1 - e^{-nwt^2}\right)^m \tag{25}$$

and $$R_{PS} = R_C(t).R_{H,NC}(t) = e^{-\beta_C t^2} - \sum_{i=0}^{m} (-1)^i \binom{m}{i} e^{-(\beta_C + inw)t^2} \tag{26}$$

The plots of Equation (26) are shown in figure 4 for the specified hypothetical values of the model parameters. These plots show the effect of varying scale parameters β_C and β_{NC} on the system reliability.

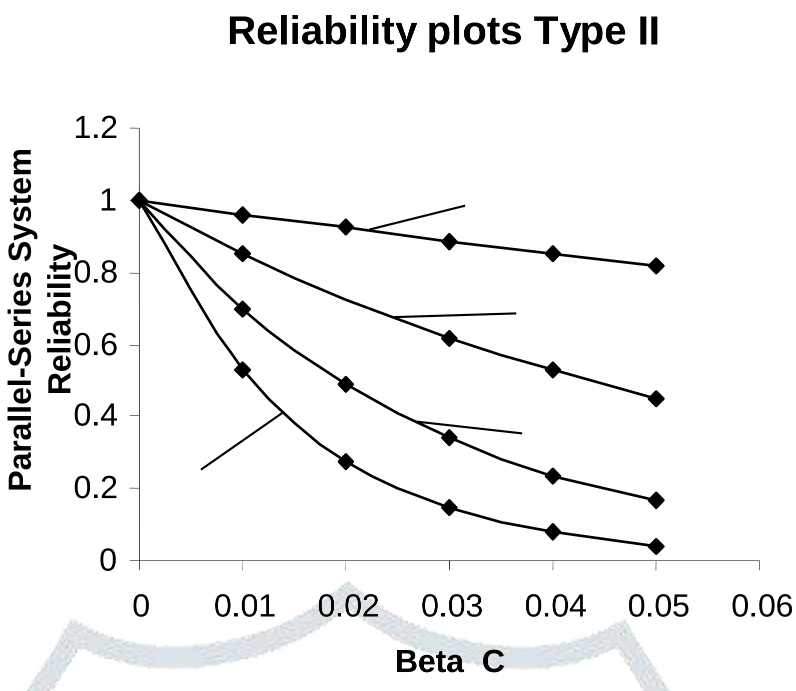

Fig 4

β_C	R_{PS} ($t = 2$)	R_{PS} ($t = 4$)	R_{PS} ($t = 6$)	R_{PS} ($t = 8$)
0.01	0.9607	0.8521	0.6976	0.5272
0.02	0.9231	0.7261	0.4867	0.2780
0.03	0.8869	0.6187	0.3395	0.1466
0.04	0.8521	0.5272	0.2369	0.0773
0.05	0.8187	0.4493	0.1652	0.0407

The system mean time to failure is

$$\text{M.T.T.F}_{PS} = \int_0^\infty R_{PS}(t)dt = \frac{1}{2}\left(\frac{\Pi}{\beta_C}\right)^{\frac{1}{2}} - \sum_{i=0}^{m}(-1)^i \binom{m}{i} \frac{1}{2}\left(\frac{\Pi}{\beta_C + inw}\right)^{\frac{1}{2}} \quad (27)$$

The failure density function of the parallel-series system with human errors is

$$f_{PS}(t) = -R'_{PS}(t) = 2\beta_C\, t\, e^{-\beta_C t^2} - \sum_{i=0}^{m}(-1)^i \binom{m}{i}.2(\beta_C + inw)t\, e^{-(\beta_C+inw)t^2} \quad (28)$$

Where $R'_{PS}(t)$ denotes the derivative of $R_{PS}(t)$ with respect to time t. Similarly, the

Hazard rate function is: $h_{PS} = \dfrac{f_{PS}(t)}{R_{PS}(t)}$

$$= \frac{2\beta_C t\left[1-\left(1-e^{-nwt^2}\right)^m\right]+2mnwte^{-nwt^2}\left(1-e^{-nwt^2}\right)^{m-1}}{1-\left(1-e^{-nwt^2}\right)^m} \tag{29}$$

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