THE BASIC STUDY ON BOUNDARY LAYER THICKNESS AND DRAG FORCE ON A FLAT PLATE IMMERSED IN FLUID FLOW

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Abstract.

In fluid dynamics, the boundary layer forms on a flat plate immersed in a fluid flow, affecting drag force and overall aerodynamic performance. This study focuses on calculating the boundary layer thickness and drag force exerted on the plate. The boundary layer thickness, δ, is determined using theoretical formulations such as the Blasius solution for laminar flows or empirical relations for turbulent flows. Drag force, which arises due to frictional forces within the boundary layer, is quantified using integral methods or through pressure distribution calculations along the surface. Practical implications of these calculations include optimizing vehicle aerodynamics, designing efficient cooling systems, and enhancing overall fluid flow efficiency in engineering applications. The results underscore the significance of boundary layer theory in understanding and predicting fluid behavior near solid surfaces, thereby influencing technological advancements across various industries.

Keywords: Boundary layer thickness, Prandtl number,fluid flow, flat plate, laminar flow, viscous fluid.

1. Introduction.

As it a known thing that, in recent days there has been tremendous application of fluid dynamics (mechanics) over a broad spectrum of disciplines of science ranging from chemical engineering to geophysics. Many chemical engineering processes like those in metallurgy and polymer extrusion involve the cooling of a molten liquid (polymer solution, molten metal, etc.,) by drawing it into a cooling liquid, sometimes referred as the ambient fluid. While drawing the extrudate into the cooling system,

As we know the fluid dynamics is an important and rich branch of science, as any living being in the universe cannot exist without fluid. The development of fluid dynamics started in quite early age of civilization. However, before the last century the discussion in fluid dynamics was mainly confined to the study of inviscid incompressible fluid flow.

As we look through the history of fluid dynamics, in the process of existence on earth, the human activities in the fields of Water supply, Irrigation, Navigation and Water power, resulted in the development of the fluid mechanics as early as 250 B. C. Archimedes formulated the basic principles of fluid dynamics which are true till today. After the fall of Roman Empire (476 A. D.) there is no record of significant progress made in fluid mechanics until the time of Leonardo da vinci (1500 A. D.) Leonardo da vinci designed the first chambered canal lock. However up to da vinci's time concepts of fluid motion was considered to be the mere arts than science.

The problems in fluid mechanics are basically not different from those in "Ordinary" mechanics (the mechanics of solids or in thermodynamics). Centuries of experience of mankind with the flow of water began to crystallize in scientific form about two hundred years ago. Two distinct schools of thought gradually evolved in the treatment of fluid mechanics one is known as classical Hydrodynamics which deals with theoretical aspects of the fluid dynamics assuming the non-existence of shearing stresses in the flow. In literature this is termed as ideal or inviscid or frictionless fluid dynamics. The other branch is known as Hydraulics. This deals with the practical aspects of fluid movement. This has been developed from experimental findings and is, therefore, more of empirical nature. The methods which work for Hydraulics problems such as the problems of canals, dams, rivers, etc are applicable, with slight modifications to the aerodynamics problems of airoplane and rockets. Notable contributions to the theoretical Hydrodynamics have been made by celebrated mathematicians like Euler, D'Alembert, Navier, Lagrange, Stokes, Kirchoff, Rayleigh, Rankine, Kelvin, Lamb and many others.

Towards the beginning of the last century a great breakthrough came in the subject with the development of boundary layer theory by German scientist Ludwig Prandtl. His significant contribution to the boundary layer theory had a tremendous influence upon the understanding of the problems having many practical applications. Other notable contributions to the modern fluid mechanics came from physicist cum mathematicians Blasius, Von-karman, Reynolds, Rouse, Froude and many others.

Today if you look around us we encounter hundreds of items made of polymer materials. All these polymer materials are essentially produced in liquid or molten or solution state. In the process of production they are passed over solid moving surfaces in some typical applications. The quality of the final product greatly depends on the rate of cooling/heating of the sheet or filament. We propose to study the role of different physical parameters associated with the cooling or heating molten viscous and viscoelastic solution.

In view of the practical applications of the stretching sheet problem the thesis presents a theoretical study of the effects visous dissipation, porous mediua, thermal conductivity,thermal radiation etc on flow, heat transfer to viscous and viscoelastic cooling liquids from a linear and non linear stretching sheet. Different types of boundary conditions for heat transfer analysis are made use of in the study.

Now we discuss some essential basics of our work

2. Fluid

A fluid in brief may be said to be substance, which is capable of flowing. It cannot preserve its shape for any length of time unless it is restricted into a particular form depending upon the shape of its surroundings. Thus it would flow in a layer of small thickness over a large area if the surrounding were removed.

Fluids are basically classified as

(a) Liquids

(b) Gases

(a) Liquids:

Fluids, which are not able to be compressed, occupy definite volume and are not affected appreciably by the change in pressure and temperature, are called liquids. In other words it may be stated as liquid has intermolecular forces, which hold it together so that it possesses volume but no definite shape. When it is poured into a container it will fill the container upto the volume of the liquid regardless of the shape of the container.

(b) Gases:

Fluids capable of being compressed and expanded are known as gases and they conform to the sphere and volume of the vessel they are contained in. Any changes in the temperature of a gas will affect its volume and pressure

3. Classification of Fluids

The fluids are classified as

(a) Ideal Fluid or Inviscid Fluid

(b) Real Fluid or Viscous Fluid

(a) Ideal Fluid or Inviscid Fluid:

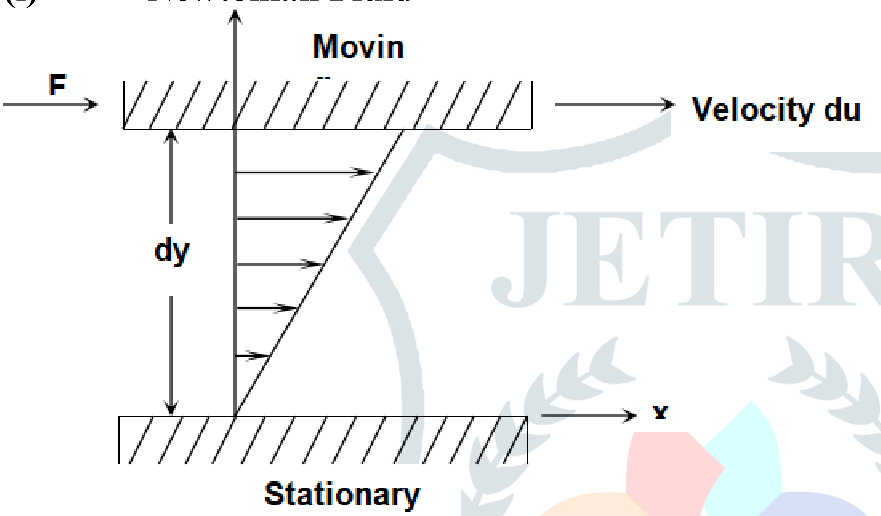
"An Ideal fluid is one, which has no property other than density. No resistance is encountered when such a fluid flows" or Ideal fluids or Inviscid fluids are those fluids in which two connecting layers experience no tangential force (shearing stress) but act on each other with normal force (pressure) when the fluids are in motion. This is equivalent of stating that inviscid fluid offers no internal resistance to chance in shape. The pressure at every point of the ideal fluid is equal in all directions, whether the fluid at rest or in motion. Inviscid fluids are also called as prefect fluid or frictionless fluid. In true sense, no such fluid exists in nature. The assumption of ideal fluids helps in simplifying the mathematical analysis. However fluids which have low viscosities such as water and air can be treated as ideal fluids under certain conditions.

(b) Viscous Fluid or Real Fluid

"Viscous fluid or real fluid are those, which have viscosity, surface tension and compressibility in addition to the density" or viscous fluid or real fluid are those when they are in motion the two contacting layers of those fluids experience tangential as well as normal stresses. This being also the case near solid wall wetted by a fluid. The property of exerting tangential or shearing stress and normal stress in a real fluid when the fluid is in motion is known as viscosity of the fluid. In viscous fluid internal friction plays an important role during the motion of the fluid. One of the important characteristics of viscous fluid is that it offers resistance to motion of the fluid. Viscosity, being the characteristics of the real fluids, exhibits a certain resistance to alter the form also. Viscous or real fluids are classified into following two categories.

(i) Newtonian Fluid
(ii) Non Newtonian Fluid

(i) Newtonian Fluid



To understand the concept of Newtonian fluid let us consider a thin layer of fluid between two parallel plates at distance dy as shown in the figure.

Here one plate is fixed and a shearing force F is applied to the other. When conditions are steady the force F will be balanced by an internal force in the fluid due to its viscosity. For a Newtonian fluid in laminar flow the shear stress is proportional to the velocity gradient. These results in

$$\frac{F}{A} = \tau \quad \alpha \quad \frac{du}{dy} \qquad \text{Or} \qquad \tau = \mu \frac{du}{dy}$$

Where μ, is known as Newtonian viscosity. It will be seen that μ is the tangential force per unit area exerted on layers of fluid a unit distance apart and having a unit velocity difference between them.

(ii) Non-Newtonian Fluids

Non-Newtonian fluids are those fluids which do not obey Newtonian law. It can also be stated as "the non-Newtonian fluids are those for which the flow curve is not linear", i.e. the 'viscosity' of a non-Newtonian fluid is not constant at a given temperature and pressure but depends on other factors such as the rate of shear in the fluid, the apparatus in which the fluid is contained or even on the previous history of the fluid.

A viscoelastic fluid is also one of the non-Newtonian fluid, which possesses both viscous and elastic properties, i.e. although the material might be viscous, it exhibits a certain elasticity of shape. This concept is perhaps most easily visualized in the case of a very viscous liquid such as pitch.

4. Magnetohydrodynamics (MHD):

When a conductor moves in a magnetic field a current is induced in the conductor in a direction mutually at right angles to both the field and the direction of motion. Conversely when a conductor currying an electric current moves in a magnetic field it experience a force tending to move it at right angles to the electric field. These two statement first enunciated by Faraday

Electromagnetic forces will be generated which may be of the same order of magnitude as the hydrodynamical and inertial forces in the case when the conductor is either a liquid or a gas. Thus the equation of motion will have to take these electromagnetic forces into account as well as the other forces. The science that treats these phenomena is called magnetohydrodynamics (MHD). Other variants of nomenclature are hydromagnetics, magneto-fluid dynamics, magneto-gas dynamics etc. As we know that MHD is relatively new but important branch of fluid dynamics. It is concerned with the interaction of electrically conduction fluid and electromagnetic fields, such interaction occurs both in nature and in new man- made device.

MHD flow occurs in the sun, the earth's interior, the ionosphere, and the stars and atmosphere, to mention a few. Engineering level experiments have been made for electric power generation by passing an ionized gas between the poles of a strong electromagnet so that an electric current would be generated at right angles to the magnetic field and to the direction of flow of the plasma, the current being collected by two spaced electrodes at right angles to the direction of the current flow. At the present time MHD generators are not a practical possibility owing to the difficulties of producing suitably efficient and stable plasmas and sufficiently refractory material to withstand the high temperatures of the plasmas.

In MHD we study dynamical behavior of electrically conducting medium, which may be a liquid or an ionized gas in presence of magnetic field. Both plasma and conducting fluids are related in common theory by assuming plasma as a continuous fluid for which the kinetic theory of gases still holds true. In MHD induced electric current produces mechanical force, which in turn modifies the motion in the fluid. Hence study of electrically conducting fluid flow in the presence of traverse magnetic field assures significance.

5. Literature Review:

Here we have given the literature review of works with viscous and viscoelastic fluids, flow through porous media and MHD flow

Works with viscous and Viscoelastic fluids:

Sakiadis [1961] was the first to initiate pioneering work on the boundary layer flow of incompressible fluid over continuous solid surface. Due to the entrainment of the ambient fluid, this boundary layer was quite different from Blasius boundary layer flow over a semi-infinite flat plate. Erickson et al. [1966] extended this problem to the case for which suction or blowing existed at the moving surface. The heat and mass transfer in the boundary layer was also being taken into account in their study. Tsou et al. [1967] later extended the work of Sakiadis [1961] to include heat transfer and verified the analytical results by experimental measurement. In this work, both surface velocity and surface temperature/surface heat fluxes were assumed to be constant.

These investigations find its application in the problem of a polymer sheet extruded from a die. It is often tacitly assumed that the sheet is inextensible, but as we know that situation arises in polymer industry, in which it is necessary to deal with a stretching plastic sheet, as pointed out by Crane [1970]. Also, final product depends to a great extent on the rate of cooling which is governed by the structure of the layer near the moving strip. A detailed knowledge of the velocity and temperature distribution in this layer is therefore of outmost important in order to obtain final product with desired characteristics. Gupta and Gupta [1977] investigated the heat and mass transfer in the flow over a stretching surface (with suction or blowing) issuing from a thin slit. They dealt with the isothermal moving plate and obtained the temperature and concentration distribution profiles for that situation. Soundalgekar and Murthy [1980] studied a thermal boundary layer flow on a continuously moving semi-infinite flat plate, whose temperature varied as algebraic power of x, where x -was measured from the leading edge of the plate. They derived the similarity equations and solved those equations numerically. Grubka and Bobba [1985] considered heat transfer occurring on a linear impermeable stretching surface with a power law surface temperature. Dutta et al [1985] analysed the temperature distribution in a flow over a stretching sheet with uniform heat flux. It was shown that temperature at a point decreased with the increase of Prandtl number. Jeng et al [1986] analysed momentum and heat transfer phenomena in the boundary layer over a continuous moving surface with arbitrary surface velocity and non-uniform surface temperature. They also obtained solution of the energy equation for isothermal and non-isothermal conditions. Dutta [1988] presented an analytical solution of the conjugate heat transfer problem for cooling of a thin stretching sheet in a viscous flow in presence of suction and blowing. The local velocity of the sheet material was assumed to be proportional to the distance from the slit. They also established the convergence criteria of the solution. Chen and Char [1988] investigated the effect of both power law surface temperature and power law heat flux variations on the heat transfer characteristics of a continuous, linearly stretching sheet subjected to suction or blowing. Ali [1995] investigated thermal boundary layer by considering a power law stretching surface. A new dimension has been added in this investigation by Elbashbeshy [2001] who examined the flow and heat transferring characteristic by considering exponentially stretching continuous surface. There are numerous works of Magyari and Keller along with Pop on power law stretching problem with general x dependent stretching velocity, power law stretching and power law shearing problems are (Magyari and Keller 1999, Magyari and Keller 1999, Magyari, Pop and Keller, 2002, Magyari and Keller 2003, Magyari, Keller and Pop 2004 and Elbashbebeshy and. Bazid 2004).

It is important to notice that the investigations mentioned above were confined to viscous fluids only. In recent years, ever increasing application of viscoelastic fluids (e.g. dilute polymer solution of 0.83% ammonium alginate in water and 5.4% polyisobutylene in cetane at 30^0 C, Markovitz and Coleman [1964]) in polymer processing industries has led to a renewed interest among researchers to investigate viscoelastic boundary layer flow over a stretching plastic sheet (Rajagopal et al. [1984,1987], Dandapat and Gupta [1989], Rollins and Vajravelu [1991], Andersson [1992], Lawerence and Rao [1992], Char [1994] and Rao [1996]). A significant effort has been directed to study the boundary layer viscous fluid flow over porous stretching sheet where the flow is influenced by suction/blowing of liquid through the porous sheet (Vajravelu and Nayfeh [1993], Vajravelu [1994], Ahmad and Mubeen [1995], Chiam [1997], Yih [1998], Acharya et al. [1999]). A new dimension has been added in this study by investigating such situation for viscoelastic fluid flow in our recent works (PRASAD et al. [2000] and Sonth et al. [2002]). Stability analysis of viscoelastic fluid flow over stretching sheet with and without magnetic field has been carried out by Dandapat et al. [1994,1998]. Sujit Kumar Khan and Emmanuel Sanjayanand [2004] considered a problem of viscoelastic boundary layer MHD flow through a porous medium over a porous quadratic stretching sheet. They have noticed that flow is enhanced by the positive values and by negative values is will suppressed flow.

Since, in reality most of the fluid considered in industrial applications is more non-Newton in nature especially of viscoelastic type than viscous types. Exhaustive literatures are available including those cited above on two dimensional viscoelastic boundary layer flows over stretching surface where the velocity of the stretched surface is assumed linearly proportional to the distance from a fixed origin. However, Gupta and Gupta [1977] have pointed out that realistically stretching of the sheet might not necessarily be linear. Also, there might be a situation of flow of linear mass flux addition or annihilation in addition to constant mass flux through the pore of the boundary sheet. This situation was dealt by Kumaran and Ramanaiah [1996] in their work on boundary layer flow over a quadratic stretching sheet. Estimation of skin friction, which is very important from the industrial application point of view, is also not presented in their analysis. Skin friction and streamline pattern might vary to a certain extent if the fluid flow takes place through porous media.

The instability of interface in porous media was considered by Saffman and Taylor [1958]. They concluded that when two super imposed fluids of different viscosity are forced through a porous medium and then subjected to small deviations (perturbations), then the stability of the interface depends on weather the direction of motion is from the more viscous to the less viscous or vice-versa. They considered two immiscible fluids, which remained completely separate at the interface in a porous medium. Raptis et al. [1984] studied unsteady flow through a porous medium bounded by infinite plate, in presence of mass transfer. Raptis [1984] investigated forced study flow through a highly porous medium bounded by a semi-infinite plate. Rapits and Thakhar [1986] studied the forced flow of a viscous incompressible fluid through a highly porous medium bounded by a semi-infinite vertical plate in the presence of mass transfer. Vafai and Thyagaraja [1987] presented a general class of problems involving interface interactions on flow and heat transfer at three different types of interface zones. These interface are (i) interface between two different porous media (ii) interface between a porous medium and a fluid (iii) interface between a porous medium and an impermeable medium. Vasantha et al. [1987] dealt with the forced convective heat transfer in a longitudinal flow over a non-isothermal circular cylinder immersed in a saturated porous medium. Lai and Kulacki [1990] studied the effect of variable viscosity on mixed convection along a vertical plate embedded in a porous medium. They examined the limiting cases of natural and forced convection. Nakayamma et al. [1990] studied the effect of non-Darcy terms on boundary layer flow and temperature fields. They obtained the asymptotic expression for local Nusselt number. Varjravelu [1994] studied flow and heat transfer in a saturated porous medium over a continuous stretching sheet with power law surface temperature/power law heat flux. They also studied the effects of frictional heating and internal heat generation.

All these works were restricted to the Newtonian fluid flow and heat transfer. However, of late, a great deal of interest has been evinced on the study of rheological effects of non-Newtonian fluid flow through porous media in oil reservoir engineering. In this application dilute aqueous solution of polymers, are used as pusher fluids in oil wells in order to enhance oil recovery by examining the properties of solution of high weight polymers and their effect on flow resistance. The majority of these studies have used concentrated polymer solution to increase the effect of viscosity. Experimental investigation reveals that, for less concentrated solutions the lager pressure drops can no longer be attributed solely to viscous effects as the increase is of an order of magnitude longer than the increase in velocity. Hence, the elastic effects must be taken in account when the relaxation time of the elastic fluid is comparable to the characteristic time of the flow. Literature review reveals that no much work has been carried out on viscoelastic fluid flow through the porous media. Qualitative mathematical analysis is essential for implementation of such fluid flow in industrial application. Keeping this view in mind recent study takes a new turning. Recently the theoretical studies of Pascal [1986, 1988] indicate that non-Newtonian displacing fluids of shear thinning behavior may minimize the instability effects on moving interface separating the displaced and displacing fluids. The viscoelastic effects in non-Newtonian steady flow through porous media are also analyzed theoretically by Pascal and Pascal [1989] by means of an approximate model of elastic behaviour. They used modified Darcy law for viscoelatic effects associated with the Maxwell model. Rudraiah et al. [1989] theoretically studied the oscillatory convection in a visco-elastic fluid through a porous layer heated from below, which is relevant to the production of some heavy crude oils., considered as viscoelastic fluid. Very recently Abel et al [2002] carried out the heat transfer characteristic of a viscoelastic fluid in a porous medium over a stretching sheet. There is also a work by Kumaran and Ramanaiah [1996] where investigation has been carried out for flow characteristics in which viscous boundary layer flow has been considered over a quadratic stretching sheet. Skin friction and streamline pattern might vary to a certain extent if the fluid flow takes place through porous media. This situation has not been considered for investigation in their work.

6. Works with MHD flows:

The flow of an electrically conducting fluid caused solely by stretching of an elastic sheet in presence of a uniform transverse magnetic field was considered by the Pavlov [1974] and obtained a similarity solution of this problem later Chakrabarati and Gupta [1979] extended Pavlov work to study temperature distribution in MHD boundary layer flow in the presence of uniform suction. Soundalgekar and Takhar [1980] they have discussed the effects of physical parameters on flow and heat transfer characteristics by considering the effects of uniform transverse magnetic field on forced and free convection flow past a semi-infinite plate taking into account of viscous dissipation and stress work. Rapits and Taxivandis [1983] carried out analytical investigations on free convective flow past an infinite vertical surface when the fluid is electrically conducting in presence of an external transverse uniform magnetic field. Hydromagnetic flow of Newtonian fluid and heat transfer over continuous moving flat surface with uniform suction has been studied by Vajravelu and Nayfeh [1993] Mahesh Kumari et al. [1990] studied the effects of induced magnetic field and source/sink on flow and heat transfer characteristics over a stretching surface. Andersson [1992] he considered in his work MHD flow of a viscoelastic fluid past a stretching surface and contribute some important information of MHD. Goral et al. [1993] investigated the effects of magnetic field strength on mixed convective flow arising from an infinitely long horizontal line source of heat when the ambient fluid considered was a non-Newtonian power-law fluid having moderately large values of Grashof number. Na and Pop [1996] investigated the boundary layer flow over a moving continuous flat paste in an electrically conducting ambient fluid with a step change in applied magnetic field. The governing equations were solved numerically using the Keller-box method. The results showed the decrease of the skin friction parameter with the increase of the magnetic parameter. Elbashbeshy [1997] investigated heat and mass transfer phenomena along a vertical plate under the combined buoyancy effects of thermal and species diffusion in presence of magnetic field. Vajravelu and Hadijinicolaou [1997] carried out the investigations of free convection and internal heat generation on flow and heat transfer characteristics in an electrically conducting fluid near an isothermal stretching sheet. Chiam [1997] presented an analytical solution of the energy equations for a boundary layer flow of an electrically conducting fluid under the influence of transverse magnetic field over a linearly stretching non-isothermal flat sheet. Ali Chamkha [1997] obtained the similarity solutions of laminar boundary layer equations describing the steady hydromagnetic two-dimension flow and heat transfer in a stationary electrically conducting and heat generating fluid driven by a continuous moving porous surface immersed in a fluid saturated porous medium. Elbashbesgy [2000] studied the flow of a viscous incompressible fluid along a heated vertical plate, taking into account the variation of viscosity and thermal diffusion with temperature in the presence of magnetic field. Very recently Sujit Kumar Khan and Emmanuel Sanjayanand. [2004] Considered a problem on viscoelastic boundary layer MHD flows through a porous medium over a porous quadratic stretching sheet. Emad M Abo-Eldahab; Mohamed a el Aziz [2004] studied the Blowing/suction effect on hydromagnetic heat transfer by mixed convection from an inclined continuously stretching surface with internal heat generation/absorption.

7. Basic Equations:

Continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum Equations

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y} \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\frac{\partial \tau_{yx}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y} \tag{3}$$

Where ρ is the constant fluid density and the individual stress components become

$$\tau_{xx}=-p+2\eta_0\frac{\partial u}{\partial x}-2k_0\left[u\frac{\partial^2 u}{\partial x^2}+v\frac{\partial^2 u}{\partial x\partial y}-2\left(\frac{\partial u}{\partial x}\right)^2-\frac{\partial u}{\partial y}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]$$

$$\tau_{yy}=-p+2\eta_0\frac{\partial v}{\partial y}-2k_0\left[u\frac{\partial^2 v}{\partial x\partial y}+v\frac{\partial^2 v}{\partial y^2}-2\left(\frac{\partial v}{\partial y}\right)^2-\frac{\partial v}{\partial x}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]$$

$$\tau_{xy}=\tau_{yx}=\eta_0\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-k_0\left[u\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 u}{\partial x\partial y}\right)+v\left(\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}\right)-2\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}-2\frac{\partial v}{\partial x}\frac{\partial u}{\partial x}\right] \tag{4}$$

Energy Equation

$$\rho c_p\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)=k\nabla^2 T+\phi+\frac{J^2}{\sigma} \tag{5}$$

Where is the c_p specific heat and k is is the thermal conductivity of the fluid.

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$$

Applying the usual boundary layer approximations we obtain the following boundary layer equations for two dimensional unsteady state viscoelastic fluid flows.

Equation of Continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

Momentum Equation

Making use of the usual boundary layer approximations, the equation of motion for unsteady flow are derived in the following simplified from

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-\frac{1}{\rho}\frac{\partial p}{\partial x}-k_0\left\{\frac{\partial^3 u}{\partial t\partial y^2}+u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}$$

(7)

For steady flow the boundary layer equation of motion is

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-\frac{1}{\rho}\frac{\partial p}{\partial x}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\} \quad (8)$$

Similarly we can obtain the Casson Fluid model as:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon(1+\frac{1}{\beta})\frac{\partial^2 u}{\partial y^2} \quad (9)$$

Equation of Energy

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial y^2}\right)+\mu\phi \quad (10)$$

Boundary layer energy equation for two-dimensional steady incompressible fluid flow is

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial y^2}\right)+\mu\phi \quad (11)$$

8. Boundary Conditions:

The boundary conditions on velocity and temperature depend on the nature of the fluid flow and geometry of the boundary wall. Here, in this thesis, we consider a steady state two dimensional boundary layer flow of incompressible, viscous and viscoelastic fluid past a flat elastic sheet. Boundary is assumed to be moving axially with a velocity by applying two equal and opposite forces along the x-axis, the sheet is being stretched with a speed proportional to the distance from the fixed origin $x=0$. The resulting motion of the fluid is thus caused solely by the moving surface. The mathematical forms of velocity boundary conditions are as follows.

(a) Linear Boundary stretching for flow:

The boundary conditions on velocity depend on the nature of the fluid flow and geometry of the boundary wall. Here, in this thesis, we consider a steady, incompressible, non-Newtonian fluid of Walters' liquid B past a flat elastic sheet, By applying two equal and opposite forces along x-axis, the sheet is being stretched with a speed proportional to the distance from the origin $x=0$. The mathematical forms of velocity boundary conditions are as follows.

$$\left.\begin{array}{lll} u=bx\ , & v=0 & at\quad y=0 \\ u\to 0, & \dfrac{\partial u}{\partial y}\to 0 & as\ y\to\infty \end{array}\right\} \quad (12)$$

(b) Exponential Boundary stretching for flow:

Some times flow in a quiescent fluid may be driven by an exponentially stretching surface subjected to suction/blowing. In this situation the boundary conditions are

$$\left.\begin{array}{ll} u=U_w(x)=U_0\exp\left(\dfrac{x}{l}\right) & at\quad y=0 \\ v=-v_w\exp\left(\dfrac{x}{2l}\right) & at\quad y=0 \\ u=0,\ \dfrac{\partial u}{\partial y}=0 & as\quad y\to\infty \end{array}\right\} \quad (13)$$

Here l is some characteristic length. The second boundary condition on v takes into account of suction of fluid as an exponential function.

(c) Linear Boundary stretching for heat transfer:

Thermal boundary conditions depend on the type of the heating process under consideration. Here, we consider two different types of heating processes, namely,

(i) Prescribed surface temperature (PST Case)

(ii) Prescribed wall heat flux (PHF Case).

(iii) Convective Boundary Conditions

Mathematical forms of such temperature boundary conditions are

PST CASE

$$\left.\begin{aligned} &T = T_w = T_\infty + A\left(\frac{x}{l}\right)^2 && at \quad y = 0, \\ &T \to T_\infty && as \quad y \to \infty \end{aligned}\right\} \qquad (14)$$

PHF CASE

$$\left.\begin{aligned} &-k\frac{\partial T}{\partial y} = q_w = D\left(\frac{x}{l}\right) && at \quad y = 0 \\ &T \to T_\infty && as \quad y \to \infty \end{aligned}\right\} \qquad (15)$$

CONVECTIVE BOUNDARY CONDITION

$$\left.\begin{aligned} &-k\frac{\partial T}{\partial y} = h_f\left(T_f - T\right) && at \quad y = 0 \\ &T \to T_\infty && as \quad y \to \infty \end{aligned}\right\},$$

(d) Exponential Boundary Stretching for Heat transfer:

The thermal Boundary Conditions for exponential boundary stretching are

(i) Boundary with prescribed exponential order surface temperature PEST

(iv) Boundary with prescribed exponential order heat flux PEHF

In PEST case we employ the following surface boundary conditions on temperature

$$\left.\begin{aligned} &T = T_w = T_\infty \exp\left(\frac{v_0 x}{2l}\right) && at \quad y = 0 \\ &T = T_\infty && as \quad y \to \infty \end{aligned}\right\} \qquad (16)$$

Where and v_0 are T_0 parameter of temperature distribution on the stretching surface and T_∞ is the temperature away from the stretching sheet.

In PEHF case we employ the following prescribed exponential law heat flux boundary conditions

$$\left.\begin{aligned} &-k\left(\frac{\partial T}{\partial y}\right)_w = T_1 \exp\left(\frac{v_1 + 1}{2l}\right)x && at \quad y = 0 \\ &T - T_\infty && as \quad y \to \infty \end{aligned}\right\} \qquad (17)$$

Where v_1 and T_1 are the parameter of temperature distribution on the stretching surface.

9. DIMENSIONLESS PARAMETERS

Here we discuss about the non-dimensional parameters which has been used in the further chapters of our work.

Dimensional analysis of any problem provides information on qualitative behavior of the physical problem. The dimensionless parameter helps us to understand the physical significance of a particular phenomenon associated with the problem.

Some of the dimensionless parameters used in this thesis are explained below

(a) VISCO-ELASTIC PARAMETER:

The Visco-elastic parameter k_1 is defined as the measure of the relative importance of elastic and viscous effects and it is written as

$$k_1 = \frac{k_0 b}{\gamma} \qquad (18)$$

where is the k_0 co-efficient of visco-elasticity and the other symbols have their predefined meanings.

(b) PRANDTL NUMBER:

In the convection problem, the mechanism of release of thermal energy associated with a rich variety of phenomena is exhibited by non-linear convection. This variety stems primarily from the dependence of the motion on the Prandtl number defined as

$$\Pr = \frac{\text{viscous force}}{\text{thermal force}}$$

$$= \frac{\mu c_p}{K} \qquad (19)$$

(c) ECKERT NUMBER :

The dimensionless quantity is Ec known as Eckert number and it is defined as

$$Ec = \frac{b^2\ l^2}{c_p\ A} \qquad (20)$$

Where b, l, c_p are some reference values of the velocity, characteristic length and specific heat at constant pressure

(d) SKIN FRICTION:

When the boundary layer equations are integrated the velocity distribution can be obtained and the position of the point of separation can be determined, this in turn permits us to calculate the viscous drag and is known as skin friction. The shearing stress at the wall is given by

$$\tau_0 = \mu \left(\frac{\partial u}{\partial y}\right) at\ y = 0 \qquad (21)$$

The viscous drag for the case of two-dimensional flow becomes

$$D_\rho = b \int_0^l \tau_0 \cos\phi\, ds \qquad (22)$$

Where b denotes the height of the cylindrical body. ϕ Is the angle between the tangent to the surface and the free stream velocity U, and S is co-ordinate measured along the surface.

(e) REYNOLDS NUMBER:

$$\mathrm{Re} = \frac{L u_0}{\nu} \qquad (23)$$

is known as local Reynolds number. Where ν is the kinematic coefficient of viscosity of the fluid and L and u_0 are characteristic length and velocity respectively. The above quantity has dimension $\frac{uL}{\nu}$. It can be easily shown that

$$\frac{\text{Inertia force}}{\text{Viscous force}} = \mathrm{Re} \qquad (24)$$

Hence Reynolds number measures the relative change of inertia force per unit change of viscous force.

If is large, Re then the inertia force will be predominant. In such a case the effect of viscosity can be confined in a thin layer known as boundary layer adjacent to the solid boundary.

(f) NON-UNIFORM HEAT SOURCE/SINK PARAMETER:

$$q''' = \left(\frac{kb}{\upsilon}\right)\left[A^*(T_w - T_\infty)\frac{u}{u_w} + B^*(T - T_\infty)\right]. \qquad (25)$$

Where A* and B* are parameters of space- and temperature-dependent internal heat generation/absorption. It is to be noted that A* > 0 and B* > 0 correspond to internal heat generation while A*< 0 and B*< 0 correspond to internal heat absorption, T_w is the temperature of sheet and T_∞ is the constant temperature for away from the sheet.

(g) RADIATION PARAMETER:

$$N = \frac{Kk^*}{4\sigma T_\infty^{\ 3}} \qquad (26)$$

Here, σ is the Stefan-Boltzmann constant and k^* is the absorption coefficient, T_∞ is the temperature far away from the wall; K is the thermal conductivity of the fluid.

CONCLUSION

In conclusion, the study of boundary layer thickness and drag force on a flat plate immersed in fluid flow contributes valuable insights into fundamental fluid mechanics. The experimental results confirm theoretical expectations regarding boundary layer growth and drag force variation, underscoring the importance of these phenomena in practical applications. Moving forward, continued research in this area promises to advance our understanding and enhance engineering designs aimed at improving efficiency and performance in various fluid dynamics applications

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