Fuzzy Tri-Magic Labeling of Some Star-Related Graphs

[1]Dr. P. Sumathi and [2]C. Monigeetha
[1]Department of Mathematics, C. Kandaswami Naidu College for Men,
Anna Nagar, Chennai 600 102, India.
[2]Department of Mathematics, Sri Muthukumaran Arts and Science College,
Chikkarayapuram, Chennai 600 069, India.

Abstract *:* A graph G consists of a finite nonempty set $V = V(G)$ of p vertices or points together with a prescribed set X of q unordered pairs of distinct points of V. Each pair $x = \{u, v\}$ of points in X is a line or edge of G, and x is set to join u and v [1]. In this paper, the concept of Fuzzy tri-magic labeling was introduced. It is proved that the star graph $S_{1,n}$ and some type of $C_3(n, m, l)$ star graphs are Fuzzy Tri-Magic.

IndexTerms - Fuzzy Labeling, Fuzzy Tri-Magic Labeling, Fuzzy Tri-Magic Star, $C_3(n, m, l)$ star graph.

I. INTRODUCTION

The graphs considered here are finite, simple, undirected, and nontrivial. Graph theory has a good development in the graph labeling and has a broad range of applications [2]. Fuzzy is a newly emerging mathematical framework to exhibit the phenomenon of uncertainty in real-life tribulations. A fuzzy set is defined mathematically by assigning a value to each possible individual in the universe of discourse, representing its grade or membership which corresponds to the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. We introduce fuzzy tri-magic labeling of graphs. If G admits fuzzy tri-magic labeling, then G is called as fuzzy tri-magic labeled graph.

In this paper, a new concept of fuzzy tri-magic labeling has been introduced. It is showed that the star graph $S_{1,n}$ and some type of $C_3(n, m, l)$ star graphs are fuzzy tri-magic.

Definition 1.1 (Fuzzy graph)

A fuzzy graph $G: (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$, where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 1.2 (Fuzzy Labeling)

Let $G = (V,E)$ be a graph, the fuzzy graph $G: (\sigma, \mu)$ is said to have a fuzzy labeling, if $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ is bijective such that the membership value of edges and vertices is distinct and $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

Definition 1.3 (Magic membership value (MMV))

Let $G: (\sigma, \mu)$ be a fuzzy graph; the induced map g:$E(G) \to [0, 1]$ defined by $g(uv)= \sigma(u)+ \mu(uv) + \sigma(v)$ is said to be a magic membership value. It is denoted by MMV.

Definition 1.4 (Fuzzy tri-magic labeling)

A fuzzy graph is said to admit tri-magic labeling if the magic membership values K_i's, $1 \leq i \leq 3$ are constants where number of K_i's and K_j's differ by at most 1 and $|K_i - K_j| \leq \frac{2}{10^r}$ for $1 \leq i, j \leq 3$, $r \geq 2$.

Definition 1.5 (Fuzzy tri-magic labeling graph)

A fuzzy labeling graph which admits a tri-magic labeling is called a fuzzy tri-magic labeling graph. The fuzzy tri-magic labeling graphs are denoted by $\tilde{T}m_0G$.

Definition 1.6 (Star Graph)

A complete bipartite graph with one vertex in one partition and n vertices in another partition is said to be a star graph and it is denoted by $S_{1,n}$.

Definition 1.7 $C_3(n,m,l)$ star graph

The graph obtained by attaching n pendent vertices in one vertex of C_3, m pendent vertices in other vertex of C_3 ,and l pendent vertices in remaining vertex of C_3 is called $C_3(n,m,l)$ star graph.

III. MAIN RESULT

Theorem 2.1: The graph $S_{1,n}$ admits fuzzy edge tri-magic labeling.

Proof:

Let G be a $S_{1,n}$ graph. $|V(G)| = n+1$ and $|E(G)| = n$. Let the vertex set of G be $V(G) = \{ v, u_1, u_2, u_3, \dots \dots \dots \dots u_n\}$ and the edge set of G be $E(G) = \{vu_i / 1 \le i \le n\}$.

Let r be any positive integer ≥ 2.

Define $\sigma : V \to [0,1]$ by $\sigma(v) = \frac{2n+3}{10^r}$ for star center and for the pendent vertices σ is defined in three cases depending upon the values of n.

Define $\mu : V \times V \to [0,1]$ by $\mu(v, u_i) = \frac{i}{10^r}$ for $1 \le i \le n$

Case (i)

If $n \equiv 0 \ (mod\ 3)$

Define $\sigma : V \to [0,1]$ by $\sigma(v) = \frac{2n+3}{10^r}$

$\sigma(u_i) = (2n+3-i)\frac{1}{10^r}$ if $1 \le i \le \frac{n}{3}$

$\sigma(u_i) = (2n+2-i)\frac{1}{10^r}$ if $\frac{n}{3}+1 \le i \le \frac{2n}{3}$

$\sigma(u_i) = (2n+1-i)\frac{1}{10^r}$ if $\frac{2n}{3}+1 \le i \le n$

Case (ii)

If $n \equiv 1 \ (mod\ 3)$

Define $\sigma : V \to [0,1]$ by

$\sigma(u_i) = (2n+3-i)\frac{1}{10^r}$ if $1 \le i \le \frac{n-1}{3}$

$\sigma(u_i) = (2n+2-i)\frac{1}{10^r}$ if $\frac{n-1}{3}+1 \le i \le \frac{2n-2}{3}$

$\sigma(u_i) = (2n+1-i)\frac{1}{10^r}$ if $\frac{2n-2}{3}+1 \le i \le n$

Case (iii)

If $n \equiv 2 \ (mod\ 3)$

Define $\sigma : V \to [0,1]$ by

$\sigma(u_i) = (2n+3-i)\frac{1}{10^r}$ if $1 \le i \le \frac{n-2}{3}$

$\sigma(u_i) = (2n+2-i)\frac{1}{10^r}$ if $\frac{n-2}{3}+1 \le i \le \frac{2n-1}{3}$

$\sigma(u_i) = (2n+1-i)\frac{1}{10^r}$ if $\frac{2n-1}{3}+1 \le i \le n$

For case (i), the MMVs are

$g(v, u_i) = \frac{4n+6}{10^r} = K_1,\ 1 \le i \le \frac{n}{3}$

$g(v, u_i) = \frac{4n+5}{10^r} = K_2,\ \frac{n}{3}+1 \le i \le \frac{2n}{3}$

$g(v, u_i) = \frac{4n+4}{10^r} = K_3,\ \frac{2n}{3}+1 \le i \le n$

For case (ii), the MMVs are

$g(v, u_i) = \frac{4n+6}{10^r} = K_1,\ 1 \le i \le \frac{n-1}{3}$

$g(v, u_i) = \frac{4n+5}{10^r} = K_2,\ \frac{n-1}{3}+1 \le i \le \frac{2n-2}{3}$

$g(v, u_i) = \frac{4n+4}{10^r} = K_3,\ \frac{2n-2}{3}+1 \le i \le n$

For case (iii), the MMVs are

$g(v, u_i) = \frac{4n+6}{10^r} = K_1,\ 1 \le i \le \frac{n-2}{3}$

$g(v, u_i) = \frac{4n+5}{10^r} = K_2,\ \frac{n-2}{3}+1 \le i \le \frac{2n-1}{3}$

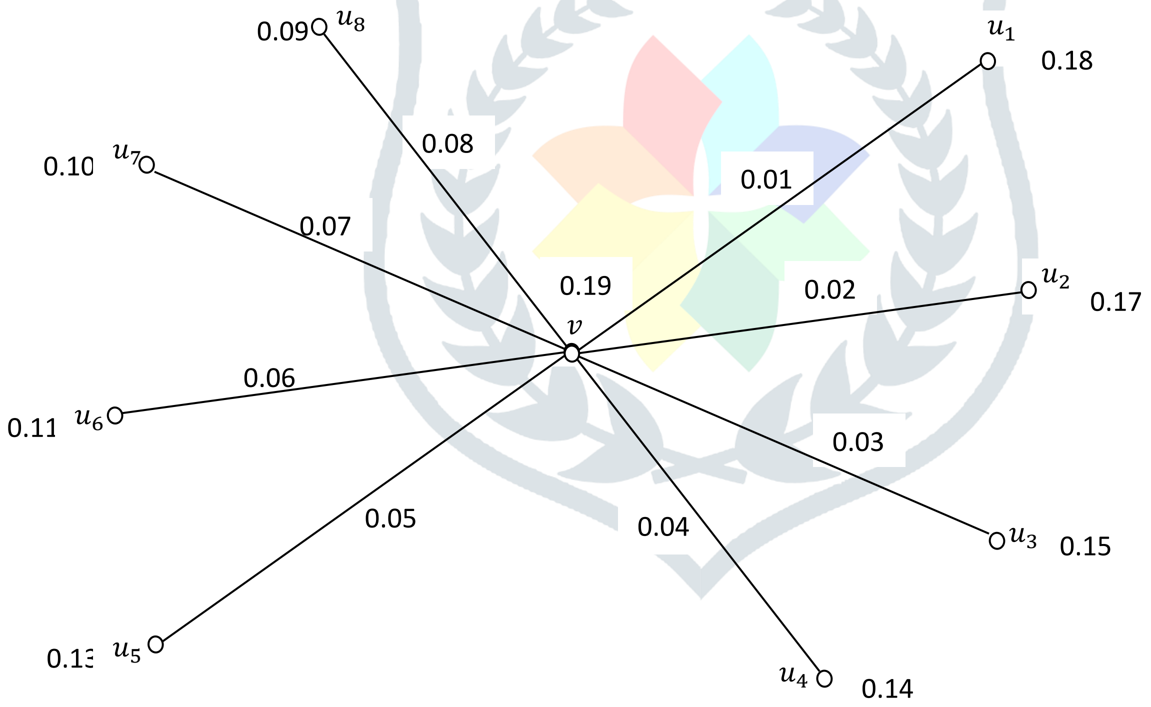
$g(v, u_i) = \frac{4n+4}{10^r} = K_3, \frac{2n-1}{3} + 1 \leq i \leq n$

From the above three situations, the magic membership values of K_i's, $1 \leq i \leq 3$ are tabulated as follows:

Nature of n	K_1	K_2	K_3	Number of K_1	Number of K_2	Number of K_3
$n \equiv 0 \ (mod\ 3)$	$\frac{4n+6}{10^r}$	$\frac{4n+5}{10^r}$	$\frac{4n+4}{10^r}$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$
$n \equiv 1 \ (mod\ 3)$	$\frac{4n+6}{10^r}$	$\frac{4n+5}{10^r}$	$\frac{4n+4}{10^r}$	$\frac{n-1}{3}$	$\frac{n-1}{3}$	$\frac{n+2}{3}$
$n \equiv 2 \ (mod\ 3)$	$\frac{4n+6}{10^r}$	$\frac{4n+5}{10^r}$	$\frac{4n+4}{10^r}$	$\frac{n-2}{3}$	$\frac{n+1}{3}$	$\frac{n+1}{3}$

Hence the maximum difference between the number of K_i's $(1 \leq i \leq 3)$ is 1 and $|K_i - K_j| \leq \frac{2}{10^r}$ for $1 \leq i, j \leq 3$. Hence star graph admits fuzzy edge tri-magic labeling.

Example 2.1.1: Fuzzy tri-magic labeling of $S_{1,8}$



Theorem 2.2: The $C_3(n,n,n)$ star graph admits fuzzy edge tri-magic labeling.

Proof:

Let G be a $C_3(n, n, n)$ star graph

$|V(G)| = 3n+3$ and $|E(G)| = 3n+3$.

Let the vertex set and edge set of G be

$V(G) = \{u_1, u_2, u_3, x_1, x_2, x_3 \dots \dots \dots \dots x_n, y_1, y_2, y_3 \dots \dots \dots \dots y_n, z_1, z_2, z_3 \dots \dots \dots \dots z_n\}$ and

$E(G) = \{u_1u_2, u_2u_3, u_1u_3\} \cup \{u_1x_i / 1 \leq i \leq n\} \cup \{u_2y_i / 1 \leq i \leq n\} \cup \{u_3z_i / 1 \leq i \leq n\}$.

Let r be any positive integer ≥ 2.

Let u_1, u_2, u_3 are the vertices of C_3 and x_i's, y_i's, z_i's are the pendent vertices.

Define $\sigma : V \rightarrow [0,1]$ by

$\sigma(x_i) = (6n + 4 - i)\,\frac{1}{10^r}$ if $1 \leq i \leq n$

$\sigma(y_i) = (5n + 4 - i)\,\frac{1}{10^r}$ if $1 \leq i \leq n$

$\sigma(z_i) = (4n + 4 - i)\,\frac{1}{10^r}$ if $1 \leq i \leq n$

$$\sigma(u_1) = (6n + 4)\,\frac{1}{10^r}$$

$$\sigma(u_2) = (6n + 5)\,\frac{1}{10^r}$$

$$\sigma(u_3) = (6n + 6)\,\frac{1}{10^r}$$

Define $\mu : V \times V \rightarrow [0,1]$ by

$$\mu(u_i, u_{i+1}) = \begin{cases} \frac{2}{10^r} \; if\ i = 2 \\ \frac{3}{10^r} \; if\ i = 1 \end{cases}$$

$$\mu(u_{i+2}, u_i) = \frac{1}{10^r}\; if\ i = 1$$

and for all $1 \leq j \leq n$

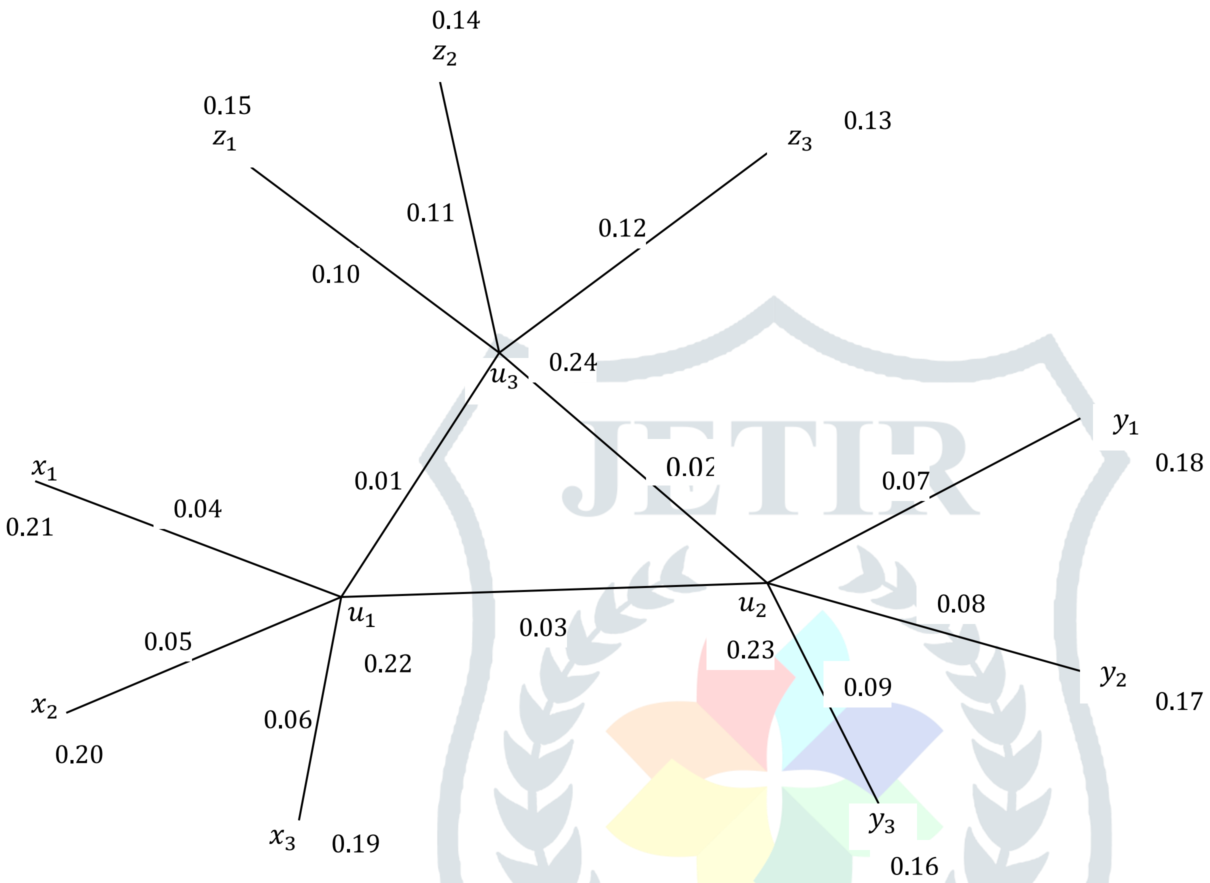
$$\mu(u_i, w_j) = \begin{cases} \frac{3 + j}{10^r} & if\ i = 1 \text{ and } w = x \\ \frac{n + 3 + j}{10^r} & if\ i = 2 \text{ and } w = y \\ \frac{2n + 3 + j}{10^r} & if\ i = 3 \text{ and } w = z \end{cases}$$

The MMVs are given in the following table:

MMV/Edges	K_1	K_2	K_3	Number of K_1	Number of K_2	Number of K_3
$u_3 z_i$ $1 \leq i \leq n$ and $u_3 u_2$	$(12n + 13)\,\frac{1}{10^r}$	–	–	$n + 1$	–	–
$u_2 y_i$ $1 \leq i \leq n$ and $u_2 u_1$	–	$(12n + 12)\,\frac{1}{10^r}$	–	–	$n + 1$	–
u_1, x_i $1 \leq i \leq n$ and u_1, u_3	–	–	$(12n + 11)\,\frac{1}{10^r}$	–	–	$n + 1$

Hence K_i's are constant and the maximum difference between the number of K_i's is 1 and $|K_i - K_j| \leq \frac{2}{10^r}$ for $1 \leq i, j \leq 3$. Hence the $C_3(n,n,n)$ star graph admits fuzzy edge tri-magic labeling.

Example 2.2.1: The $C_3(3,3,3)$ star graph admits fuzzy edge tri-magic labeling.



Theorem 2.3: The $C_3(n, 2n, 3n)$ star graph admits fuzzy edge tri-magic labeling.

Proof:

Let G be a $C_3(n, 2n, 3n)$ star graph

$|V(G)| = 6n+3$ and $|E(G)| = 6n+3$. Let the vertex set and edge set of G be

$V(G) = \{u_1, u_2, u_3, x_1, x_2, x_3 \dots \dots \dots \dots x_{2n}, y_1, y_2, y_3 \dots \dots \dots \dots y_{3n}, z_1, z_2, z_3 \dots \dots \dots \dots z_n\}$ and

E (G) = $\{u_1u_2, u_2u_3, u_1u_3\} \cup \{u_1x_i / 1 \leq i \leq 2n\} \cup \{u_2y_i / 1 \leq i \leq 3n\} \cup \{u_3z_i / 1 \leq i \leq n\}$.

Let r be any positive integer ≥ 2.

Let u_1, u_2, u_3 are the vertices of C_3 and x_i's, y_i's, z_i's are the pendent vertices.

Define $\sigma : V \rightarrow [0, 1]$ by

$\sigma(x_i) = (12n + 6 - i)\,\frac{1}{10^r}$ if $1 \leq i \leq 2n$

$\sigma(y_i) = (10n + 6 - i)\,\frac{1}{10^r}$ if $1 \leq i \leq 2n$

$\sigma(z_i) = (7n + 5 - i)\,\frac{1}{10^r}$ if $1 \leq i \leq n$

$\sigma(u_1) = (12n + 6)\,\frac{1}{10^r}$

$\sigma(u_2) = (12n + 7)\,\frac{1}{10^r}$

$\sigma(u_3) = (12n + 8)\,\frac{1}{10^r}$

define $\mu : V \times V \to [0, 1]$ by

$$\mu(u_i, u_{i+1}) = \begin{cases} \frac{2}{10^r} & \text{if } i = 2 \\ \frac{3}{10^r} & \text{if } i = 1 \end{cases}$$

$$\mu(u_{i+2}, u_i) = \frac{1}{10^r} \text{ if } i = 1$$

and for all $1 \le j \le n$

$$\mu(u_i, w_j) = \begin{cases} \frac{3 + j}{10^r} & if\ i = 1, 1 \le j \le 2n \text{ and } w = x \\ \frac{2n + 3 + j}{10^r} & if\ i = 2, 1 \le j \le 2n \text{ and } w = y \\ \frac{2n + 4 + j}{10^r} & if\ i = 2, 2n + 1 \le j \le 3n \text{ and } w = y \\ \frac{5n + 4 + j}{10^r} & if\ i = 3, 1 \le j \le n \text{ and } w = z \end{cases}$$

The MMVs are given in the following table:

MMV/Edges	K_1	K_2	K_3	Number of K_1	Number of K_2	Number of K_3
u_3z_i $1 \le i \le n$ u_2y_i $2n+1 \le i \le 3n$ and u_3u_2	$(24n + 17)\,\frac{1}{10^r}$	–	–	$2n + 1$	–	–
u_2y_i $1 \le i \le 2n$ and u_2u_1	–	$(24n + 16)\,\frac{1}{10^r}$	–	–	$2n + 1$	–
u_1, x_i $1 \le i \le 2n$ and u_1, u_3	–	–	$(24n + 15)\,\frac{1}{10^r}$	–	–	$2n + 1$

Hence K_i's are constant and the maximum difference between the number of K_i's is 1 and $|K_i - K_j| \le \frac{2}{10^r}$ for $1 \le i, j \le 3$. Hence the $C_3(n, 2n, 3n)$ star graph admits fuzzy edge tri-magic labeling.

CONCLUSION

In this paper, we have shown that the star graph $S_{1,n}$ and some type of $C_3(n,m,l)$ star graphs are fuzzy tri-magic. We are working in fuzzy tri-magic labeling of some more special cases of graphs.

REFERENCES

[1] Harary, F. 1988. *Graph theory*. New Delhi: Narosa Publishing House Pvt. Ltd.

[2] Gallian, JA. 2017. A dynamic survey of graph labeling. The Electronics Journal of Combinatorics 17: DS6.