ON NEUTROSOPHIC FEEBLY CONTINUOUS FUNCTIONS

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ABSTRACT

In this paper, we introduce a new class of functions using neutrosophic feebly open sets and neutrosophic feebly closed sets, namely neutrosophic feebly continuous also we study some basic properties and the characterizations of above mentioned function.

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INTRODUCTION

Neutrosophy, as a new branch of Philosophy has been introduced by Smrandache [9, 10, 11] and explained, neutrosophic set is a generalization of Intuitionistic fuzzy set [2]. Smrandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy and non membership values respectively, where $]^-0, 1^+[$ is non standard unit interval. In 2012, Salama, Alblowi [17] introduced the concept of neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of Intuitionistic fuzzy topological spaces and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non- membership of each element.

Let T, I, F be real standard or not standard subset of $]^-0,1^+[$, with

$sup\_T = t\_sup, inf\_ = t\_inf$

$sup\_I = i\_sup, inf\_ = i\_inf$

$sup\_F = f\_sup, inf\_ = f\_inf$

$$n - sup = t\_sup + i\_sup + f\_sup$$

$n - inf = t\_inf + i\_inf + f_{inf}, T, I, F$ are called neutrosophic components.

In 2012, Salama, Alblowi[16], introduced the concept of neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element. In 2014, Salama, Smarandache and Valeri[18] were introduced the concept of neutrosophic closed sets and neutrosophic continuous functions.

The Section I consists of the basic definitions of neutrosophic sets and operations which are used in the later sections. The Section II deals with the concept of *neutrosophic feebly continuous functions* in neutrosophic topological space and study their properties.

I. PRELIMINARIES

In this section, we give the basic definitions for neutrosophic sets and its operations.

Definition 1.1 [17] Let X be a non-empty fixed set. A neutrosophic set (NF for short) A is an object having the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark 1.2 [17] A neutrosophic set $A =\{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A \rangle$ in $]^-0,1^+[$ on X.

Remark 1.3 [17] For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ for the neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$.

Example 1.4 [17] Every intuitionistic fuzzy Set A is a non-empty set in X is obviously on neutrosophic set having the form $A = \{ \langle x, \mu_A(x), 1- (\mu_A(x) + \gamma_A(x)), \gamma_A(x) \rangle : x \in X \}$. Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the neutrosophic set 0_N and 1_N in X as follows :

0_N may be defined as:

(0_1) $0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$
(0_2) $0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$
(0_3) $0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$
(0_4) $0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$

1_N may be defined as:

(1_1) $1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$
(1_2) $1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$
(1_3) $1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$
(1_4) $1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$

Definition 1.5 [17] Let $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$ be a NF on X. Then the complement of the set A (C(A) for short) may be defined as three kinds of complements :

(C_1) C(A) = { ⟨ x, 1 - $\mu_A(x)$, 1 - $\sigma_A(x)$, 1 - $\gamma_A(x)$⟩ : x ∈X }
(C_2) C(A) = { ⟨ x, $\gamma_A(x)$, $\sigma_A(x)$, $\mu_A(x)$⟩ : x ∈X }
(C_3) C(A) = { ⟨ x, $\gamma_A(x)$, 1 - $\sigma_A(x)$, $\mu_A(x)$⟩ : x ∈X }
One can define several relations and operations between neutrosophic set follows :

Definition 1.5 [17] Let x be a non-empty set, and neutrosophic set A and B in the form A = { ⟨ x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ ⟩ : x∈X } and B = { ⟨ x, $\mu_B(x)$, $\sigma_B(x)$, $\gamma_B(x)$ ⟩ : x ∈ X }. Then we may consider two possible definitions for subsets (A ⊆ B).
(A ⊆ B) may be defined as :
(1) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x)$, $\sigma_A(x) \le \sigma_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ ∀ x ∈ X
(2) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x)$, $\sigma_A(x) \ge \sigma_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ ∀ x ∈X

Proposition 1.6 [17] For any neutrosophic set A the following are holds :
(1) $0_N \subseteq A$, $0_N \subseteq 0_N$
(2) $A \subseteq 1_N$, $1_N \subseteq 1_N$

Definition 1.7 [17] Let X be a non-empty set, and A = ⟨ x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ ⟩, B = ⟨ x, $\mu_B(x)$, $\sigma_B(x)$, $\gamma_B(x)$ ⟩ are neutrosophic set. Then
(1) A ∩ B may be defined as:
(I_1) A ∩ B = ⟨ x, $\mu_A(x) \wedge \mu_B(x)$, $\sigma_A(x) \wedge \sigma_B(x)$ and $\gamma_A(x) \vee \gamma_B(x)$ ⟩
(I_2) A ∩ B = ⟨ x, $\mu_A(x) \wedge \mu_B(x)$, $\sigma_A(x) \vee \sigma_B(x)$ and $\gamma_A(x) \vee \gamma_B(x)$ ⟩
(2) A ∪ B may be defined as:
(U_1) A ∪ B =⟨ x, $\mu_A(x) \vee \mu_B(x)$, $\sigma_A(x) \vee \sigma_B(x)$ and $\gamma_A(x) \wedge \gamma_B(x)$ ⟩
(U_2) A ∪ B =⟨ x, $\mu_A(x) \vee \mu_B(x)$, $\sigma_A(x) \wedge \sigma_B(x)$ and $\gamma_A(x) \wedge \gamma_B(x)$ ⟩

We can easily generalize the operations of intersection and union in Definition 1.8 to arbitrary family of neutrosophic set as follows:

Definition 1.8 [17] Let { A_j: j ∈ J }be a arbitrary family of neutrosophic set in X. Then
(1) ∩ A_j may be defined as:
(i) $\cap A_j$= ⟨ x, $\wedge_{j\in J}\, \mu_{A_j}(x)$, $\wedge_{j\in J}\, \sigma_{A_j}(x)$, $\vee_{j\in J}\, \gamma_{A_j}(x)$ ⟩
(ii) $\cap A_j$= ⟨ x, $\wedge_{j\in J}\, \mu_{A_j}(x)$, $\vee_{j\in J}\, \sigma_{A_j}(x)$, $\vee_{j\in J}\, \gamma_{A_j}(x)$ ⟩
(2) ∪ A_j may be defined as:
(i) $\cup A_j$= ⟨ x, ∨, ∨, ∧ ⟩
(ii) $\cup A_j$= ⟨ x, ∨, ∧, ∧ ⟩

Definition 1.9 [17] A neutrosophic topology (NT for short) is a non-empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms :
(NT_1) 0_N, $1_N \in \tau$,
(NT_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(NT_3) $\cup G_i \in \tau$ for every { G_i : i ∈ J } ⊆ τ.
In this case the pair (X, τ) is called a neutrosophic topological space (NTS for short). The elements of τ are called neutrosophic open sets (NOS for short). A neutrosophic set F is closed if and only if it C (F) is neutrosophic open.

Example 1.10 [17] Any fuzzy topological space (X, τ_0) in the sense of Chang is obviously a NTS in the form τ = { A : $\mu_A \in \tau_0$ } wherever we identify a fuzzy set in X whose membership function is μ_A with its counterpart.

Definition 1.11 [17] The complement of a neutrosophic open set A(C (A) for short) is called a neutrosophic closed set (NCS for short) in X.

Now, we define neutrosophic closure and interior operations in neutrosophic topological spaces.

Definition 1.12 [17] Let (X, τ) be NTS and A = $\langle$ x, μ_A(x), σ_A(x), γ_A(x) $\rangle$ be a NF in X. Then the neutrosophic closure and neutrosophic interior of A are defined by

$NFcl$ (A) = $\cap$ { K : K is a NCS in X and A $\subseteq$ K }

$Nint$ (A) = $\cup$ {G : G is a NOS in X and G $\subseteq$ A }.

It can be also shown that NCl (A) is NCS and NInt (A) is a NOS in X. That is,

a) A is NCS in X if and only if A = $NFcl$ (A).

b) A is NOSin X if and only if A = $Nint$ (A).

Proposition 1.13 [17] For any neutrosophic set A in (X, τ) we have

(a) $NFcl$ (C (A))= C ($Nint$ (A),

(b) $Nint$ (C (A)) = C ($NFcl$ (A)).

Proposition 1.14 [17] Let (X, τ) be a NTS and A,B be two neutrosophic sets in X. Then the following properties holds :

(a) NInt (A) $\subseteq$ A,

(b) A $\subseteq$ Ncl (A),

(c) A $\subseteq$ B $\Rightarrow$ $Nint$ (A) $\subseteq$ $Nint$ (B),

(d) A $\subseteq$ B $\Rightarrow$ Ncl (A) $\subseteq$ Ncl (B),

(e) $Nint$ (A $\cap$ B)) = $Nint$ (A) $\wedge$ $Nint$ (B),

(f) Ncl (A $\cup$ B) = Ncl (A) $\vee$ Ncl (B),

(g) $Nint$ (1_N) = 1_N,

(h) Ncl (0_N) = 0_N,

(i) A $\subseteq$ B $\Rightarrow$ C (B) $\subseteq$ C (A),

(j) Ncl (A $\cap$ B) $\subseteq$ Ncl (A) $\cap$ Ncl (B),

(k) $Nint$ (A $\cup$ B) $\supseteq$ $Nint$ (A) $\cup$ $Nint$ (B),

Definition 1.15.[18] Let X and Y be two neutrosophic sets and $f: X \rightarrow Y$ be a function.

(i) If $B = \langle y, \mu_B(y), \sigma_B(y), \gamma_B(y)\rangle$ is a neutrosophic sets in Y, then the pre image of B under f is denoted and defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}$.

(ii) If $A = \{ \langle x, \alpha_A(x), \delta_A(x), \lambda_A(x) \rangle : x \in X \}$ is a neutrosophic sets in X, then the image of A under f is denoted and defined by

$$f(A) = \{\langle y, f(\alpha_A)(y), f(\delta_A)(y), f\_(\lambda_A)(y)\rangle : y \in Y\} \text{where} f\_(\lambda_A) = (f(\lambda_A{}^C))^C.$$

In (i), (ii), since μ_B, σ_B, γ_B, α_A, δ_A, λ_A are neutrosophic sets, we explain that $f^{-1}(\mu_B)(x) = \mu_B(f(x))$, and $f(\alpha_A)(y) = \begin{cases} \sup \alpha_A(x) \text{ if } y \in f^{-1}(x) \\ 0 \quad \text{Otherwise} \end{cases}$.

Lemma 1.16. [18] Let $f: X \to Y$ be a function. Then the following statements hold.

(i) If A and B are neutrosophic subsets of X such that $A \leq B$, then $f(A) \leq f(B)$,

(ii) If A and B are neutrosophic subsets of Y such that $A \leq B$, then $f^{-1}(A) \leq f^{-1}(B)$.

Lemma 1.17 [3] Let $f: X \to Y$ be a function. If A is a neutrosophic subset of X and μ is a neutrosophic subset of Y. Then

(i) $f(f^{-1}(A)) \leq A$

(ii) $f(f^{-1}(A)) = A \Leftrightarrow f$ is surjective.

(iii) $f^{-1}(f(A)) \geq A$

(iv) $f^{-1}(f(A)) = A$ whenever f is injective.

Definition 1.18. [6] Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ be the two neutrosophic functions. Then the neutrosophic product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is defined by $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2))$ for all $(x_1, x_2) \in X_1 \times X_2$.

Definition 1.19 [6] Let $A, A_i (i \in J)$ be neutrosophic subsets in X and $B, Bj\ (j \in K)$ be neutrosophic subsets in Y and $f: X \to Y$ be the neutrosophic function. Then

(i) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

(ii) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

(iii) $f^{-1}(1_N) = 1_N$, $f^{-1}(0_N) = 0_N$

(iv) $f^{-1}(B^C) = (f^{-1}(B))^c$

(v) $f(\cup A_i) = \cup f(A_i)$.

Definition 1.20. [3] Let $f : X \to Y$ be the function. Then the neutrosophic graph $g : X \to X \times Y$ of f is defined by $g(x) = (x, f(x))$, for all $x \in X$.

Lemma 1.21. [3] Let $f_i : X_i \to Y_i$ $(i = 1, 2)$ be the functions and A, B be neutrosophic subsets of Y_1, Y_2 respectively. Then $(f_1 \times f_2)^{-1}(A \times B) = f_1^{-1}(A) \times f_2^{-1}(B)$.

Lemma 1.22. Let $g: X \to X \times Y$ be the neutrosophic graph of the function $f: X \to Y$. If A is the neutrosophic set of X and B is the neutrosophic set of Y, then $g^{-1}(A \times B)(x) = (A \cap f^{-1}(B))(x)$.

Definition 1.23. [18] Let (X, τ) and (Y, σ) be neutrosophic topological spaces. Then a map $f : (X, \tau) \to (Y, \sigma)$ is called neutrosophic continuous (in short N-continuous) function if the inverse image of every neutrosophic open set in (Y, σ) is neutrosophic open set in (X, τ).

Definition 1.24[12] A Neutrosophic subset A of a Neutrosophic topological Space (X , τ) is Neutrosophic feebly open if there is a Neutrosophic open set U in X such that U≤ A≤NSClU .

Definition 1.25 [12] A Neutrosophic subset A of a Neutrosophic topological Space (X , τ) is Neutrosophic feebly closed if there is a Neutrosophic closed set U in X such that NSInt U≤A≤U.

2. Neutrosophic Feebly Continuity

Definition 2.1. Let (X, τ) and (Y, σ) be two neutrosophic topological spaces. Then a function $f: X \to Y$ is said to be neutrosophic feebly continuous function if $f^{-1}(G)$ is neutrosophic feebly open set in X for all neutrosophic open set G in Y.

Example 2.2 Let X= {x_1, x_2}, Y={y_1,y_2} and $f: X \to Y$ be a function such that A={(x_1,0.4,0.2,0.2), (x_2,0.5,0.4,0.6)}, B={(y_1,0.2,0.4,0.8), (y_2,0.5,0.7,0.1)} and $f(x_1)=y_1$, $f(x_2)=y_2$. Then, τ={$1_\aleph$, $0_\aleph$, A} and σ={$1_\aleph$, $0_\aleph$, B} are two neutrosophic topologies over X and over Y, respectively. Hence, $f: (X, \tau) \to (Y, \sigma)$ is a neutrosophic feebly continuous function.

Theorem 2.3 Every neutrosophic continuous function is neutrosophic feebly continuous function.

Proof. Let $f: (X, \tau) \to (Y, \sigma)$ be neutrosophic continuous function. Let V be a neutrosophic open set in (Y, σ). Then $f^{-1}(V)$ is neutrosophic open set in (X, τ). Since every neutrosophic open set is neutrosophic feebly open set, $f^{-1}(V)$ is neutrosophic feebly open set in (X, τ). Hence f is neutrosophic feebly continuous function.

Remark 2.4 The converse of the above theorem is need not be true as shown by following example.

Example 2.5 Let X = Y = { a , b, c }. Define the neutrosophic sets as follows :

A = ⟨ (0.4, 0.5, 0.2), (0.3, 0.2, 0.1), (0.9, 0.6, 0.8) ⟩

B = ⟨ (0.2, 0.4, 0.5), (0.1, 0.1, 0.2), (0.6, 0.5, 0.8) ⟩

C = ⟨ (0.5, 0.4, 0.2), (0.2, 0.3, 0.1), (0.6, 0.9, 0.8) ⟩ and

D = ⟨ (0.4, 0.2, 0.5), (0.1, 0.1, 0.2), (0.5, 0.6, 0.8) ⟩ . Now T = { 0_N, A, B, 1_N } and S = { 0_N, C, D, 1_N } are neutrosophic topologies on X. Thus (X,τ) and (Y,σ) are neutrosophic topological spaces. Also we define f:(X,τ)→(Y,σ) as follows: $f\ (a) = b, f(b) = a, f(c) = c$. Clearly f is NS-continuous function. But f is not N-continuous function. Since E = ⟨ (0.5, 0.6, 0.1), (0.4, 0.3, 0.1), (0.9, 0.8, 0.5) ⟩ is a neutrosophic open in (Y,σ), f^{-1}(E) is not neutrosophic open set in (X,τ).

Theorem 2.6 Let (X, τ) and (Y, σ) and (Z, ψ) be three neutrosophic topological spaces. If $f: (X, \tau) \to (Y, \sigma)$ is a neoutrosophic feebly continuous function and $g: (Y, \sigma) \to (Z, \psi)$ is neoutrosophic continuous, then $gof: (X, \tau) \to (Z, \psi)$ is a neutrosophic feebly continuous function.

Proof. Let G be a neutrosophic open set in Z. Since $g: (Y, \sigma) \to (Z, \psi)$ is neoutrosophic continuous, $g^{-1}(G)$ is neutrosophic open set in Y. Since f is a neutrosophic feebly continuous , $f^{-1}(g^{-1}(G))$ is neutrosophic feebly open set in X and $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$. Then $(gof)^{-1}(G)$ is neutrosophic feebly open set in X. Hence, (gof) is a neutrosophic feebly continuous function.

Theorem 2.7. Let (X, τ) and (Y, σ) be two neutrosophic topological spaces. If $f: X \to Y$ is a neutrosophic feebly continuous if and only if $f^{-1}(A)$ is neutrosophic feebly cloed set in X for all neutrosophic cloed set A in Y.

Proof . Let A be a neutrosophic cloed set in Y. By Definition 1.1.15, A^C is neutrosophic open in Y. Since f is neutrosophic feebly continuous, By Definition 4.1.1, $f^{-1}(A^C)$ is a neutrosophic feebly open set in X. By Definition 1.3.3(iv), $f^{-1}(A^C) = (f^{-1}(A))^C$, $f^{-1}(A)$ is neutrosophic feebly closed set in X.

Let A be a neutrosophic open set in Y. Then A^C is neutrosophic cloed in Y. By assumption $f^{-1}(A^C)$ is neutrosophic feebly closed in X. By Definition 1.3.3, $f^{-1}(A^C) = (f^{-1}(A))^C$, $f^{-1}(A)$ is neutrosophic feebly open set in X. Hence, f is a neutrosophic feebly continuous.

Theorem 2.8. Let (X, τ) and (Y, σ) be two neutrosophic topological spaces and $f: X \to Y$ be a function. Then, f is a neutrosophic feebly continuous function if and only if $f(NFcl(A)) \subseteq NFcl(f(A))$ for all neutrosophic set A in X.

Proof . Let A neutrosophic set in X and f be a neutrosophic continuous function. From Theorem 1.3.2, $A \subseteq NFcl(A)$ implies $f(A) \subseteq NFcl(f(A))$. By Theorem 2.2, we have $A \subseteq f^{-1}\big(f(A)\big) \subseteq$ and $NFcl(A) \subseteq NFcl(f^{-1}(NFcl(f(A))))$. Since f is a neutrosophic feebly continuous function and $NFcl(f(A))$ is a neutrosophic feebly closed set,$NFcl(f^{-1}(NFcl(f(A))) = f^{-1}(NFcl\, f\, (A)))$. Hence, $f(NFcl(A) \subseteq NFcl(f(A))$.

Conversely, $f(NFcl(A)) \subseteq NFcl(f(A))$, for all neutrosophic set in X. Let F be a neutrosophic closed set in Y.Then$NFcl(f(f^{-1}(F))) \subseteq NFcl(F) = F$. By assumption, $f(NFcl(f^{-1}(F))) \subseteq NFcl(f(f^{-1}(F))) \subseteq F$and hence $NFcl(f^{-1}(F)) \subseteq f^{-1}(F)$. Since $f^{-1}(F) \subseteq NFcl(f^{-1}(F))$, $NFcl\big(f^{-1}(F)\big) = f^{-1}(F)$. By using $NFcl\, F = F$ if and only if F is neutrosophic feebly closed, $f^{-1}(F)$ neutrosophic feebly closed set in X. From the above theorem, f is a neutrosophic feebly continuous function.

Theorem 2.9. Let (X, τ) and (Y, σ) be two neutrosophic topological spaces, and $f: X \to Y$ be a function. Then, f is a neutrosophic feebly continuous function if and only if $NFcl(f^{-1}(B)) \subseteq f^{-1}(NFcl(B))$ for all neutrosophic set B in Y.

Proof. Let B be any neutrosophic set in Y and f be a neutrosophic feebly continuous function. From Proposition 3.1.7, and Lemma 1.2, we have $f^{-1}(B) \subseteq f^{-1}(NFcl(B))$. Then, $NFcl(f^{-1}(B)) \subseteq NFcl(f^{-1}(NFcl(B)))$. Since $NFcl(B)$ neutrosophic feebly closed set in Y, by Theorem 2.7, $f^{-1}(NFcl(B))$ is neutrosophic feebly closed set in X. Thus, $NFcl(f^{-1}(B)) \subseteq NFcl(f^{-1}(NFcl(B))) = f^{-1}(NFcl(B))$.

Conversely, $NFcl(f^{-1}(B)) \subseteq f^{-1}(NFcl(B))$ for all neutrosophic set B in Y. Let F be a neutrosophic closed set in Y. Since every neutrosophic closed set is neutrosophic feebly closed set, $NFcl(f^{-1}(F)) \subseteq f^{-1}(NFcl(F)) = f^{-1}(F)$. From Theorem 4.1.7, f is a neutrosophic feebly continuous function.

Theorem 2.10. Let (X, τ) and (Y, σ) be two neutrosophic topological spaces, $f: (X, \tau) \to (Y, \sigma)$ be a bijective function. Then f is neutrosophic feebly continuous if and only if $NFint(f(A)) \subseteq f(NFint(A))$ for all neutrosophic set A in X.

Proof. Let A be any neutrosophic set in X and f be a bijective and neutrosophic feebly continuous function. Let $f(A) = B$. From Theorem 2.2 and Theorem 2.1[12], Lemma 2.2, Preposition 3.1.7[12], $f^{-1}(NFint(B)) \subseteq f^{-1}(B)$. Since f is an injective function, $f^{-1}(B) = A$, so that $f^{-1}(NFint(B)) \subseteq A$. Therefore, $NFint(f^{-1}(NFint(B))) \subseteq NFint(A)$. Here, $f^{-1}(NFint(B))$ neutrosophic feebly open set in X and $f^{-1}(NFint(B)) \subseteq NFint(A)$, then $f(f^{-1}(NFint(B)) \subseteq f(NFint(A))$. Since f is a surjective function, Lemma 1.3.3. Hence, $NFint(f(A)) \subseteq f(NFint\, (A))$.

Conversely, $NFint(f(A)) \subseteq f(NFint(A))$ for all neutrosophic set A in X. Let V be a neutrosophic open set in Y. Since f is surjective and Preposition 3.2[12], $V = NFint(V) = NFint(f(f^{-1}(V))) \subseteq$

$f(NFint(f^{-1}(V))$. It follows that, $f^{-1}(V) \subseteq NFint\big(f^{-1}(V)\big)$. Therefore $f^{-1}(V)$is neutrosophic feebly open set in X. Hence by Definition 2.1, f is a neutrosophic feebly continuous function.

Theorem 2.11. Let (X,τ) and (Y,σ) be two neutrosophic topological spaces and $f: X \to Y$ be a function. Then f is a neutrosophic feebly continuous function if and only if $f^{-1}(NFint(B)) \subseteq NFint\,(f^{-1}(B))$ for all neutrosophic set in B in Y.

Proof. Let B be any neutrosophic set in Y and f be a neutrosophic feebly continuous function. By Lemma 1.2 and Proposition 3.7[12], $f^{-1}(NFint(B)) \subseteq f^{-1}(B)$ implies $NFint(f^{-1}(NFint(B)) \subseteq NFint(f^{-1}(B))$. Since $NFint(B)$is neutrosophic feebly open set in Y and f is neutrosophic feebly continuous, $f^{-1}(NFint(B))$ is neutrosophic feebly open set in X. So that, $NFint(f^{-1}(NFint(B))) = f^{-1}(NFint(B)) \subseteq NFint\,(f^{-1}(B))$.

Conversely, $f^{-1}(NFint(B)) \subseteq NFint(f^{-1}(\,B))$ for all neutrosophic set B in Y. Let G be any neutrosophic open set in Y. Since by Theorem 2.3, G is neutrosophic open set. Then $f^{-1}(G) = f^{-1}(NFint(G)) \subseteq NFint(f^{-1}(G))$ and therefore f^{-1}(G)= $NFint(f^{-1}(G))$. So that $f^{-1}(G)$ is neutrosophic feebly open set in X. Hence f is a neutrosophic feebly continuous function.

Definition 2.12 Let A be a neutrosophic subset of a neutrosophic topological space (X, τ). Then the neutrosophic feebly frontier of λ is defined as $NFFr\,\lambda = NFcl\,\lambda \wedge NFcl\,(\lambda^{C})$.

Theorem 2.13. Let (X,τ) and (Y,σ) be two neutrosophic topological spaces and $f: X \to Y$ be a bijective function. Then f is a neutrosophic feebly continuous function if and only if $f(NFfr(A)) \subseteq NFfr(f(A))$ for all neutrosophic set A in X.

Proof. Let f be a bijective and neutrosophic feebly continuous function. Let A be neutrosophic set in X. From Definition 2.2, $NFfr(A) = NFcl(A) \cap (NFint(A))^{C}$. By Theorem 2.7, $f(NFint(A)) \subseteq NFint(f(A))$ and from Theorem 2.5, $f\big(NFcl(A)\big) \subseteq NFcl\big(f(A)\big)$, $f(NFfr(A))$=$f(NFcl(A) \cap f(NFint(A))^{C}) \subseteq f(NFcl(A)) \cap (\,f\big(NFcl(A^{C})\big) = NFcl\big(f(A)\big) \cap NFcl\big(f(A^{C})\big) = NFcl\big(f(A)\big) \cap (NFint(\big(f(A)\big))^{C} = NFfr\big(f(A)\big)$.

Conversely, $f(NFfr(A)) \subseteq NFfr(f(A))$ for all neutrosophic set A in X. Then$f(NFcl(A)) = f(NFint(A)) \cup f(NFfr(A)) \subseteq f(A) \cup NFfr(f(A)) \subseteq fNFcl(f(A))$. By Theorem 2.8, f is neutrosophic continuous function.

Theorem 2.13. Let (X,τ) and (Y,σ) be two neutrosophic topological spaces and $f: X \to Y$ be a bijective function. Then, f is a neutrosophic continuous function if and only if $NFfr(f^{-1}(B)) \subseteq f^{-1}(NFfr(B))$ for all neutrosophic set B in Y.

Proof. Let f be a bijective and neutrosophic continuous function. Let B be a neutrosophic set in Y.By Theorem 2.4 and Theorem 2.6, $NFcl(f^{-1}(B)) \subseteq f^{-1}(NFcl(B))$ and $f^{-1}(NFint(B)) \subseteq NFint(f^{-1}(B))$. $f^{-1}(NFfr(B)) = f^{-1}(NFcl(B)) \subseteq \left(NFint(B)\right)^{C}) = f^{-1}(NFcl(B)) \subseteq f^{-1}((NFint(B))^{C}) =$ $f^{-1}(NFcl(B)) = NFint(B)$ From Theorem 3.3 and Theorem 3.6 $NFcl(f^{-1}(B)) \subseteq f^{-1}(NFcl(B))$ and $(NFint(f^{-1}(B)))^{C} \subseteq (f^{-1}(NFint(B)))^{C}$; hence, $f^{-1}(B) \subseteq f^{-1}(NFfr(B))$.

Since $NFfr(f^{-1}(B)) \subseteq f^{-1}(NFfr(B))$ for all $B \in \boldsymbol{N}\,(V)$. Then, $NFfr(f^{-1}(B)) \subseteq f^{-1}(B) \subseteq f^{-1}(NFfr(B)) \subseteq f^{-1}(B)$. Hence, $NFcl(f^{-1}(B)) \subseteq f^{-1}(NFfr(B) \subseteq B) = f^{-1}(NFcl(B))$ From Theorem 3.4, f is a neutrosophic continuous function.

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