VARIATION OF FLOW RATE IN LAMINAR FLOW OVER A MOVING INFINITE VERTICAL POROUS PLATE IN THE CONSTANT HEAT FLUX

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ABSTRACT:

In this paper the case of variation of flow rate in situation of incompressible, laminar, viscous fluid flow past an infinite vertical porous plate under the influence of aligned magnetic field in the presence of thermal radiation and constant heat flux has been examined. The governing equations of the flow are solved analytically and the impact of various flow parameters on velocity, temperature and flow rate variation has been analyzed through graphs. It is observed that as the pore size increases the flow rate is found to be increasing. Also, increase in the intensity of the magnetic field gives to the decrease in flow rate. Further, increase in Grashof number gives to the decrease in the flow rate.

Key words: Vertical plate, laminar flow, porous, heat flux.

1. INTRODUCTION:

Usually in industrial situations like nuclear power plants, gas turbines and propulsion devices for air craft, missiles and space vehicles are some conditions where in radiative convective heat transfer happens more often. The applications wide in general and are found mostly in fossil fuel combustion, energy processes, astrophysical flows, cooling chambers, solar power technology and space vehicle re-entry. Generally, radiative heat transfer is found to have a vital role in manufacturing sectors for the design of highly precision equipment.

Stokes [1] studied the problem of viscous incompressible fluid past an impulsively started infinite horizontal moving plate in its own plane. Thereafter Brinkman [2] investigated the viscous force imparted by flowing fluid. Subsequently, Stewartson [3] studied the problem of viscous flow past and impulsively started semi-infinite horizontal plate for an analytical solution. The case of two dimensional steady state flow of an incompressible fluid with rigid parallel porous walls was examined by Berman [4]. Thereafter, Mori [5] studied the flow between two vertical plates which are electrically non-conducting, while the wall temperature varies linearly in the direction of the flow. Later, the flow in renal tubules was investigated by Macy [6]. Subsequently, Hall [7] examined similar problems using method of finite differences while Chang et al [8] analyzed the radioactive heat transfer effects on free convection regimes in specialized applications that occur in geophysics and geothermal reservoirs. Later, Mahajan et al [9] examined the effect of viscous heat dissipating in natural convective flows. It was established that the heat transfer rates are raised by a decrease in the dissipation parameter. Afterwards, Soundalgekar and Thaker [10] observed thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Later, Das et al [11] examined higher order numerical approximation for mass transfer effects on the study flow past an accelerated vertical porous plate. Using Rossland's approximation Hossain et al [12] examined the radiation effects on a mixed convective flow along a vertical plate with a uniform surface temperature. Raptis and Perdikis [13] studied the effect of thermal radiation and convective flow past a moving infinite vertical plate. The effects of thermal radiation on the flow past a semi-infinite vertical isothermal plate with uniform heat flux was studied exhaustively by Chandrakala and Antony Raj [14]. Rashad and Bakier [15] investigated a steady two-dimensional laminar forced flow and heat transfer of a viscous incompressible electrically conducting and heat-generating fluid past a permeable wedge embedded in non-Darcy high-porosity ambient medium with uniform surface heat flux. Makinde [16] examined the effects of mass transfer on the free convection flow of a viscous incompressible electrically conducting fluid past a vertical porous flat plate embedded in a porous medium with constant heat flux in the presence of a transversely imposed magnetic field. The effect of thermophoresis and chemical reaction on MHD natural convective heat and mass transfer flow in a rotating fluid considering heat and mass fluxes in the presence of strong magnetic field and viscous dissipation has been studied by Ferdows et al[17]. An unsteady three dimensional free convection flow with heat and mass transfer over a vertical plate embedded in a porous medium with constant heat flux and transverse sinusoidal permeability is studied by Jain et al [18]. An analysis was carried out to investigate effects of radiation on a free convection flow bounded by a vertical surface embedded in a porous medium with constant heat flux by Sudhakar Reddy et al [19]. Manjulatha et al [20] analyses the effects of radiation absorption and mass transfer on the steady MHD flow past an infinite vertical flat plate through a porous medium with an aligned magnetic field.

In all above papers the behavior of velocity with reference to all parameters that appears in the field equations were studied. The most important factor i.e. flow rate has been completely ignored and was not addressed by any of the investigators. Also, not much of main importance was given to the bounding surface and factors influencing the flow rate. The flow rate is found to have a main application in several situations where heat and mass transfer take place. Hence, such an important concept cannot be ignored while studying the situation of thermal radiation. Hence, in this paper the influence of various critical parameters which appear in the field equations and their effects on the flow rate is examined. Such an analysis provides the information of the transfer of fluid from one reaction chamber to another chamber and the amount of fluid that adheres to the walls of the container.

2. FORMULATION OF THE PROBLEM:

The physical model of the problem is shown in figure 1. The x^* axis is taken along the plate in the vertical upward direction and the y^* axis is taken normal to the plate.

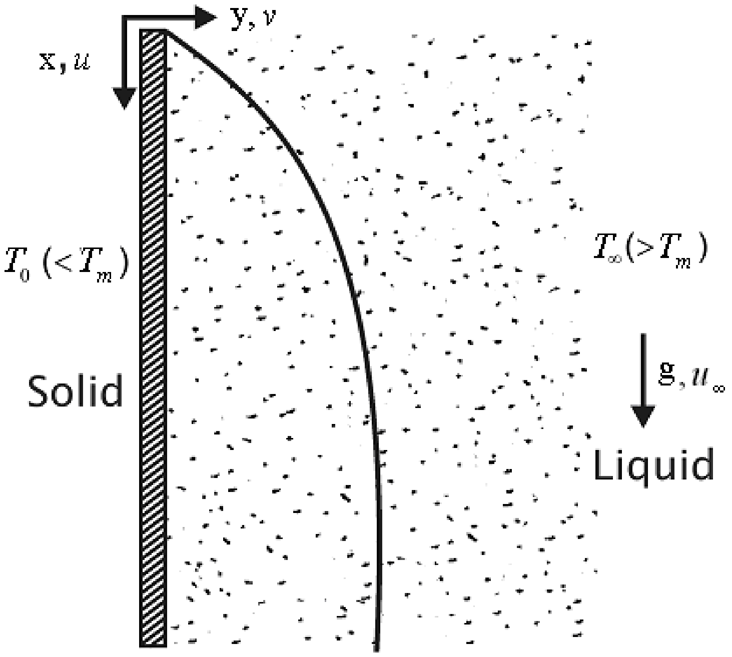

Figure 1:Physical model of the problem

We made the following assumptions:

1. The fluid is to be incompressible, laminar, viscous radiating fluid past an impulsively started infinite vertical plate.
2. When $t^* \leq 0$, the plate and fluid are maintained at same temperature.
3. At time t > 0, the plate is given an impulsive motion in the vertical direction and in opposite to the gravitational field with constant velocity u_0.
4. Viscous dissipation is neglected in the energy equation as the motion is due to free convection only.
5. The fluid is considered to be absorbing, emitting radiation.

Using Boussinesq's approximation we have

$$\frac{\partial u^*}{\partial t^*} = \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\upsilon}{K^*} u^* - \frac{\sigma B_0^2 \sin^2 \phi}{\rho} u^* + g\beta(T - T_\infty) \quad (1)$$

$$\frac{\partial T}{\partial t^*} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^{*2}} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y^*} \quad (2)$$

In view of Rossland approximation q_r is given by

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \tag{3}$$

While, the initial and boundary conditions are:

When $t^* \le 0, \quad u^* = 0, \quad T = T_\infty$ for all y^*

and if $t^* > 0, \quad u^* = u_0, \quad \frac{\partial T}{\partial y^*} = -\frac{q}{k}$ at $y^* = 0$

$$u^* = 0, \quad T \to T_\infty \quad \text{at } y^* \to \infty \tag{4}$$

Assuming that the temperature differences within the flow are negligible than T^4 can be expressed as a linear function of the temperature. By neglecting higher order terms T^4 in Taylor's series about T_∞ can be expressed as

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{5}$$

By using equations (3) and (5), equation (2) reduces to

$$\frac{\partial T}{\partial t^*} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^{*2}} + \frac{16\sigma T_\infty^3}{3k^*\rho c_p}\frac{\partial^2 T}{\partial y^{*2}} \tag{6}$$

Where u^* is the velocity, ρ the fluid density, p^* is the pressure, t^* the time, y^* is the dimensionless coordinate axis normal to the plate, y coordinate axis normal to the plate, υ is the kinematic viscosity coefficient, K^* the porous medium permeability coefficient, σ is the conductivity of the fluid, B_0 is electromagnetic induction, g is the acceleration due to gravity, β the coefficient of volume expansion due to temperature, T the fluid temperature, T_∞ is the temperature of the fluid far away from the plate, k the thermal conductivity, c_p is the specific heat of constant pressure, q_r is the radiative heat flux in the y direction, u_0 the velocity of the fluid plate, q is the heat flux per unit area, k^* mean absorption coefficient.

So as to make the Governing equations, initial and boundary conditions we introduce the following non – dimensional terms as

$$t = \frac{t^* u_0^2}{\upsilon}, \quad y = \frac{y^* u_0}{\upsilon}, \quad u = \frac{u^*}{u_0}, \quad \theta = \frac{(T - T_\infty)u_o k}{q\upsilon}, \quad \Pr = \frac{\mu c_p}{k}, \quad Gr = \frac{g\beta q\upsilon^2}{u_0^4 k},$$
$$M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2}, \quad N = \frac{k^* k}{4\sigma T_\infty^3}, \quad K = \frac{u_0^2 K^*}{\upsilon^2} \tag{7}$$

Where *Gr* is thermal Grashof number, *Pr* is the Prandtl number, K is porosity, *M* the Hartmann number, N is radiation parameter, t time, θ is the dimensionless temperature.

Using the above dimensionless quantities into equations (1) and (6), we have

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - (M\sin^2\phi + \frac{1}{K})u + Gr\theta \tag{8}$$

$$\frac{\partial \theta}{\partial t} = \left(\frac{3N+4}{3N\Pr}\right)\frac{\partial^2 \theta}{\partial y^2} \tag{9}$$

The initial and boundary conditions in non-dimensionless form are

When $t \le 0, \quad u = 0, \quad \theta = 0$ for all y

If $t > 0, \quad u = 1, \quad \frac{\partial \theta}{\partial y} = -1$ at $y = 0$

$$u = 0, \quad \theta \to 0 \quad \text{at } y \to \infty \tag{10}$$

3. SOLUTION OF THE PROBLEM:

In order to solve the equations (8) and (9) with boundary conditions (10), we assuming the solutions for equations (8) and (9) as:

$$u(x,t) = u_0(y)e^{i\omega t}$$
$$\theta(x,t) = \theta_0(y)e^{i\omega t} \tag{11}$$

The equations (8) and (9) give

$$u_0'' - \left(M\sin^2\phi + \frac{1}{K} + i\omega \right)u_0 = -Gr\theta_0 \tag{12}$$

$$\theta_0'' - \left(\frac{3i\omega N \Pr}{3N+4} \right)\theta_0 = 0 \tag{13}$$

The corresponding boundary conditions are,

When $t \le 0, \quad u_0 = 0, \quad \theta_0 = 0$ for all y

$$t > 0; \quad u_0 = e^{-i\omega t}, \qquad \frac{d\theta}{dy} = -e^{-i\omega t} \quad \text{at } y = 0$$
$$u_0 = 0, \quad \theta_0 \to 0 \qquad \text{at} \quad y \to \infty \tag{14}$$

On solving equations (12) and (13) and using the boundary conditions (14), we get

$$\theta_0(y) = \frac{e^{-i\omega t}}{m_1}e^{-m_1 y} \tag{15}$$

$$u_0(y) = \frac{-Gre^{-i\omega t}}{R_1}e^{-m_1 y} + \left(e^{-i\omega t} + \frac{Gre^{-i\omega t}}{R_1} \right)e^{-m_2 y} \tag{16}$$

Subustituting the above solutions (15) and (16) in (11), we get the final form of velocity and temperature distribution in the boundary layer as follows:

$$u(y,t) = \frac{-Gr}{R_1}e^{-m_1 y} + e^{-m_2 y} + \frac{Gr}{R_1}e^{-m_2 y} \tag{17}$$

$$\theta(y,t) = \frac{e^{-m_1 y}}{m_1} \tag{18}$$

Flow Rate:

$$Q = \int_0^h u(y,t)dy \tag{19}$$

From equation (19)

$$Q = \frac{Gr}{R_1}\left[\frac{e^{-m_1 h} - 1}{m_1} + \frac{1 - e^{-m_2 h}}{m_2} \right] + \frac{1 - e^{-m_2 h}}{m_2} \tag{20}$$

4. RESULTS AND DISCUSSIONS:

In this section we discuss the effects of various parameters like Hartmann number *M*, thermal Grashof number *Gr*, Prandtl number Pr, permeability of porous medium K, radiation parameter *N*, and aligned magnetic parameter ϕ on velocity, temperature profiles and flow rate and have been shown in graphs. In the present work we have chosen $M = 1$, $Gr = 1$, $N = 1$, $Pr = 0.71$ and $\phi = \dfrac{\pi}{3}$.

Figure 2 represents the dimensionless velocity profiles for different values of magnetic field parameter. Due to the effect of magnetic field, the velocity of the flow field gets decelerated. Figure 3 depicts the influence of Prandtl number over the velocity distribution. As Prandtl number increases the velocity decreases. The influence of radiation parameter over velocity distribution is presented in figure 4. It is observed that the radiation parameter decreases the velocity. Figure 5 shows the effect of thermal buoyancy force parameter Gr on the velocity. We can be seen from this figure, the velocity field increases with increase in the values of thermal buoyancy. Figure 6 shows that the effect of porosity parameter on the velocity profile. It is observed that an increase in the porosity parameter increases the velocity profile. Aligned angle ϕ have reducing effects on the dimensionless velocity as shown in figure 7. However the effect is less significant.

Increasing the Prandtl number reductions the temperature of the flow field and is seen in figure 8. Also it is interesting to see that the thermal boundary layer thickness is reduced due to increase in Prandtl number. Figure 9 plots the temperature distribution for various values of radiation parameter. It is observed that radiation parameter decreases the thermal boundary layer thickness.

The influence of magnetic field with respect to thermal Grashof number (Gr) for different Pore sizes of the fluid bed are illustrated in Figures10 and 11. In general it is observed as the pore size increase the flow rate is found to be increasing. As the magnetic intensity increases the flow rate is found to be decreasing. This can be endorsed to the fact that the magnetic field suppresses the velocity field hence the flow rate is found to be decreasing. Further, it is seen that for a constant value of magnetic and porosity parameters, increase in Grashof number (Gr) contributes to the decrease in the flow rate.

Figure 12 and Figure 13 shows the influence of applied magnetic field with respect to the porosity and the Prandtl number (Pr). In general it is observed that, as the porosity increases the flow rate is also found to be increasing. Increase in the intensity of the magnetic field decreases the flow rate. For a constant porosity and applied magnetic intensity change in the Prandtl number (Pr) influence the flow rate. In all these situations increase in Prandtl number (Pr) contributes to the decrease in the flow rate.

When the pore size of the fluid bed is held constant, the effect of the magnetic field with respect to the frequency of excitation has been shown in Figure 14 and Figure 15. In general, it is seen that as the pore size increases the flow rate is found to be increasing. As the magnetic intensity increases the flow rate is found to be decreasing. Also, it is seen that for a constant value of magnetic and porosity parameters, increase in frequency of excitation (ω) contributes to the decrease in the flow rate.

The effect of thermal Grashoff number (Gr) with respect to the Prandtl number (Pr) on the flow rate has been illustrated in Figure 16and Figure 17. It is observed that as the thermal Grashof number (Gr) increases the flow rate is observed to be decreasing. The magnetic field suppresses the flow rate. Increase in the Prandtl number (Pr) contributes to the decreases in the flow rate.

The effect of the magnetic intensity with respect to the Grashof number (Gr) has been illustrated in Figure 18. It is observed that as the magnetic intensity increases the flow rate is decreased. Further, increase in the Grashof number (Gr) contributes to the decrease in the flow rate.

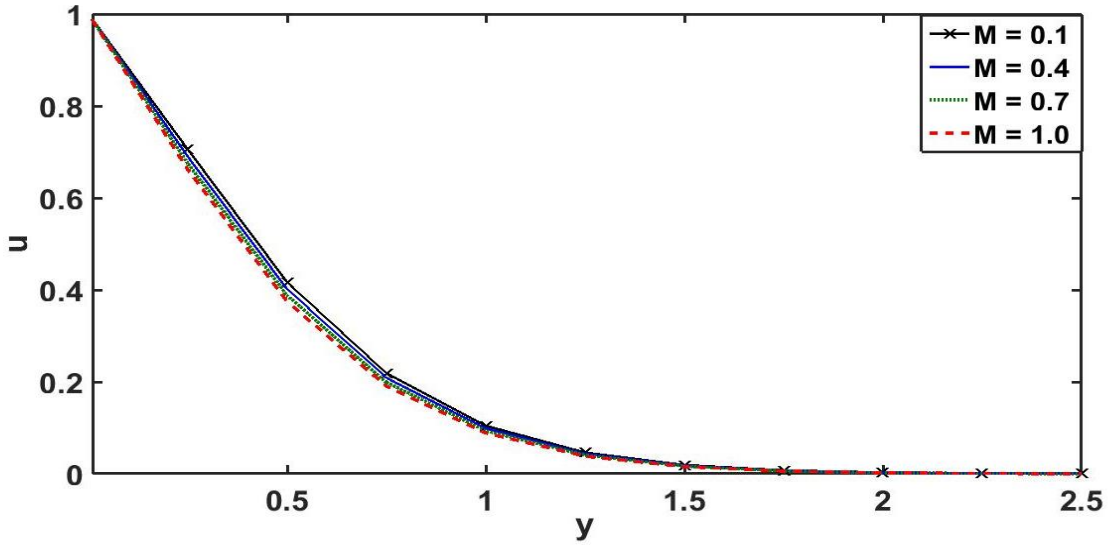

Figure 2: Effect of M for different values on Velocity

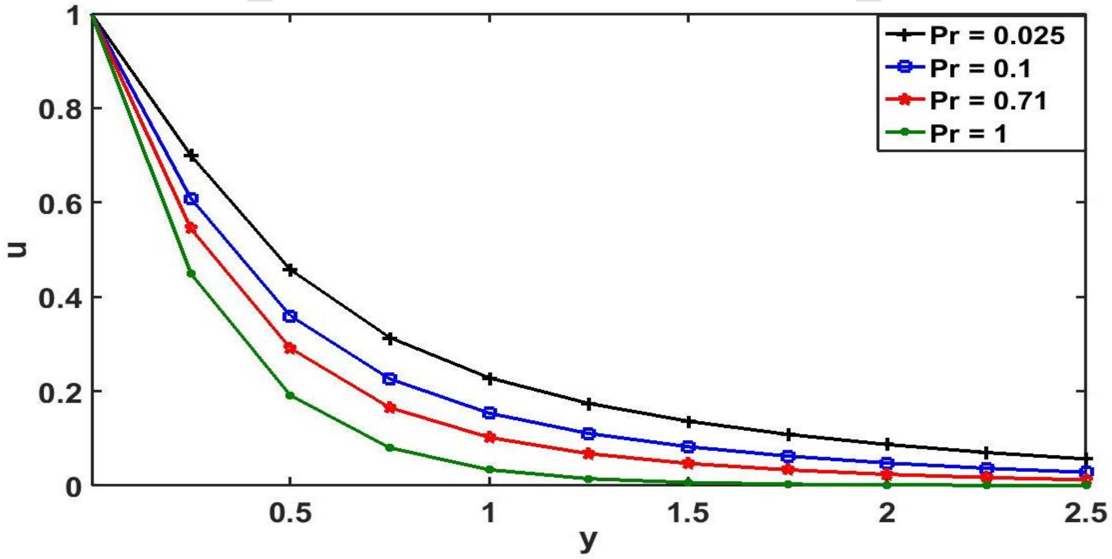

Figure 3: Effect of Pr for different values on Velocity

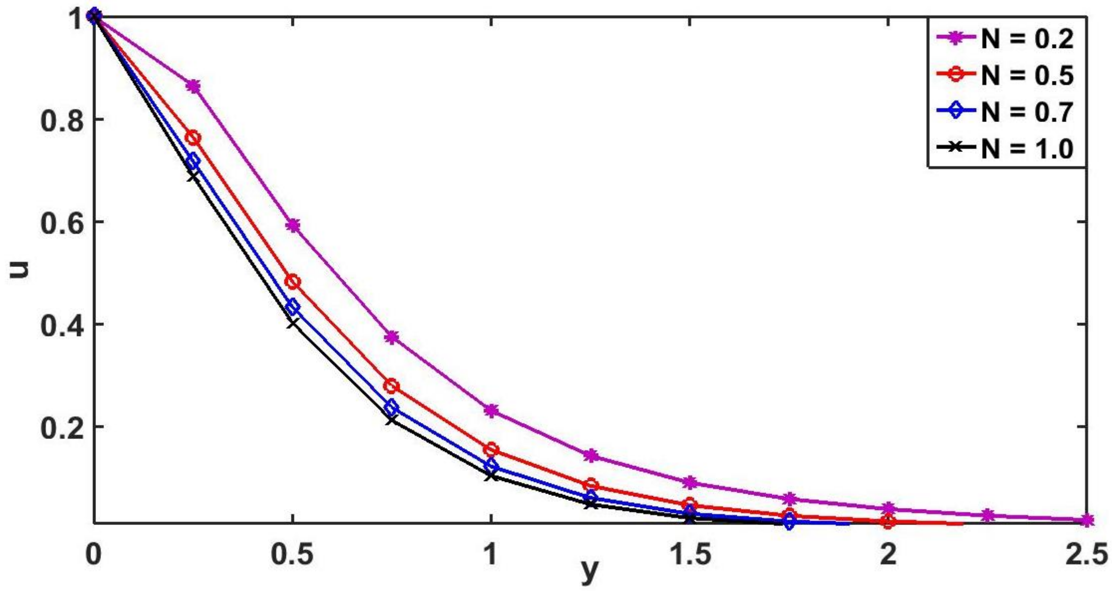

Figure 4: Effect of N for different values on Velocity

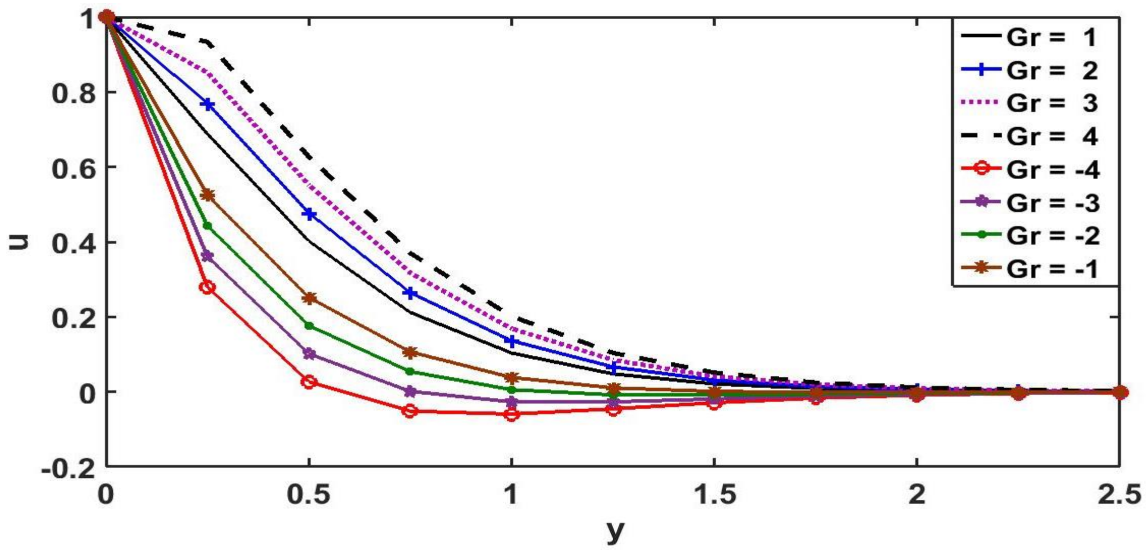

Figure 5: Effect of Gr for different values on Velocity

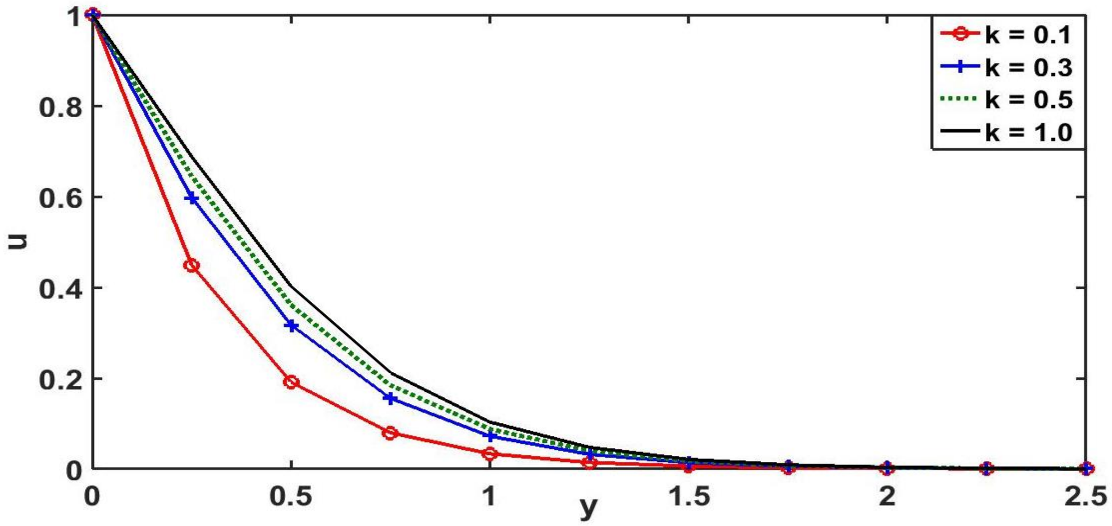

Figure 6: Effect of k for different values on Velocity

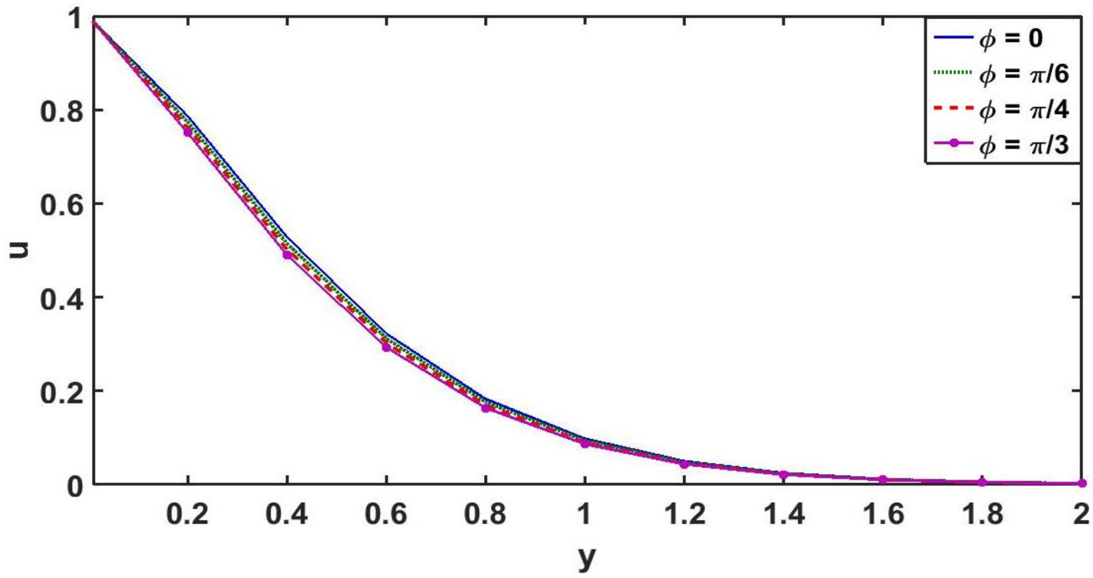

Figure 7: Effect of ϕ for different values on Velocity

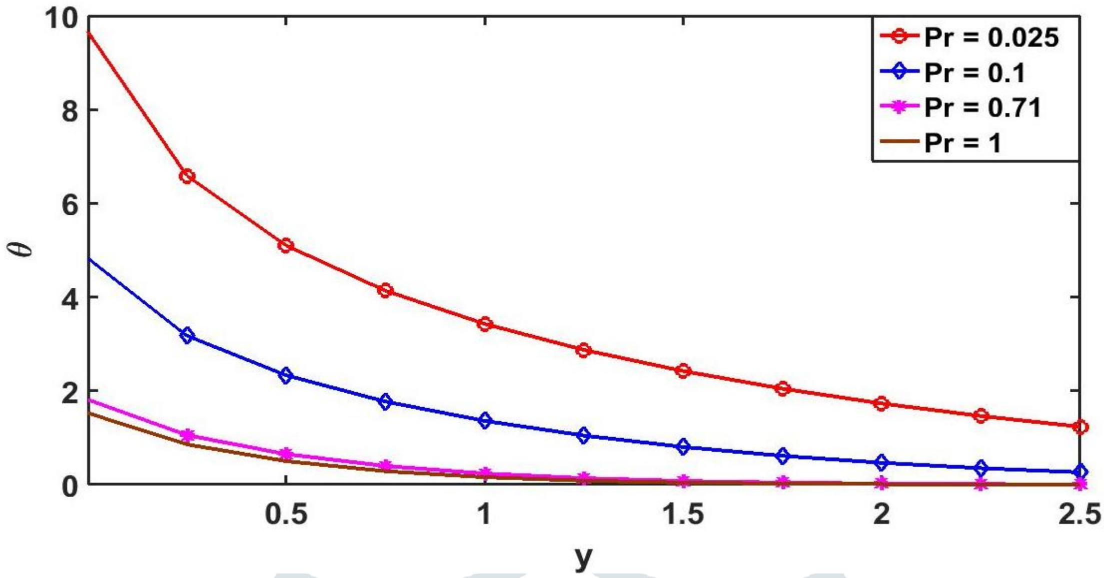

Figure 8: Effect of Pr for different values on Temperature

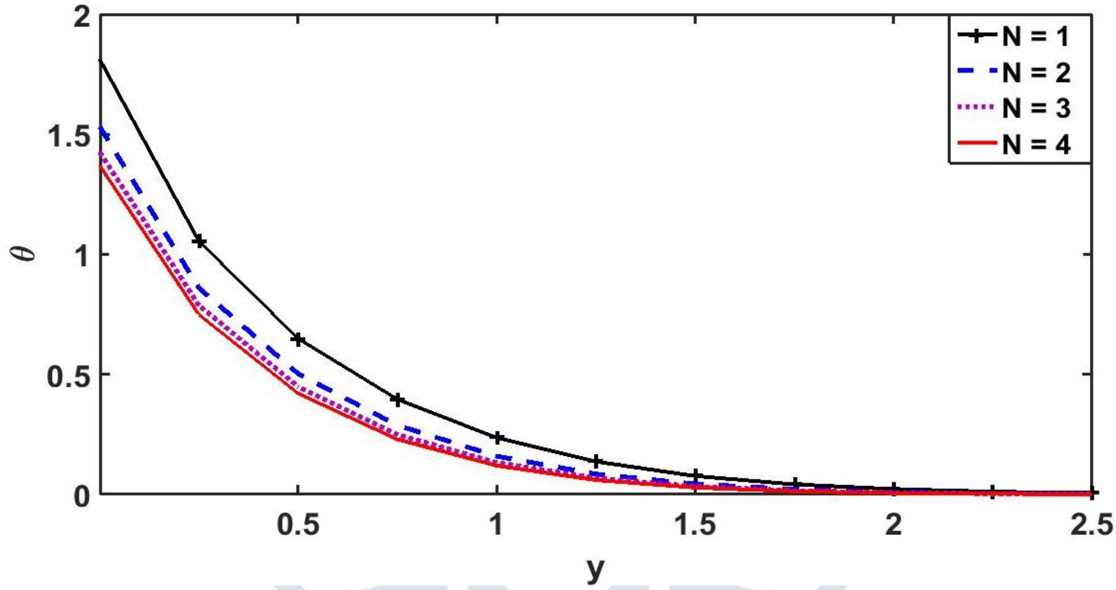

Figure 9: Effect of N for different values on Temperature

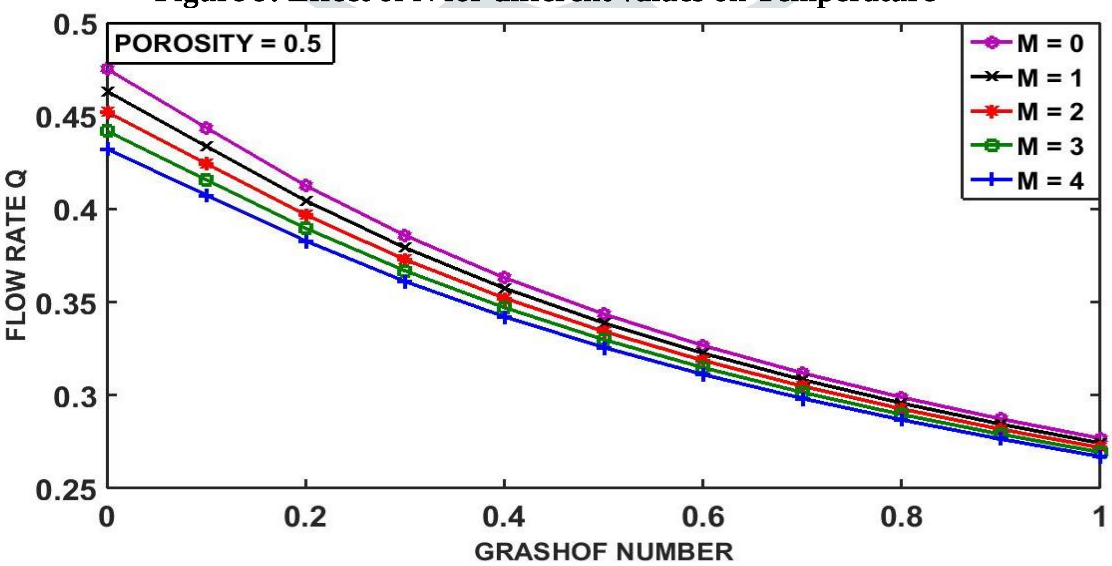

Figure 10: Effect of magnetic field M on flow rate

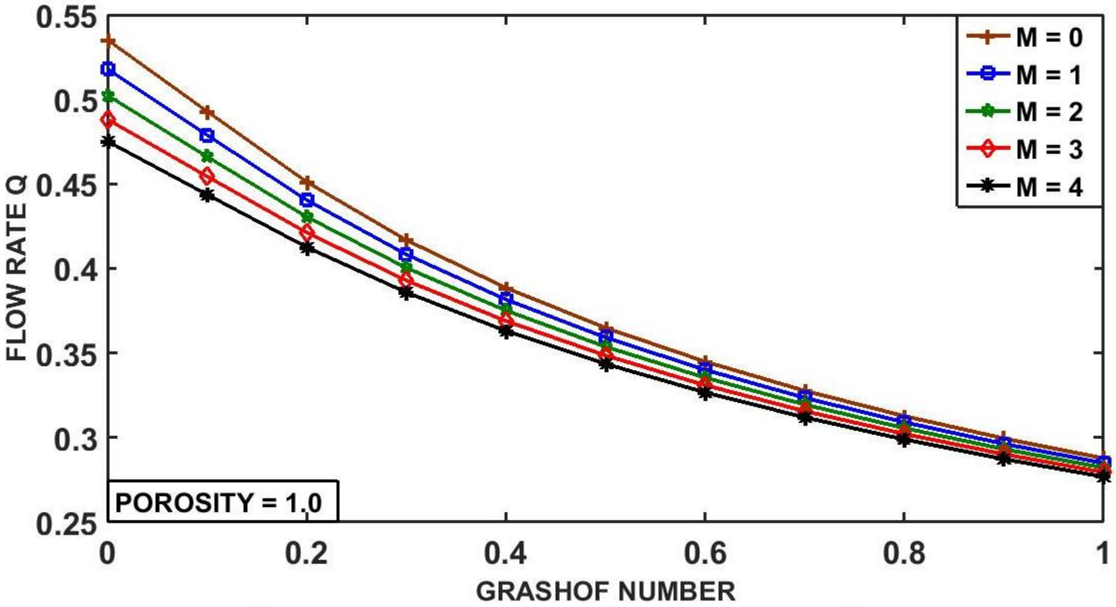

Figure 11: Influence of magnetic field M on flow rate

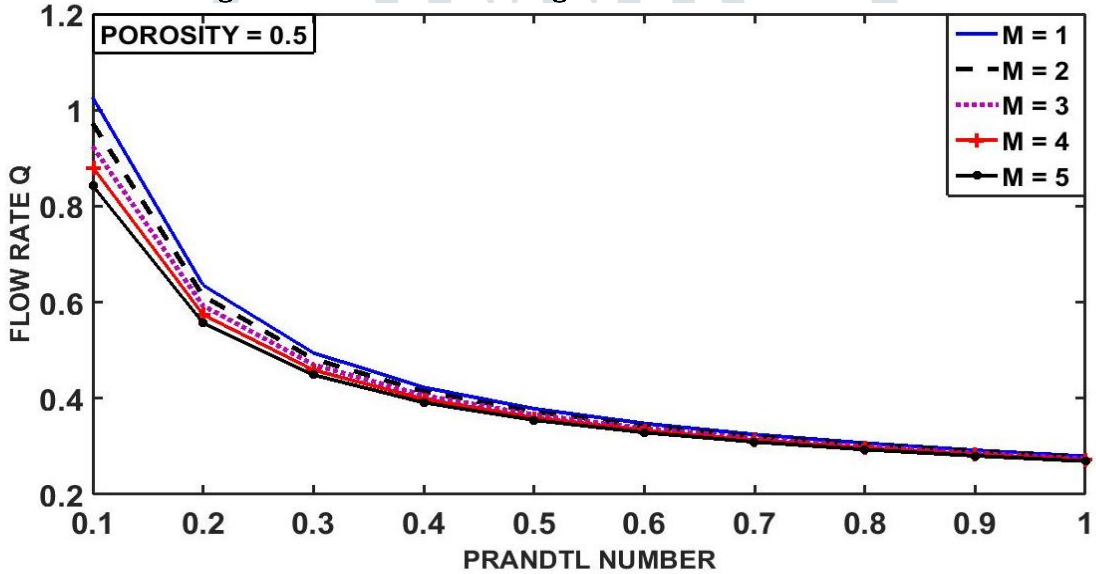

Figure 12: Influence of magnetic field M and Prandtl number Pr on flow rate

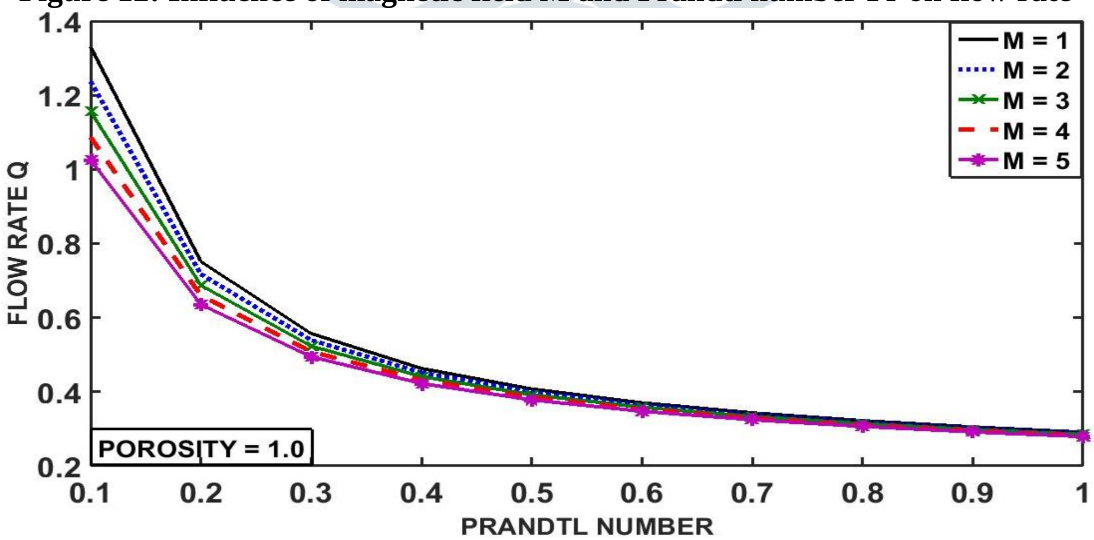

Figure 13: Effect of magnetic field M and Prandtl number Pr on flow rate

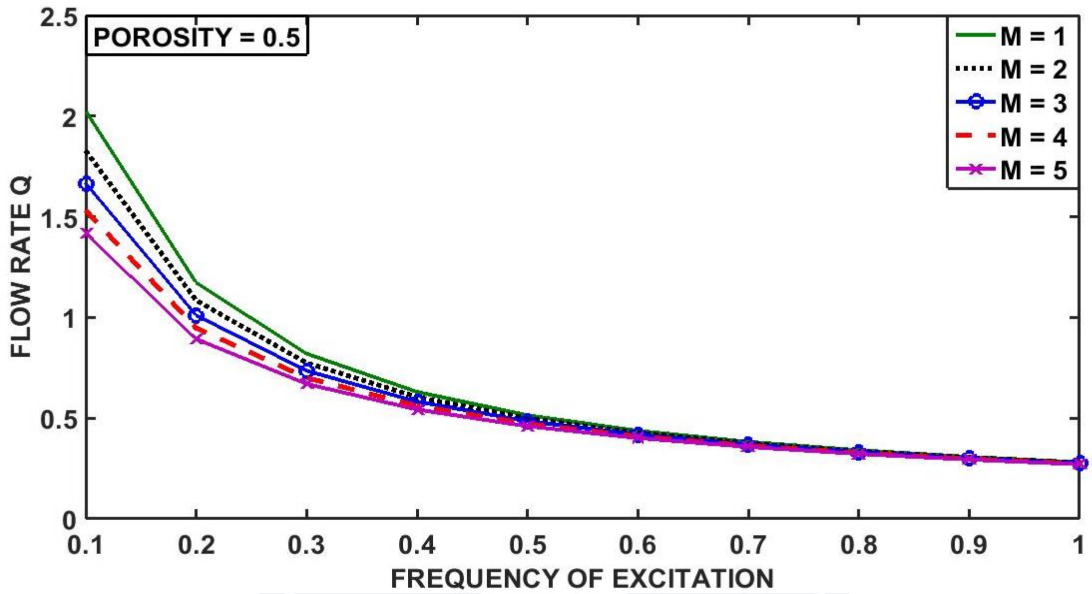

Figure 14: Combined effect of frequency excitation and magnetic field on flow rate

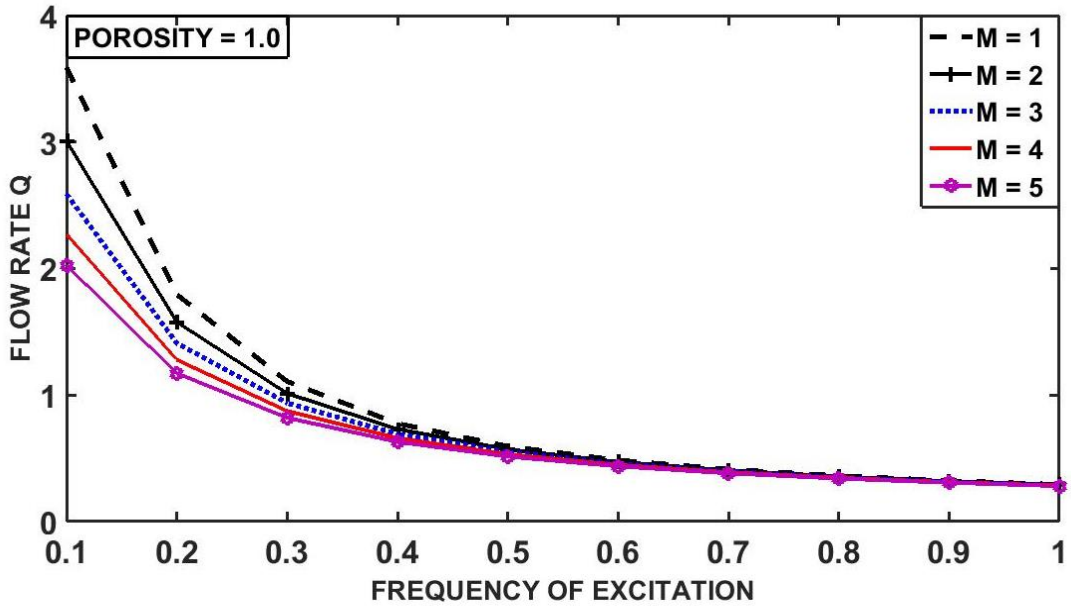

Figure 15: Effect of frequency excitation and magnetic field on flow rate

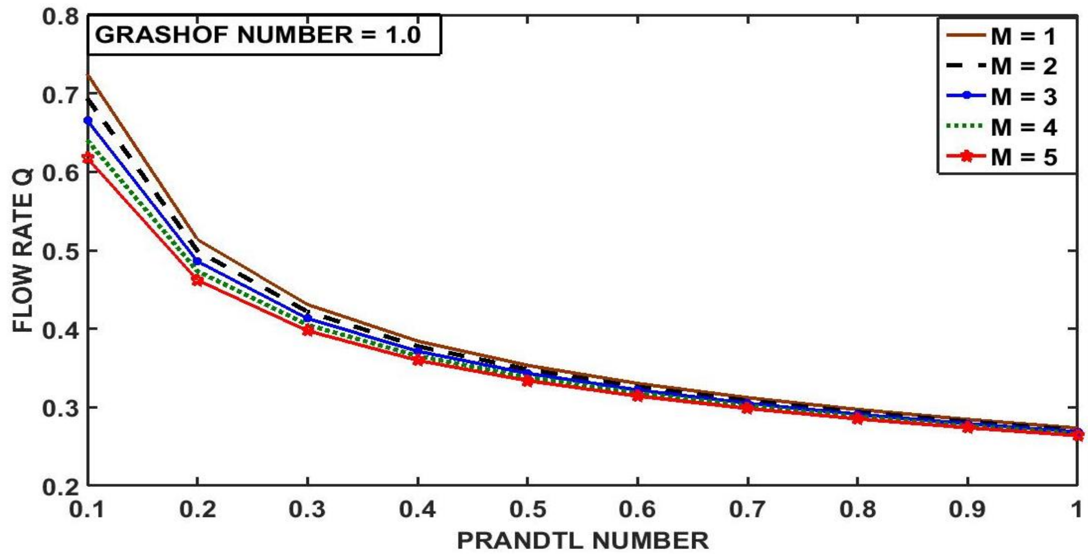

Figure 16: Consolidated effect of Prandtl number and Grashof number on flow rate

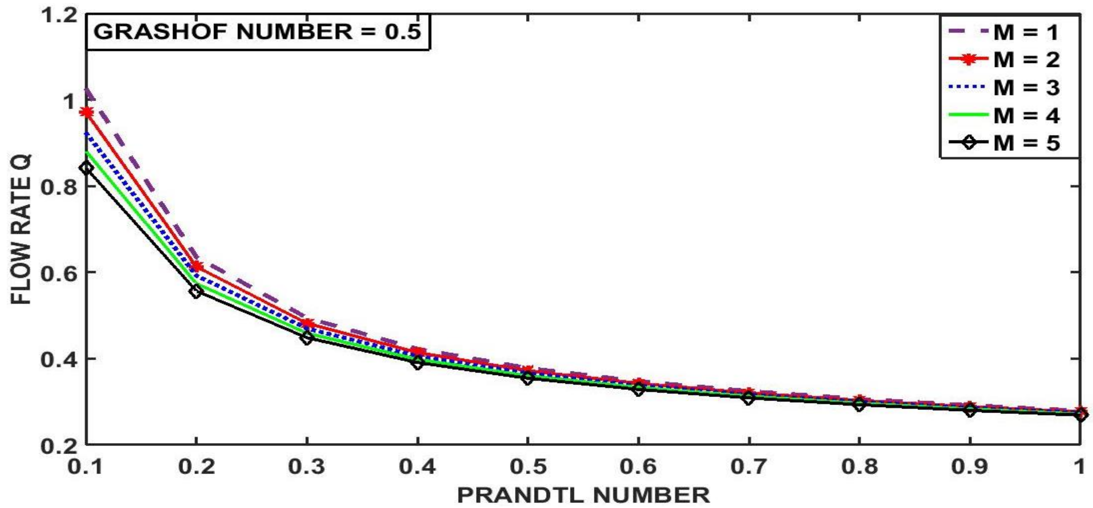

Figure 17: Combined effect of Prandtl number and Grashof number on flow rate

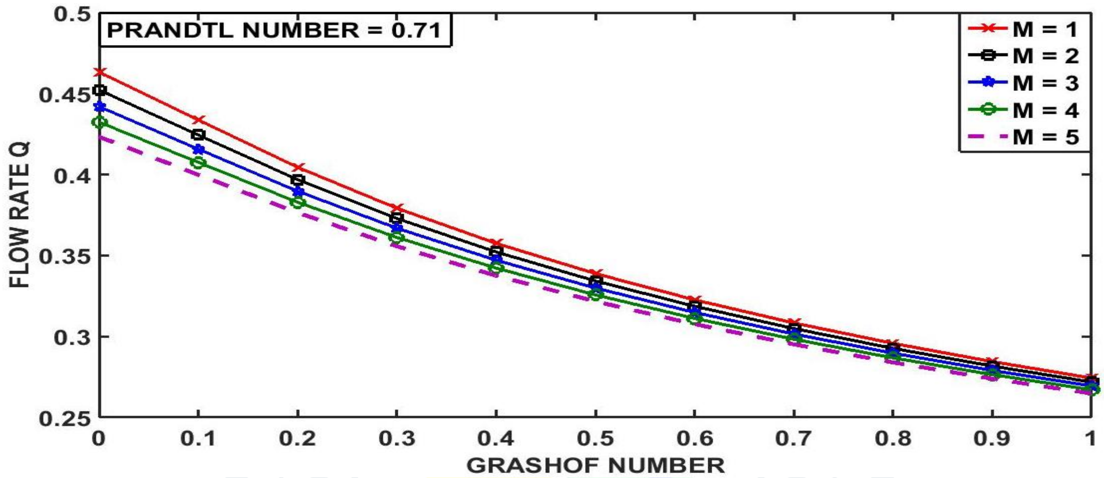

Figure 18: Effect of magnetic field with respect to Grashof number on flow rate

5. CONCLUSIONS:

In this paper we have studied the case of variation of flow rate with respect to various flow entities in situation of viscous, incompressible fluid flow over a moving infinite vertical porous plate in the presence of aligned magnetic, thermal radiation and constant heat flux. From the present investigation the following conclusions is arrived.

- The influence of magnetic field is to decelerate the velocity for its increasing values.
- The Prandtl number has the same effect on velocity and temperature so as to decrease for increase in Prandtl number.
- An increase in thermal radiation parameter, the velocity and temperature profiles decrease.
- The velocity profile decreases as an increase of aligned magnetic parameter.
- The Grashof number and porosity parameter enhances the velocity.
- Increase in Grashof number or magnetic field or Prandtl number or frequency of excitation leads to reduction in the flow rate.
- When the pore size increase the flow rate is found to be increasing.

6. APPENDIX:

$$m_1 = \sqrt{\frac{3N\Pr i\omega}{3N+4}}, \quad m_2 = \sqrt{M + \frac{1}{k} + i\omega}\ , \quad R_1 = m_1(m_1^2 - m_2^2)$$

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