A SPECIAL NEAR-RING STRUCTURE

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Abstract: In this paper we introduce a new type of near-ring. In [1] R.Balakrishnan and S.Silviya defined a right near-ring N to be a B_1 near-ring if for every a ∈ N, there exists x ∈N^* such that Nax = Nxa. Motivated by this, we introduce the concept of β_1 near-rings by defining that N is $\beta_1$if xNy= Nxy for all x, y in N. We discuss the properties of this newly introduced structure. We prove that in a β_1 near-ring with mate functions, the set of all N-subgroups is a Boolean algebra under the usual set inclusion.

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***IndexTerms* - near-ring, ideal, Boolean algebra**.

I. INTRODUCTION

Near-rings are generalized rings. If in a ring (N, +,·) with two binary operations '+' and '·', we ignore the commutativity of '+' and one of the distributive laws, (N, +, ·) becomes a near-ring. If we do not stipulate the left distributive law, (N, +, ·) becomes a right near-ring. Throughout this paper, N stands for a right near-ring (N, +, ·) with at least two elements. Obviously, 0n = 0 for all n in N, where '0' denotes the identity of the group (N, +). As in [4], a subgroup (M, +) of (N, +) is called (i) a left N-subgroup of N if MN⊆M, (ii) an N-subgroup of N if NM⊆M and (iii) an invariant N – subgroup of N if M satisfies both (i) and (ii). Again in [4], a normal subgroup (I, +) of (N, +) is called (i) a left ideal if n(n ′ + i) – nn′ ∈ I for all n, n′ ∈ N and i∈ I (ii) a right ideal if IN ⊆ I and (iii) an ideal if I satisfies both (i) and (ii). An ideal I of N is called (i) a prime ideal if for all ideals J, K of N, JK ⊆ I ⇒ J ⊆I or K ⊆ I. (ii) a completely semiprime ideal if for a ∈ N, a^2∈ I ⇒a ∈ I. (iii) an IFP ideal, if for a, b ∈ N, ab∈ I ⇒ anb∈ I for all n in N. (iv) a semiprime ideal if for all ideals J of N, J^2⊆ I ⇒ J ⊆ I. If {0} is a semiprime ideal, then N is called a semiprime near-ring [2.87, p.67 of Pilz [4]]. The concept of a mate function in N has been introduced in [5] with a view to handling the regularity structure with considerable ease. A map '*f*' from N into N is called a mate function for N if *x = xf(x)x* for all x in N. Also the existence of mate functions is preserved under homomorphisms. By identity 1of N, we mean only the multiplicative identity of N. Basic concepts and terms used but left undefined in this paper can be found in Pilz [4].

II. NOTATIONS

A. E denotes the set of all idempotent of N (e in N is called an idempotent if e^2 = e)
B. L denotes the set of all nilpotent of N (a in N is nilpotent if a^k = 0 for some positive integer k)
C. N_d = {n∈N / n(x+y) = nx + ny for all x, y in N} – set of all distributive elements of N.
D. C(N) = {n∈N / nx = xn for all x in N} – Centre of N.
E. N_0 = {n∈ N / n0 = 0} – zero-symmetric part of N.
F. (0: A) = {n∈ N / nA = {0} } – annihilator of A.

III. PRELIMINARY RESULTS

We freely make use of the following results and designate them as R(1),R(2), ...etc

R(1) N has no non-zero nilpotent elements (i.e) L = {0}if and only if x^2= 0⇒x = 0 for all x in N

R(2) If *f* is a mate function for N, then for every x in N, *xf(x), f(x)x ∈E* and *Nx =Nf(x)x, xN = xf(x)N* (Lemma 3.2 of [5])

R(3) If L={0} and N =N_0 then (i) xy = 0⇒yx = 0 for all x, y in N (ii) N has Insertion of Factors Property– IFP for short– i.e. for x, y in N, xy=0⇒xny=0 for all n in N. If N satisfies (i) and (ii) then N is said to have (∗, IFP) (Lemma 2.3 of [5] &[6])

R(4) N has strong IFP if and only if for all ideals I of N, and for x, y ∈N, xy∈I ⇒ xny∈I for all n∈N. (Proposition 9.2, p.289 of Pilz [4])

R(5) For any n in N, (0 : n) is a left ideal of N (1.43, p.21 of Pilz [4])

R(6) If N is zero-symmetric, then every left ideal is an N-subgroup (Proposition 1.34(b), p.19 of Pilz [4])

R(7) A zero-symmetric near-ring N has IFP if and only if (0: S) is an ideal where S is any non-empty subset of N (Proposition 9.3, p.289 of Pilz [4])

R(8) If L={0} and N =N_0 , and e is an idempotent in N, then for any a,b ∈N, abe = aeb.(Section 2 of Lemma 3 of [3])

IV. Definition 4.1

Let N be a right near-ring. If for every x, y in N, $xNy = Nxy$ then we say N is a β_1 near-ring.
Examples: **(i)** Let $(N, +)$ be the Klein's four group $\{0,a,b,c\}$.The near-ring$(N,+,\cdot)$where '$\cdot$' is defined as per scheme 4, p.408, Pilz [4], which forms a part of Clay [2] is given as follows.

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	c	b
c	0	a	b	c

This near-ring is a β_1 near-ring. It is worth noting that this near- ring does not admit mate functions.
(ii) The near-ring $(N,+,\cdot)$ where $(N,+)$ is the group of integers modulo 5 and '$\cdot$'defined as per scheme 6, p.408, Pilz[4], is given as follows.

.	0	1	2	3	4
0	0	0	0	0	0
1	0	0	4	1	0
2	0	0	3	2	0
3	0	0	2	3	0
4	0	0	1	4	0

Then N is not a β_1 near-ring, since $2N2 \neq N22$
Remark: A β_1 near-ring with identity 1 is zero-symmetric. But the converse is not valid.
For example, Let $(N, +)$ be the group of integers modulo 6. We define '$\cdot$'as per scheme 36, p.409,Pilz [4] as follows.

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	4	2	0	4	2
2	0	2	4	0	2	4
3	0	0	0	0	0	0
4	0	4	2	0	4	2
5	0	2	4	0	2	4

This near-ring $(N, +,\cdot)$ is a zero-symmetric β_1 near-ring with no identity.

Proposition 4.2: If N is a β_1 near-ring, then $xNx= Nx^2$ for all x in N.
Proof : When N is a β_1 near-ring, by definition, for all x, y in N, $xNy= Nxy$ (4.2.1)
The desired result follows by replacing y by x in Equation (4.2.1)

Remark 4.3: The converse of proposition 4.2 is not true.

For example, the near-ring (N,+,·) where (N, +) is the Klein's four group {0, a, b, c} and '·' defined as per scheme 8, p.408, Pilz[4] is as follows.

.	0	a	b	c
0	0	0	0	0
a	0	0	0	a
b	0	a	b	b
c	0	a	b	c

This near-ring satisfies the condition $xNx= Nx^2$ for all x in N. But it is not a β_1 near-ring.

Proposition 4.4 : Every zero-symmetric β_1 near-ring has strong *IFP*.

Proof: Let N be a β_1 near-ring.Then

$$xNy= Nxy \text{ for all } x, y \text{ in } N \quad (4.4.1)$$

Let I be an ideal of N.Since N is zero-symmetric

$$NI \subseteq I \quad (4.4.2)$$

Let $ab \in I$. Now, for any $n \in N$, $anb \in aN\,b = Nab$ [by Equation 4.4.1] $\subseteq NI \subseteq I$ [by Equation 4.4.2].

(*i.e*) $anb \in I$.

Now, R(4) guarantees that N has strong *IFP*.

Theorem 4.5: Let N be a zero-symmetric β_1 near-ring with a mate function f. Then we have,

(i) Every N-subgroup of N is an ideal.

(ii) $(0 : x) = (0 : x^2)$ for every x in N.

(iii) $N = (0: x) \oplus Nx$ where $(0 : x)$ and Nx are ideals of N.

(iv) $(0 : x) = eN$ where e is an idempotent and $1 \in N$.

Proof:

(i) Since N is a β_1 near-ring, we have by Proposition 4.2

$$xNx=Nx^2 \quad (4.5.1)$$

Let $x \in N$. Since f is a mate function for N, $x = xf(x)x$. Let $f(x)x = e \in E$ [by R(2)] and

$$Ne=Nx \quad \text{[by R(2)]} \quad (4.5.2)$$

Let $S=\{n-ne/n\in N\}$.We claim that $(0:S)=Ne$. Since $(n-ne)e=0$ for all $n\in N$,$(n-ne)Ne=\{0\}$[by R(3))] which implies $(n-ne)Nx =\{0\}$[by Equation (4.5.2)] Consequently,

$$Nx\subseteq(0 :S) \quad (4.5.3)$$

For the reverse inclusion, Let $z \in (0 :S)$. Then since f is a mate function for N,

$z = zf(z)z \in zNz= Nz^2$ [by Equation 4.5.1].

Then $z = yz^2$ for some $y\in N$ (4.5.4)

Now, $yz\in N$ implies $yz - yze \in S$. Since $z\in(0:S)$, $z(yz- yze) = 0$.

By R(3) , $(yz- yze)z = 0$. This implies that $yz^2 - yzez= 0$ and $yz^2 - yz^2e = 0$. [by R(8)]

Therefore, by Equation (4.5.4) $z - ze= 0$. Hence $z = ze \in N\,e$ and $z \in Nx$[by Equation 4.5.2]

It follows that $(0 :S)\subseteq Nx$ (4.5.5)

Combining Equations (4.5.3) and (4.5.5), we get $(0 :S) = Nx$. Using R(7) , we get Nx is an ideal of N. Now, if M is any N-subgroup of N, then $M=\sum Nx$ for $x \in M$.Thus M becomes an ideal of N.

(ii) Let $x \in N$ and $y \in (0 :x)$ for some y in N. Then $yx \in N$. Now, $yx^2 = yx.x = 0.x = 0$.

This implies $y \in (0 :x^2)$,

Therefore, $(0:x) \subseteq (0:x^2)$ (4.5.6)

On the other hand, let $u \in (0:x^2)$. Then $ux^2 = 0$

Now, $(xux)^2 = (xux)(xux) = x(ux^2)ux = x.0.ux = 0$ [since $N = N_0$].

Now, since L={0} , by R(1) ,we have $xux = 0$.

Also, $(ux)^2 = (ux)(ux) = u(xux) = u.0 = 0$ [since $N = N_0$], Again L={0}implies $ux = 0$. Therefore, $u \in (0:x)$.

Thus, $(0:x^2) \subseteq (0:x)$ (4.5.7)

Combining Equations (4.5.6) and (4.5.7) we get the desired result.

(iii) Since f is a mate function for N, we have, $x \in Nx^2$ for all x in N. Then $x = n^1x^2$ for some n^1in N.

This implies $nx = nn^1x^2$ for all n in N. (*i.e.*), $nx = n_1x^2$ where $nn^1 = n_1$ and hence $(n-n_1x)x = 0$.

Therefore, $n - n_1x \in (0:x)$. Since $n = (n - n_1x) + n_1x$, we have $N = (0:x) + Nx$.

Next we claim that $(0:x) \cap Nx = \{0\}$.

Let $0 \neq y \in (0:x) \cap Nx$. Then $yx = 0$ and $y = zx$ for some $z \in N$. Now, $zx^2 = zx.x = yx = 0$.

Therefore, $z \in (0:x^2) = (0:x)$. This implies $zx = 0$ (*i.e.*), $y = 0$.

Thus$(0:x) \cap Nx = \{0\}$. Also, $(0:x)$ is an ideal of N. [by R(7)]. And Nx is an ideal of N [by (i)].

Consequently, (iii)follows.

(iv) We have, $N = (0:x) \oplus Nx$for all x in N. [by (iii)]. Then there exist some $y \in (0:x)$ and $z \in Nx$ such that

$$1 = y+z \quad (4.5.8)$$

By (iii), $(0:x)$ and Nx are ideals of N, it follows that $yz, zy \in (0:x) \cap Nx = \{0\}$.Hence $yz=0$ and $zy=0$.

Now, $y = 1.y = (y+z)y$[by Equation (4.5.8)] $= y^2 + zy = y^2$.

And $z = 1.z = (y+z)z$ [by Equation 4.5.8)] $= yz + z^2 = z^2$. Therefore, y and z are idempotent. By (ii) we get,

$$yN \subseteq (0:x) \quad (4.5.9)$$

For the reverse inclusion, let $u \in (0:x)$. Then $ux = 0$.Now,

$u = 1.u = (y+z)u$[by Equation (4.5.8)] $= yu$. Therefore, $u \in yN$. Thus

$$(0:x) \subseteq yN \quad (4.5.10)$$

From Equations(4.5.9)and (4.5.10),we get $(0:x) = yN$ where y is an idempotent.

Theorem 4.6: Let N be a zero-symmetric β_1 near-ring with mate functions and P be a proper ideal of N. Then the following are equivalent.

(i) P is a prime ideal. (ii) P is a completely prime ideal. (iii) P is a maximal ideal.

Proof. (i) ⇒ (ii): Let $xy \in P$, $NxNy = Nxy \subseteq NP \subseteq P$. [by R(6)]

By Theorem 4.5(i), Nx and Ny are ideals in N. Since P is prime,

$NxNy \subseteq P$ implies $Nx \subseteq P$ or $Ny \subseteq P$.

Since f is a mate function for N, for all x, y in N,

$$x = xf(x)x \in Nx \subseteq P \text{ and } y = yf(y)y \in Ny \subseteq P.$$

Therefore either $x \in P$ or $y \in P$.

Hence (ii)follows.

(ii) ⇒ (i) is obvious.

(i) ⇒ (iii): Let J be an ideal of N such that $J \neq P$ and that $P \subseteq J \subseteq N$. Let $x \in J - P$. Since f is a mate function for N, for any x in N, $x = x(f(x)x) = f(x)xx$ .Thus for all n in N, $nx = nf(x)x^2$ and this implies $(n - nf(x)x)x = 0$.

Since N has $(*, IFP)$, we get $(n - nf(x)x)zx = 0$ and $z(n - nf(x)x)zx = z.0 = 0$. [since$N = N_0$] for all $z \in N$.

Consequently, $N(n - nf(x)x)Nx = N0 = \{0\}$ [since $N = N_0$] If $y = n - nf(x)x$, then $NyNx = \{0\} \subseteq P$.

Since P is a prime ideal and Nx, $Ny$are ideals in N.[by Theorem 4.5(i)], $Nx \subseteq P$ or $Ny \subseteq P$

If $Nx \subseteq P$, then $x = xf(x)x \in Nx \subseteq P$. Therefore, $x \in P$ which is a contradiction. Hence $Ny \subseteq P$. Then $Ny \subseteq J$ and this demands that for all y in N, $y = yf(y)y \in Ny \subseteq J$. Therefore $y \in J$. (*i.e.*), $n - nf(x)x \in J$.

Now, since $x \in J$, $nf(x)x \in NJ \subseteq J$ and therefore $n \in J$. Hence $J = N$ and (iii) follows.

(iii) ⇒ (i) is obvious.

The following Lemma is required to prove the main theorem of this paper.

Lemma 4.7: Let N be an abelian near-ring and let $E \subseteq C(N)$. If $e_1, e_2 \in E$, then $Ne_1 + Ne_2 = Ne$ where

$e = e_1 + e_2 - e_1e_2 \in E$.

Proof: Let $e = e_1 + e_2 - e_1e_2$ where $e_1, e_2 \in E$.

$= e_1^2 + e_2^2 + e_1^2e_2^2 + 2e_1e_2 - 2e_2e_1e_2 - 2e_1e_1e_2$

$= e_1 + e_2 + e_1e_2 + 2e_1e_2 - 2e_1e_2 - 2e_1e_2$ [since $E \subseteq C(N)$]

$= e_1 + e_2 - e_1e_2 = e$

Thus $e \in E$.

Let $n_1e_1 + n_2e_2 \in Ne_1 + Ne_2$ for all $n_1, n_2 \in N$.

Then $(n_1e_1 + n_2e_2)e = n_1e_1e + n_2e_2e$

$= n_1e_1(e_1 + e_2 - e_1e_2) + n_2e_2(e_1 + e_2 - e_1e_2)$

$= n_1(e_1^2 + e_1e_2 - e_1^2e_2) + n_2(e_2e_1 + e_2^2 - e_2e_1e_2)$ [since $E \subseteq C(N) \Rightarrow E \subseteq N_d$]

$= n_1(e_1 + e_1e_2 - e_1e_2) + n_2(e_2e_1 + e_2 - e_2e_1)$[since $E \subseteq C(N)$]

$= n_1e_1 + n_2e_2$ [since $(N, +)$ is abelian] Therefore, $n_1e_1 + n_2e_2 = (n_1e_1 + n_2e_2)e \in Ne$

This implies that $Ne_1 + Ne_2 \subseteq Ne$ (4.7.1)

For any $n \in N, ne = en = (e_1 + e_2 - e_1e_2)n$

$= e_1n + e_2n - e_1e_2n$

$= ne_1 + ne_2 - ne_1e_2$ [since $E \subseteq C(N)$]

$= ne_1 + (n - ne_1)e_2$

$\in Ne_1 + Ne_2$

Therefore, $Ne \subseteq Ne_1 + Ne_2$ (4.7.2)

From Equations (4.7.1) and (4.7.2), we get $Ne_1 + Ne_2 = Ne$.

As an immediate consequence of Lemma 4.7 we have the following.

Theorem 4.8 : Let N bea zero-symmetric β_1 near-ring with a mate function f. Then for every $x, y \in N$, there exists some $z \in N$ such that $Nx + Ny = Nz$.

Proof: Let Nx and Ny be any principal N-subgroups of N.

We need to prove $Nx + Ny = Nz$ for some z in N.

Now, $Nx + Ny = Nf(x)x + Nf(y)y$ [by[5]] $= Ne_1 + Ne_2$ where $e_1 = f(x)x$ and $e_2 = f(y)y$. [Since $E \subseteq C(N)$ and by Theorem 8.11, p.252,Pilz[4]], it follows that $(N, +)$ is abelian.

Hence by Lemma 4.7, $Ne_1 + Ne_2 = Nz$ where $z = e_1 + e_2 - e_1e_2 \in E$.

Thus $Nx + Ny = Nz$.

We now furnish below the main theorem of this paper.

Theorem 4.9: Let N be a zero-symmetric β_1 near-ring with a mate function f. Then the set $\mathfrak{I}$ of all N-subgroups is a Boolean Algebra under the usual set inclusion.

Proof: Let $\mathfrak{I} = \{Nx / x \in N\}$. By Theorem 8.11, p.252,Pilz[4], N is an abelian near-ring.

Also $E \in C(N)$. Again $Nx \cap Ny = Nxy$ and by Theorem 4.8, $Nx + Ny = Nz$ for some z in N.

Hence $\mathfrak{I}$ is a lattice under the usual set inclusion.

For every $x, y, z \in N$, since f is a mate function for N, $x = xf(x)x$, $y = y(y)y$ and $z = zf(z)z$ for some $f(x), f(y), f(z) \in N$ and $f(x)x$, $f(y)y$, $f(z)z \in E$. We also observe that $Nx = Nf(x)x$, $Ny = Nf(y)y$, $Nz = Nf(z)z$ [by R(2)] Further,

$(f(x)xf(y)y)^2 = (f(x)xf(y)y)(f(x)xf(y)y)$

$= f(x)x(f(y)yf(x)x)f(y)y$

$= f(x)x(f(x)xf(y)y)f(y)y$ [since $E \subseteq C(N)$]

$= (f(x)x)^2(f(y)y)^2$

$= (f(x)x)(f(y)y)$

Hence $f(x)x)(f(y)y \in E$

Similarly, $f(x)x)(f(z)z \in E$

To prove $\mathfrak{I}$ is a distributive lattice. For all Nx, Ny, Nz in $\mathfrak{I}$, we have

$Nx \cap (Ny + Nz) = Nf(x)x \cap (Nf(y)y + Nf(z)z)$

$= Nf(x)x \cap N(f(y)y + f(z)z - f(y)yf(z)z)$ [by Lemma 4.7]

$= Nf(x)x(f(y)y + f(z)z - f(y)yf(z)z)$

$= N(f(x)xf(y)y + f(x)xf(z)z - f(x)xf(y)yf(z)z)$ [since $E \subseteq C(N) \Rightarrow E \subseteq N_d$]

$= N(f(x)xf(y)y + f(x)xf(z)z - (f(x)x)^2 f(y)yf(z)z)$ [since $f(x)x \in E$]

$= N(f(x)xf(y)y + f(x)xf(z)z - f(x)xf(y)yf(x)xf(z)z)$ [since $E \subseteq C(N)$]

$= Nf(x)xf(y)y + Nf(x)xf(z)z$ [by Lemma 4.7]

$= Nf(x)x \cap Nf(y)y + Nf(x)x \cap Nf(z)z = (Nx \cap Ny) + (Nx \cap Nz)$. Hence $\mathfrak{I}$ is a distributive lattice.

We shall prove that if $Nx \subseteq Ny \subseteq Nz$, then there exists some $w \in N$ such that $Ny \cap Nw = Nx$ and $Ny + Nw = Nz$.

Now, $Nx \subseteq Ny \subseteq Nz$ implies $Nf(x)x \subseteq Nf(y)y \subseteq Nf(z)z$.

Then as $f(x)x \in Nf(x)x \subseteq Nf(y)y \subseteq Nf(z)z$, there exists $n_1, n_2 \in N$ such that $f(x)x = n_1 f(y)y = n_2 f(z)z$.

Hence $f(x)xf(y)y = (n_1 f(y)y)f(y)y = n_1 f(y)y$ [since $f(y)y \in E$] $= f(x)x$

Similarly $f(x)xf(z)z = f(x)x$

Now, $f(y)yf(x)x = f(y)y(n_1 f(y)y) = n_1 f(y)y$ [since $f(y)y \in E$ and $E \subseteq C(N)$] $= f(x)x$

Similarly $f(z)zf(x)x = f(x)x$

Collecting all these pieces we get,

$$f(x)xf(y)y = f(y)yf(x)x = f(x)xf(z)z = f(z)zf(x)x = f(x)x \quad (4.9.1)$$

Similarly, since $f(y)y \in N$, $Nf(y)y \subseteq Nf(z)z$, there exists $n_3 \in N$ such that $f(y)y = n_3 f(z)$

Hence $f(y)yf(z)z = (n_3 f(z)z)f(z)z = n_3 f(z)z$ [since $f(z)z \in E$] $= f(y)y$

And $f(z)zf(y)y = f(z)z(n_3 f(z)z) = n_3 f(z)z$ [since $f(z)z \in E$ and $E \subseteq C(N)$] $= f(y)y$

Therefore, $f(y)yf(z)z = f(z)zf(y)y = f(y)y$ (4.9.2)

Let $w = f(x)x + f(z)z - f(y)y$ (4.9.3)

Now, $w^2 = (f(x)x + f(z)z - f(y)y)^2$

$= (f(x)x + f(z)z - f(y)y)\,(f(x)x + f(z)z - f(y)y)$

$= f(x)x(f(x)x + f(z)z - f(y)y) + f(z)z(f(x)x + f(z)z - f(y)y) - f(y)y(f(x)x + f(z)z - f(y)y)$

$= (f(x)x + f(z)z - f(y)y)\, f(x)x + (f(x)x + f(z)z - f(y)y)\, f(z)z - (f(x)x + f(z)z - f(y)y)\, f(y)y$ [since $E \subseteq C(N)$]

$= f(x)xf(x)x + f(z)zf(x)x - f(y)yf(x)x + f(x)xf(z)z + f(z)zf(z)z - f(y)yf(z)z - f(x)xf(y)y + f(z)zf(y)y - f(y)yf(y)y$.

$= f(x)x + f(x)x - f(x)x + f(x)x + f(z)z - f(y)y - f(x)x + f(y)y - f(y)y$ [by Equations (4.9.1) and (4.9.2)]

$= f(x)x + f(z)z - f(y)y = w$

Hence $w \in E$.

And $f(y)yw = wf(y)y$ [since $E \subseteq C(N)$]

$= (f(x)x + f(z)z - f(y)y)f(y)y$

$= f(x)xf(y)y + f(z)zf(y)y - f(y)yf(y)y$

$= f(x)x + f(y)y - f(y)y$

$= f(x)x$ [since $(N,+)$ is abelian]

(*i.e.*) $f(y)yw = f(x)x$ (4.9.4)

Collecting all these results, we get, $Nf(y)y \cap Nw = Nf(y)yw = Nf(x)x$. [by Equation 4.9.4] (*i.e.*) $Ny \cap Nw = Nx$

Further, $Nf(y)y + Nw = N(f(y)y + w - f(y)yw)$ [by Theorem 4.8]

$= N(f(y)y + w - f(x)x)$ [by Equation (4.9.4)]

$= Nf(z)z$ [by Equation 4.9.3] (*i.e.*) $Ny + Nw = Nz$.

Collecting all the pieces proved so far, $\mathfrak{I}$ is a Boolean algebra.

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