# UNSTEADY MHD FLOW OF TWO IMMISCIBLE VISCO-ELASTIC FLUIDS OF HIGHER ORDER WITH PERIODIC PRESSURE GRADIENT

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## ABSTRACT

In this paper an attempt has been made to study the flow of two immiscible visco-elastic fluids of higher order bounded by a rectilinear pipe of uniform cross section in presence of transverse magnetic field under periodic pressure gradient. Towards solving the problem variable separation technique has been applied. The analytical solution of the problem has been utilised to find out the solution of the corresponding problems in the cases of visco-elastic fluids: (i) Maxwell fluids of first and second order, (ii) Oldroyd fluids of first and second order, (iii) Rivlin-Ericksen fluids of first and second order, (iv) Walters fluids and finally in case of ordinary viscous fluids also. Numerical computation of the velocity profiles have also been derived in the investigation.

## 1.INTRODUCTION

Due to the consideration of other fluids available in polymer science and polymeric liquids we have marked that non-Newtonian fluids, micropolar fluids, conducting fluids in presence of magnetic fields are gradually getting more appreciation from theoretical and practical stand points. Several hydrodynamic problems can be found in the monographs of Cowling[1], Ferraro-Plumpton[2], Cabannes[3] and Jeffrey[4]. In the area of non-Newtonian fluids the works of Bhatnagar[5] and Joseph[6] are also very worthy to mention. Bagchi[7] studied the flow of two immiscible visco-elastic Maxwell fluids under transient pressure gradient. The flow of an incompressible viscous fluid in a long rectangular channel due to a pressure gradient was studied by Drake[8]. The problem of unsteady flow of two immiscible visco-elastic fluids under a certain pressure gradient between two fixed plates was studied by Kapur and Shukla[9]. Sengupta and Raymahapatra[10] investigated the flow of two immiscible visco-elastic Maxwell fluids with transient pressure gradient through a rectangular tube. Chakraborty and Sengupta[11] studied the hydromagnetic flow of two immiscible visco-elastic Walter conducting liquid between two inclined parallel plates. Sengupta and Kundu[12] investigated the unsteady MHD flow of visco-elastic Oldroydian fluids with periodic pressure gradient in a porous rectangular duct.

Following the above investigations and methods the authors of the present paper have investigated the unsteady MHD flow of two immiscible visco-elastic fluids of higher order under periodic pressure gradient.

## 2. GENERAL MODEL OF VISCO ELASTIC FLUIDS

A new general model of visco-elastic fluid has been suggested by P.R. Sengupta in the following form:

$$\left.\begin{aligned}
&\tau_{ij} = -p\delta_{ij} + \tau'_{ij} \\
&(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j})\tau'_{ij} = 2\mu(1+\mu_j \frac{\partial^j}{\partial t^j})e_{ij} \\
&e_{ij} = \frac{1}{2}(v_{ij}+v_{ji})
\end{aligned}\right\} \quad \ldots(1)$$

Where $\tau_{ij}$ is the stress tensor, $\tau'_{ij}$ is the deviatoric stress tensor, $e_{ij}$ is the rate of strain tensor, p is the fluid pressure, $\lambda_j$ are new material constants of which the greatest value $\lambda_1$ represents the relaxation time parameter and $\lambda_2, \lambda_3, \ldots\ldots, \lambda_n$ are additional material constants; $\mu_j$ are also new material constants of which the greatest value $\mu_1$ represents the strain rate retardation time parameter and $\mu_2, \mu_3, \ldots\ldots, \mu_n$ are additional material constants representing the behaviour of a very wide class of visco-elastic fluids, $\delta_{ij}$ is the metric tensor in Cartesian co-ordinates and $\mu$ the co-efficient of viscosity and $v_i$ are the velocity components. The material constants $\lambda_j$ and $\mu_j$ designating visco-elasticity satisfy the following conditions $\lambda_1 > \lambda_2 > \ldots\ldots > \lambda_n$ and $\mu_1 > \mu_2 > \ldots\ldots > \mu_n$ i.e. they are arranged in descending order of magnitudes.
Now from above:

$$\tau'_{ij} = 2\mu\left[\frac{1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j}}{1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j}}\right] \text{ and } v^* = \frac{\mu^*}{\rho} = \frac{\mu}{\rho}\left[\frac{1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j}}{1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j}}\right] = v\left[\frac{1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j}}{1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j}}\right]$$

The fundamental Navier-Stokes equation of motion is:

$$\frac{dq}{dt} = -\frac{1}{\rho}\vec{\nabla}p + v^*\nabla^2\vec{q} + \vec{F} \quad \ldots(2)$$

Where $\vec{q}$ is velocity of the fluid, ρ is the density, p is the pressure, $\vec{F}$ is the body force vector and $v^* = \frac{\mu^*}{\rho}$, then (2) takes the form:

$$\frac{dq}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \frac{\mu}{\rho}\left[\frac{1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j}}{1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j}}\right]\nabla^2\vec{q} + \vec{F}$$

$$i.e.\,(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j})\frac{dq}{dt} = -\frac{1}{\rho}(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j})\vec{\nabla}p + v(1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j})\nabla^2\vec{q} + (1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j})\vec{F}$$

Where $v = \frac{\mu}{\rho}$ is the kinematical co-efficient of viscosity.

## 3. MATHEMATICAL FORMULATION

With reference to rectangular Cartesian co-ordinate system we consider the boundary of the walls of the channel as $x = \pm a$ and $y = \pm b$. The z-axis is chosen on the surface of the fluids and towards the direction of motion of both fluids, the x-axis perpendicular to the interface drawn into the upper fluid and the y-axis in the plane of the interface.
Let$[\rho_1, \{\lambda_1, \lambda_2, \lambda_3, \ldots\ldots, \lambda_n\}, \{\mu_1, \mu_2, \mu_3, \ldots\ldots, \mu_n\}, \mu_L, \sigma_1]$
and$[\rho_2, \{\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3, \ldots\ldots, \bar{\lambda}_n\}, \{\bar{\mu}_1, \bar{\mu}_2, \bar{\mu}_3, \ldots\ldots, \bar{\mu}_n\}, \mu_U, \sigma_2]$ be the density, relaxation time, co-efficient of viscosity and electrical conductivity of the lower and upper fluids each occupying height 'a'. We also suppose that two media have approximately the same permeability $\mu_e$ throughout and thus the same magnetic field $H_0$ is interacting to both the conducting fluids, the velocities of the lower and upper fluids are respectively $w_i(x,y,t)$ [i=1,2], in the z-direction.

The equations of motion of visco-elastic conducting lower and upper fluids of higher order in the presence of a transverse magnetic field in view of the above assumptions become:

$$(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t_j})\frac{\partial w_1}{\partial t} = -\frac{1}{\rho_1}(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t_j})\frac{\partial p}{\partial z} + \nu_1(1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j})\nabla^2 w_1 + \frac{\sigma_1 B_0^{\ 2}}{\rho_1}(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t_j})w_1 \quad \ldots(2.1)$$

$$(1+\sum_{j=1}^{n}\overline{\lambda}_j \frac{\partial^j}{\partial t_j})\frac{\partial w_2}{\partial t} = -\frac{1}{\rho_2}(1+\sum_{j=1}^{n}\overline{\lambda}_j \frac{\partial^j}{\partial t_j})\frac{\partial p}{\partial z} + \nu_2(1+\sum_{j=1}^{n}\overline{\mu}_j \frac{\partial^j}{\partial t^j})\nabla^2 w_2 + \frac{\sigma_2 B_0^{\ 2}}{\rho_2}(1+\sum_{j=1}^{n}\overline{\lambda}_j \frac{\partial^j}{\partial t_j})w_2 \quad \ldots(2.2)$$

Where $\nu_1 = \frac{\mu_L}{\rho_1}$ and $\nu_2 = \frac{\mu_U}{\rho_2}$ are the kinematical co-efficient of viscosity and $B_0=\mu_e H_0$ is the magnetic induction vector.

## 4. SOLUTION OF THE PROBLEM

Here we consider the solution of (2.1) and (2.2) as: $w_l = \mathrm{Re}\{ f_l(x,y)e^{i\omega t}\}, [l=1,2]$ …(3)

The boundary conditions of the lower fluid are:

$$\left.\begin{aligned} &V_1 = 0 \ \ when \ \ x=-a, \ -b \le y \le b \\ &V_1 = V_0 \ when \ \ x=0, \ \ -b \le y \le b \\ &w_1 = 0 \ \ when \ \ y=\pm b, -a \le x \le a \end{aligned}\right\} \quad \ldots(4)$$

The boundary conditions of the upper fluid are:

$$\left.\begin{aligned} &V_2 = 0 \ \ when \ \ x=-a, \ -b \le y \le b \\ &V_2 = V_0 \ when \ \ x=0, \ \ -b \le y \le b \\ &w_2 = 0 \ \ when \ \ y=\pm b, -a \le x \le a \end{aligned}\right\} \quad \ldots(5)$$

Equation (3) will satisfy last boundary condition of (4) and (5) and thus we have:

$$m = (2n+1)\frac{\pi}{2b} \quad \ldots(6)$$

So, all possible solution of (2.1) and (2.2) can be taken as:

$$\left.\begin{aligned} &f_l(x,y) = \sum_{n=0}^{\infty} V_l(x)\cos my \\ &\text{assuming } \frac{\partial p}{\partial z} = -P\cos\omega t \end{aligned}\right\}, \ \omega > 0, [l=1,2] \quad \ldots(7)$$

Now under the boundary condition (4) we get the solution of (2.1) for the lower fluid as:

$$V_1(x) = \frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_L\alpha^2}\left\{\frac{1+\sum_{j=1}^{n}\lambda_j(i\omega)^j}{1+\sum_{j=1}^{n}\mu_j(i\omega)^j}\right\}\left\{1-\frac{\sinh\alpha(1+\frac{x}{a})-\sinh\frac{\alpha x}{a}}{\sinh\alpha}\right\}+V_0\frac{\sinh\alpha(1+\frac{x}{a})}{\sinh\alpha}$$ Where, $-a \le x \le 0$

…(8.1)

Again under the boundary condition (5) we get the solution of (2.2) for the upper fluid as:

$$V_2(x) = \frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_U\overline{\alpha}^2}\left\{\frac{1+\sum_{j=1}^{n}\overline{\lambda}_j(i\omega)^j}{1+\sum_{j=1}^{n}\overline{\mu}_j(i\omega)^j}\right\}\left[1-\frac{\sinh\overline{\alpha}(1-\frac{x}{a})+\sinh\frac{\overline{\alpha}x}{a}}{\sinh\overline{\alpha}}\right]+V_0\frac{\sinh\overline{\alpha}(1-\frac{x}{a})}{\sinh\overline{\alpha}}$$

Where, $0 \le x \le a$ …(8.2)

Where, $$\alpha^2 = \frac{\sigma_1 B_0^{\,2} a^2}{\mu_L}\left\{\frac{1+(i\omega)^n \lambda_n}{1+(i\omega)^n \mu_n}\right\} + m^2 a^2 + \frac{(i\omega)^n a^2}{\nu_1}\left\{\frac{1+(i\omega)^n \lambda_n}{1+(i\omega)^n \mu_n}\right\} \quad \ldots(9)$$

$$\bar{\alpha}^2 = \frac{\sigma_2 B_0^{\,2} a^2}{\mu_U}\left\{\frac{1+(i\omega)^n \bar{\lambda}_n}{1+(i\omega)^n \bar{\mu}_n}\right\} + m^2 a^2 + \frac{(i\omega)^n a^2}{\nu_2}\left\{\frac{1+(i\omega)^n \bar{\lambda}_n}{1+(i\omega)^n \bar{\mu}_n}\right\} \quad \ldots 10)$$

Hence, for the immiscible fluids we finally get:

$$w_1 = \mathrm{Re}\sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_L \alpha^2}\left\{\frac{1+\sum_{j=1}^{n}\lambda_j (i\omega)^j}{1+\sum_{j=1}^{n}\mu_j (i\omega)^j}\right\}\left\{1-\frac{\sinh\alpha\,(1+\frac{x}{a}) - \sinh\frac{\alpha\, x}{a}}{\sinh\alpha}\right\} + V_0\frac{\sinh\alpha\,(1+\frac{x}{a})}{\sinh\alpha}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{i\omega t} \;\; ; \; -a \le x \le 0 \quad \ldots(11)$$

$$w_2 = \mathrm{Re}\sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_U \bar{\alpha}^2}\left\{\frac{1+\sum_{j=1}^{n}\bar{\lambda}_j (i\omega)^j}{1+\sum_{j=1}^{n}\bar{\mu}_j (i\omega)^j}\right\}\left\{1-\frac{\sinh\bar{\alpha}\,(1-\frac{x}{a}) + \sinh\frac{\bar{\alpha}\, x}{a}}{\sinh\bar{\alpha}}\right\} + V_0\frac{\sinh\bar{\alpha}\,(1-\frac{x}{a})}{\sinh\bar{\alpha}}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{i\omega t} \;\; ; \; 0 \le x \le a \quad \ldots(12)$$

**For [j=1]**

$$w_1 = \sum_{n=0}^{\infty}\Big[\frac{(-1)^n 4a^2 P}{(2n+1)\pi\mu_L(1+\mu_1^{\,2}\omega^2)\alpha_1^{\,2}}\{1-\frac{\sinh\alpha_1(1+\frac{x}{a}) - \sinh\alpha_1\frac{x}{a}}{\sinh\alpha_1}\}\{(1+\lambda_1\mu_1\omega^2) -$$

$$(\lambda_1-\mu_1)\omega\tan\omega t\} + \frac{V_0\sinh\alpha_1(1+\frac{x}{a})}{\sinh\alpha_1}\Big]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ,\text{-a} \le x \le 0 \quad \ldots(13)$$

$$w_2 = \sum_{n=0}^{\infty}\Big[\frac{(-1)^n 4a^2 P}{(2n+1)\pi\mu_U(1+\bar{\mu}_1^{\,2}\omega^2)\bar{\alpha}_1^{\,2}}\{1-\frac{\sinh\bar{\alpha}_1(1+\frac{x}{a}) - \sinh\bar{\alpha}_1\frac{x}{a}}{\sinh\bar{\alpha}_1}\}\{(1+\bar{\lambda}_1\bar{\mu}_1\omega^2) -$$

$$(\bar{\lambda}_1-\bar{\mu}_1)\omega\tan\omega t\} + \frac{V_0\sinh\bar{\alpha}_1(1+\frac{x}{a})}{\sinh\bar{\alpha}_1}\Big]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ,0 \le x \le a \quad \ldots(14)$$

$$\alpha_1^{\,2} = \left\{m^2 - \frac{(\lambda_1-\mu_1)\omega^2}{\nu_1(1+\mu_1^{\,2}\omega^2)} + \frac{\sigma_1 B_0^{\,2}(1+\lambda_1\mu_1\omega^2)}{\mu_L(1+\mu_1^{\,2}\omega^2)}\right\}a^2$$

And $$\bar{\alpha}_1^{\,2} = \left\{m^2 - \frac{(\bar{\lambda}_1-\bar{\mu}_1)\omega^2}{\nu_2(1+\bar{\mu}_1^{\,2}\omega^2)} + \frac{\sigma_2 B_0^{\,2}(1+\bar{\lambda}_1\bar{\mu}_1\omega^2)}{\mu_U(1+\bar{\mu}_1^{\,2}\omega^2)}\right\}a^2$$

**For [j=2]**

$$w_1=\sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2P}{(2n+1)\pi\mu_L\{(1+\mu_2{}^2\omega^2)^2+\mu_1{}^2\omega^2\}\alpha_1{}^2}\{1-\frac{\sinh\alpha_1(1+\frac{x}{a})-\sinh\alpha_1\frac{x}{a}}{\sinh\alpha_1}\}\{(1-\lambda_2\omega^2)(1-\mu_2\omega^2)$$

$$+\lambda_1\mu_1\omega^2-\{(\lambda_1\omega(1-\mu_2\omega^2)-\mu_1\omega(1-\lambda_2\omega^2)\tan\omega t\}+\frac{V_0\sinh\alpha_1(1+\frac{x}{a})}{\sinh\alpha_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ...(15)$$

$$w_2=\sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2P}{(2n+1)\pi\mu_U\{(1+\bar{\mu}_2{}^2\omega^2)^2+\bar{\mu}_1{}^2\omega^2\}\bar{\alpha}_1{}^2}\{1-\frac{\sinh\bar{\alpha}_1(1+\frac{x}{a})-\sinh\bar{\alpha}_1\frac{x}{a}}{\sinh\bar{\alpha}_1}\}\{(1-\bar{\lambda}_2\omega^2)(1-\bar{\mu}_2\omega^2)$$

$$+\bar{\lambda}_1\bar{\mu}_1\omega^2\{(\bar{\lambda}_1\omega(1-\bar{\mu}_2\omega^2)-\bar{\mu}_1\omega(1-\bar{\lambda}_2\omega^2)\tan\omega t\}+\frac{V_0\sinh\bar{\alpha}_1(1+\frac{x}{a})}{\sinh\bar{\alpha}_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ...(16)$$

$$\alpha_1^2=[\frac{1}{(1-\omega^2\mu_2)^2+\omega^2\mu_1^2}][\{(1-\omega^2\lambda_2)(1-\omega^2\mu_2)+\omega^2\lambda_1\mu_1\}\{\frac{\sigma_1B_0^2}{\mu_L}-\frac{\omega^2}{\nu_1}\}-\frac{\omega^2}{\nu_1}(\lambda_1-\mu_1)]$$

$$\bar{\alpha}_1^2=[\frac{1}{(1-\omega^2\bar{\mu}_2)^2+\omega^2\bar{\mu}_1^2}][\{(1-\omega^2\bar{\lambda}_2)(1-\omega^2\bar{\mu}_2)+\omega^2\bar{\lambda}_1\bar{\mu}_1\}\{\frac{\sigma_2B_0^2}{\mu_U}-\frac{\omega^2}{\nu_2}\}-\frac{\omega^2}{\nu_2}(\bar{\lambda}_1-\bar{\mu}_1)]$$

Similarly we can obtain the velocities putting j=3,4,….etc.

## 5. INTERFACE VELOCITY, FLUX AND SKIN FRICTION

The interface velocity $w'=\mathrm{Re}\sum_{n=0}^{\infty}V_0\cos(2n+1)\frac{\pi y}{2b}e^{i\omega t}$ is obtained from the condition that the tangential stress is continuous at the interface for both fluids. Thus we get,

$$\left[\mathrm{Re}\sum_{n=0}^{\infty}\mu_L\left\{\frac{1+(i\omega)^n\mu_n}{1+(i\omega)^n\lambda_n}\right\}\frac{\partial V_1}{\partial x}\cos my\right]=\left[\mathrm{Re}\sum_{n=0}^{\infty}\mu_U\left\{\frac{1+(i\omega)^n\bar{\mu}_n}{1+(i\omega)^n\bar{\lambda}_n}\right\}\frac{\partial V_2}{\partial x}\cos my\right]_{x=0}$$

Using this condition we get from (11) and (12):

$$V_0=\frac{\sum_{n=0}^{\infty}\left[\frac{(-1)^n2Pa^2}{(2n+1)\pi}\left\{\frac{\tanh\frac{\alpha}{2}}{\frac{\alpha}{2}}+\frac{\tanh\frac{\bar{\alpha}}{2}}{\frac{\bar{\alpha}}{2}}\right\}\cos(2n+1)\frac{\pi y}{2b}\right]}{\mathrm{Re}\sum_{n=0}^{\infty}\left[\alpha\mu_L\coth\alpha\left\{\frac{1+(i\omega)^n\mu_n}{1+(i\omega)^n\lambda_n}\right\}+\bar{\alpha}\mu_U\coth\bar{\alpha}\left\{\frac{1+(i\omega)^n\bar{\mu}_n}{1+(i\omega)^n\bar{\lambda}_n}\right\}\right]\cos(2n+1)\frac{\pi y}{2b}} \quad ...(17)$$

The total flux Q is given by:

$$Q=Q_1+Q_2=\int_{-b}^{b}\int_{-a}^{0}w_1dxdy+\int_{-b}^{b}\int_{0}^{a}w_2dxdy \quad ...(18)$$

From (17) using (11) and (12) we have:

$$Q = \mathrm{Re}\sum_{n=0}^{\infty}\left[\frac{16Pa^3b}{(2n+1)^2\pi^2\mu_L\alpha^2}\left\{\frac{1+(i\omega)^n\lambda_n}{1+(i\omega)^n\mu_n}\right\}\left\{1-\frac{\tanh\frac{\alpha}{2}}{\frac{\alpha}{2}}\right\}+\frac{(-1)^n 2abV_0}{(2n+1)\pi}\frac{\tanh\frac{\alpha}{2}}{\frac{\alpha}{2}}\right]e^{i\omega t}$$

$$+\mathrm{Re}\sum_{n=0}^{\infty}\left[\frac{16Pa^3b}{(2n+1)^2\pi^2\mu_U\alpha^2}\left\{\frac{1+(i\omega)^n\bar{\lambda}_n}{1+(i\omega)^n\bar{\mu}_n}\right\}\left\{1-\frac{\tanh\frac{\bar{\alpha}}{2}}{\frac{\bar{\alpha}}{2}}\right\}+\frac{(-1)^n 2abV_0}{(2n+1)\pi}\frac{\tanh\frac{\bar{\alpha}}{2}}{\frac{\bar{\alpha}}{2}}\right]e^{i\omega t} \quad ...(19)$$

The skin friction on the wall x = -a is:

$$(\tau_1)_{x=-a} = \mu_L\,\mathrm{Re}\sum_{n=0}^{\infty}\left\{\frac{1+(i\omega)^n\mu_n}{1+(i\omega)^n\lambda_n}\right\}\left[\frac{(-1)^{n+1}4aP}{(2n+1)\pi\alpha\mu_L}\left\{\frac{1+(i\omega)^n\lambda_n}{1+(i\omega)^n\mu_n}\right\}\tanh\frac{\alpha}{2}+\frac{V_0\alpha}{a\sinh\alpha}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{i\omega t} \quad ...(19.1)$$

The skin friction on the wall x = a is:

$$(\tau_2)_{x=a} = \mu_U\,\mathrm{Re}\sum_{n=0}^{\infty}\left\{\frac{1+(i\omega)^n\bar{\mu}_n}{1+(i\omega)^n\bar{\lambda}_n}\right\}\left[\frac{(-1)^{n+1}4aP}{(2n+1)\pi\alpha\mu_U}\left\{\frac{1+(i\omega)^n\bar{\lambda}_n}{1+(i\omega)^n\bar{\mu}_n}\right\}\tanh\frac{\bar{\alpha}}{2}-\frac{V_0\bar{\alpha}}{a\sinh\bar{\alpha}}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{i\omega t} \quad ...(19.2)$$

The total skin friction on the wall y = b is given by:

$$(\tau)_{y=b} = (\tau_1)_{y=b}+(\tau_2)_{y=b}$$

$$= \mathrm{Re}\sum_{n=0}^{\infty}\left[\begin{array}{l}\frac{2Pa^2}{b}\left\{\frac{1}{\alpha^2}\left(1-\frac{\sinh\alpha(1+\frac{x}{a})-\sinh\frac{\alpha x}{a}}{\sinh\alpha}\right)+\frac{1}{\bar{\alpha}^2}\left(1-\frac{\sinh\bar{\alpha}(1-\frac{x}{a})+\sinh\frac{\bar{\alpha}x}{a}}{\sinh\bar{\alpha}}\right)\right\}\\ +V_0\left\{\frac{(-1)^n(2n+1)\pi}{2b}\left(\frac{\mu_L\sinh\alpha(1+\frac{x}{a})}{\sinh\alpha}\times\frac{1+(i\omega)^n\mu_n}{1+(i\omega)^n\lambda_n}+\right)+\left(\frac{\mu_U\sinh\bar{\alpha}(1-\frac{x}{a})}{\sinh\bar{\alpha}}\times\frac{1+(i\omega)^n\bar{\mu}_n}{1+(i\omega)^n\bar{\lambda}_n}+\right)\right\}\end{array}\right]e^{i\omega t} \quad ...(20)$$

Replacing b by –b in (20) we get the total friction on the wall y = -b.

## 6.DEDUCTION FOR VARIOUS VISCO-ELASTIC FLUIDS

**Case I:** In case of two immiscible Maxwell fluids we take $\lambda_1$, $\bar{\lambda}_1>0$ and $\lambda_j= \bar{\lambda}_j= 0$ (j=2,3,…..,n); $\mu_j= 0= \bar{\mu}_j$ (j=1,2,…..,n).So the velocities of the lower and upper fluids are:

$$w_1 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4a^2P}{(2n+1)\pi\mu_L\alpha_1^2}\left\{1-\frac{\sinh\alpha_1(1+\frac{x}{a})-\sinh\alpha_1\frac{x}{a}}{\sinh\alpha_1}\right\}(1-\lambda_1\omega\tan\omega t)\right.$$

$$\left.+\frac{V_0\sinh\alpha_1(1+\frac{x}{a})}{\sinh\alpha_1}\right]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ,-a\le x\le 0 \quad ...(21)$$

$$w_2 = \sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2 P}{(2n+1)\pi\mu_U \bar{\alpha}_1{}^2}\{1-\frac{\sinh\bar{\alpha}_1(1+\frac{x}{a})-\sinh\bar{\alpha}_1\frac{x}{a}}{\sinh\bar{\alpha}_1}\}(1-\bar{\lambda}_1\omega\tan\omega t)$$

$$+\frac{V_0\sinh\bar{\alpha}_1(1+\frac{x}{a})}{\sinh\bar{\alpha}_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ,0\le x\le a \qquad ...(22)$$

$$\alpha_1{}^2 = \left\{m^2-\frac{\lambda_1\omega^2}{\nu_1}+\frac{\sigma_1 B_0{}^2}{\mu_L}\right\}a^2 \text{ and } \bar{\alpha}_1{}^2 = \left\{m^2-\frac{\bar{\lambda}_1}{\nu_2}+\frac{\sigma_2 B_0{}^2}{\mu_U}\right\}a^2$$

**Case II:** In case of two immiscible Maxwell fluids of second order we take $\lambda_1>\lambda_2>0$, $\bar{\lambda}_1>\bar{\lambda}_2>0$ and $\lambda_j=\bar{\lambda}_j=0$ (j=3,4…..,n); $\mu_j=0=\bar{\mu}_j$ (j=1,2,…..,n).So the velocities of the lower and upper fluids are:

$$w_1 = \sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2 P}{(2n+1)\pi\mu_L \alpha_1{}^2}\{1-\frac{\sinh\alpha_1(1+\frac{x}{a})-\sinh\alpha_1\frac{x}{a}}{\sinh\alpha_1}\}(1-\lambda_1\omega+\lambda_2\omega^2)$$

$$+\frac{V_0\sinh\alpha_1(1+\frac{x}{a})}{\sinh\alpha_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \qquad ...(23)$$

$$w_2 = \sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2 P}{(2n+1)\pi\mu_U \bar{\alpha}_1{}^2}\{1-\frac{\sinh\bar{\alpha}_1(1+\frac{x}{a})-\sinh\bar{\alpha}_1\frac{x}{a}}{\sinh\bar{\alpha}_1}\}(1-\bar{\lambda}_1\omega-\bar{\lambda}_2\omega^2)$$

$$+\frac{V_0\sinh\bar{\alpha}_1(1+\frac{x}{a})}{\sinh\bar{\alpha}_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \qquad ...(24)$$

$$\alpha_1{}^2 = [(1-\omega^2\lambda_2)\{\frac{\sigma_1 B_0{}^2}{\mu_L}-\frac{\omega^2}{\nu_1}\}-\frac{\omega^2\lambda_1}{\nu_1}] \text{ and } \bar{\alpha}_1{}^2 = [(1-\omega^2\bar{\lambda}_2)\{\frac{\sigma_2 B_0{}^2}{\mu_U}-\frac{\omega^2}{\nu_2}\}-\frac{\omega^2\bar{\lambda}_1}{\nu_2}]$$

**Case III:** In case of two immiscible Oldroyd fluids we take $\lambda_1$, $\bar{\lambda}_1$, $\mu_1$, $\bar{\mu}_1>0$ and $\lambda_j=\bar{\lambda}_j=0$ (j=2,3,…..,n); $\mu_j=0=\bar{\mu}_j$ (j=2,3,…..,n).So the velocities of the lower and upper fluids are:

$$w_1 = \sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2 P}{(2n+1)\pi\mu_L(1+\mu_1{}^2\omega^2)\alpha_1{}^2}\{1-\frac{\sinh\alpha_1(1+\frac{x}{a})-\sinh\alpha_1\frac{x}{a}}{\sinh\alpha_1}\}\{(1+\lambda_1\mu_1\omega^2)-$$

$$(\lambda_1-\mu_1)\omega\tan\omega t\}+\frac{V_0\sinh\alpha_1(1+\frac{x}{a})}{\sinh\alpha_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ,-a\le x\le 0 \qquad ...(25)$$

$$w_2 = \sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2 P}{(2n+1)\pi\mu_U(1+\bar{\mu}_1{}^2\omega^2)\bar{\alpha}_1{}^2}\{1-\frac{\sinh\bar{\alpha}_1(1+\frac{x}{a})-\sinh\bar{\alpha}_1\frac{x}{a}}{\sinh\bar{\alpha}_1}\}\{(1+\bar{\lambda}_1\bar{\mu}_1\omega^2)-$$

$$(\bar{\lambda}_1-\bar{\mu}_1)\omega\tan\omega t\}+\frac{V_0\sinh\bar{\alpha}_1(1+\frac{x}{a}}{\sinh\bar{\alpha}_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ,0\le x\le a \qquad ...(26)$$

$$\alpha_1{}^2 = \left\{m^2-\frac{(\lambda_1-\mu_1)\omega^2}{\nu_1(1+\mu_1{}^2\omega^2)}+\frac{\sigma_1 B_0{}^2(1+\lambda_1\mu_1\omega^2)}{\mu_L(1+\mu_1{}^2\omega^2)}\right\}a^2$$

$$\bar{\alpha}_1{}^2 = \left\{m^2-\frac{(\bar{\lambda}_1-\bar{\mu}_1)\omega^2}{\nu_2(1+\bar{\mu}_1{}^2\omega^2)}+\frac{\sigma_2 B_0{}^2(1+\bar{\lambda}_1\bar{\mu}_1\omega^2)}{\mu_U(1+\bar{\mu}_1{}^2\omega^2)}\right\}a^2$$

**Case IV:** In case of two immiscible Oldroyd fluids of second order we take $\lambda_1>\lambda_2>0$, $\bar{\lambda}_1>\bar{\lambda}_2>0$; $\mu_1>\mu_2>0$, $\bar{\mu}_1>\bar{\mu}_2>0$ and $\lambda_j=\bar{\lambda}_j=0$ (j=3,4…..,n); $\mu_j=0=\bar{\mu}_j$ (j=3,4,…..,n).So the velocities of the lower and upper fluids are:

$$w_1=\sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2P}{(2n+1)\pi\mu_L\{(1+\mu_2{}^2\omega^2)^2+\mu_1{}^2\omega^2\}\alpha_1{}^2}\{1-\frac{\sinh\alpha_1(1+\frac{x}{a})-\sinh\alpha_1\frac{x}{a}}{\sinh\alpha_1}\}\{(1-\lambda_2\omega^2)(1-\mu_2\omega^2)$$
$$+\lambda_1\mu_1\omega^2-\lambda_1\omega(1-\mu_2\omega^2)-\mu_1\omega(1-\lambda_2\omega^2)\tan\omega t\}+\frac{V_0\sinh\alpha_1(1+\frac{x}{a})}{\sinh\alpha_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ...(27)$$

$$w_2=\sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2P}{(2n+1)\pi\mu_U\{(1+\bar{\mu}_2{}^2\omega^2)^2+\bar{\mu}_1{}^2\omega^2\}\bar{\alpha}_1{}^2}\{1-\frac{\sinh\bar{\alpha}_1(1+\frac{x}{a})-\sinh\bar{\alpha}_1\frac{x}{a}}{\sinh\bar{\alpha}_1}\}\{(1-\bar{\lambda}_2\omega^2)(1-\bar{\mu}_2\omega^2)$$
$$+\bar{\lambda}_1\bar{\mu}_1\omega^2-\bar{\lambda}_1\omega(1-\bar{\mu}_2\omega^2)-\bar{\mu}_1\omega(1-\bar{\lambda}_2\omega^2)\tan\omega t\}+\frac{V_0\sinh\bar{\alpha}_1(1+\frac{x}{a})}{\sinh\bar{\alpha}_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ...(28)$$

$$\alpha_1^2=[\frac{1}{(1-\omega^2\mu_2)^2+\omega^2\mu_1^2}][\{(1-\omega^2\lambda_2)(1-\omega^2\mu_2)+\omega^2\lambda_1\mu_1\}\{\frac{\sigma_1B_0{}^2}{\mu_L}-\frac{\omega^2}{\nu_1}\}-\frac{\omega^2}{\nu_1}(\lambda_1-\mu_1)]$$
$$\bar{\alpha}_1^2=[\frac{1}{(1-\omega^2\bar{\mu}_2)^2+\omega^2\bar{\mu}_1^2}][\{(1-\omega^2\bar{\lambda}_2)(1-\omega^2\bar{\mu}_2)+\omega^2\bar{\lambda}_1\bar{\mu}_1\}\{\frac{\sigma_2B_0{}^2}{\mu_U}-\frac{\omega^2}{\nu_2}\}-\frac{\omega^2}{\nu_2}(\bar{\lambda}_1-\bar{\mu}_1)]$$

**Case V:** In case of two immiscible Rivlin-Ericksen fluids we take $\mu_1$, $\bar{\mu}_1>0$ and $\mu_j=\bar{\mu}_j=0$ (j=2,3,…..,n); $\lambda_j=0=\bar{\lambda}_j$ (j=1,2,…..,n).So the velocities of the lower and upper fluids are:

$$w_1=\sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2P}{(2n+1)\pi\mu_L(1+\mu_1{}^2\omega^2)\alpha_1{}^2}\{1-\frac{\sinh\alpha_1(1+\frac{x}{a})-\sinh\alpha_1\frac{x}{a}}{\sinh\alpha_1}\}(1+\mu_1\omega\tan\omega t)$$
$$+\frac{V_0\sinh\alpha_1(1+\frac{x}{a})}{\sinh\alpha_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ,-a\le x\le 0 \quad ...(29)$$

$$w_2=\sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2P}{(2n+1)\pi\mu_U(1+\bar{\mu}_1{}^2\omega^2)\bar{\alpha}_1{}^2}\{1-\frac{\sinh\bar{\alpha}_1(1+\frac{x}{a})-\sinh\bar{\alpha}_1\frac{x}{a}}{\sinh\bar{\alpha}_1}\}(1+\bar{\mu}\omega\tan\omega t)$$
$$+\frac{V_0\sinh\bar{\alpha}_1(1+\frac{x}{a}}{\sinh\bar{\alpha}_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ,0\le x\le a \quad ...(30)$$

$$\alpha_1{}^2=\left\{m^2+\frac{\mu_1\omega^2}{\nu_1(1+\mu_1{}^2\omega^2)}+\frac{\sigma_1B_0{}^2}{\mu_L(1+\mu_1{}^2\omega^2)}\right\}a^2$$
$$\bar{\alpha}_1{}^2=\left\{m^2+\frac{\bar{\mu}_1\omega^2}{\nu_2(1+\bar{\mu}_1{}^2\omega^2)}+\frac{\sigma_2B_0{}^2}{\mu_U(1+\bar{\mu}_1{}^2\omega^2)}\right\}a^2$$

**Case VI:** In case of two immiscible Rivlin-Ericksen fluids of second order we take $\mu_1>\mu_2>0$, $\bar{\mu}_1>\bar{\mu}_2>0$ and $\lambda_j=\bar{\lambda}_j=0$(j=1,2,…..,n); $\mu_j=0=\bar{\mu}_j$ (j=3,4,…..,n).So the velocities of the lower and upper fluids are:

$$w_1=\sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2P}{(2n+1)\pi\mu_L\{(1+\mu_2{}^2\omega^2)^2+\mu_1{}^2\omega^2\}\alpha_1{}^2}\{1-\frac{\sinh\alpha_1(1+\frac{x}{a})-\sinh\alpha_1\frac{x}{a}}{\sinh\alpha_1}\}\{(1-\mu_2\omega^2)+$$

$$\mu_1\omega\tan\omega t\}+\frac{V_0\sinh\alpha_1(1+\frac{x}{a})}{\sinh\alpha_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \qquad ...(31)$$

$$w_2=\sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2P}{(2n+1)\pi\mu_U\{(1+\bar{\mu}_2{}^2\omega^2)^2+\bar{\mu}_1{}^2\omega^2\}\bar{\alpha}_1{}^2}\{1-\frac{\sinh\bar{\alpha}_1(1+\frac{x}{a})-\sinh\bar{\alpha}_1\frac{x}{a}}{\sinh\bar{\alpha}_1}\}\{(1-\bar{\mu}_2\omega^2)+$$

$$+\bar{\mu}_1\omega\tan\omega t\}+\frac{V_0\sinh\bar{\alpha}_1(1+\frac{x}{a})}{\sinh\bar{\alpha}_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \qquad ...(32)$$

$$\alpha_1{}^2=[\frac{1}{(1-\omega^2\mu_2)^2+\omega^2\mu_1{}^2}][(1-\omega^2\mu_2)\{\frac{\sigma_1B_0{}^2}{\mu_L}-\frac{\omega^2}{\nu_1}\}+\frac{\omega^2\mu_1}{\nu_1}]$$

$$\bar{\alpha}_1{}^2=[\frac{1}{(1-\omega^2\bar{\mu}_2)^2+\omega^2\bar{\mu}_1{}^2}][(1-\omega^2\bar{\mu}_2)\{\frac{\sigma_2B_0{}^2}{\mu_U}-\frac{\omega^2}{\nu_2}\}+\frac{\omega\bar{\mu}_1{}^2}{\nu_2}]$$

**Case VII:** In case of two immiscible Walters fluids we take $\mu_1=-\beta$, $\bar{\mu}_1=-\bar{\beta}$ ($\beta$, $\bar{\beta}>0$) and $\lambda_j=\bar{\lambda}_j=0$ (j=1,2,3,…..,n); $\mu_j=0=\bar{\mu}_j$ (j=2,3,…..,n).So the velocities of the lower and upper fluids are:

$$w_1=\sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2P}{(2n+1)\pi\mu_L(1+\beta^2\omega^2)\alpha_1{}^2}\{1-\frac{\sinh\alpha_1(1+\frac{x}{a})-\sinh\alpha_1\frac{x}{a}}{\sinh\alpha_1}\}(1+\beta\omega\tan\omega t)$$

$$+\frac{V_0\sinh\alpha_1(1+\frac{x}{a})}{\sinh\alpha_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ,-a\le x\le 0 \qquad ...(33)$$

$$w_2=\sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2P}{(2n+1)\pi\mu_U(1+\bar{\beta}^2\omega^2)\bar{\alpha}_1{}^2}\{1-\frac{\sinh\bar{\alpha}_1(1+\frac{x}{a})-\sinh\bar{\alpha}_1\frac{x}{a}}{\sinh\bar{\alpha}_1}\}(1+\bar{\beta}\omega\tan\omega t)$$

$$+\frac{V_0\sinh\bar{\alpha}_1(1+\frac{x}{a}}{\sinh\bar{\alpha}_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ,0\le x\le a \qquad ...(34)$$

$$\alpha_1{}^2=\left\{m^2-\frac{\beta\omega^2}{\nu_1(1+\beta^2\omega^2)}+\frac{\sigma_1B_0{}^2}{\mu_L(1+\beta^2\omega^2)}\right\}a^2$$

$$\bar{\alpha}_1{}^2=\left\{m^2-\frac{\bar{\beta}\omega^2}{\nu_2(1+\bar{\mu}_1{}^2\omega^2)}+\frac{\sigma_2B_0{}^2}{\mu_U(1+\bar{\beta}^2\omega^2)}\right\}a^2$$

**Case VIII:** In case of two immiscible purely viscous fluids we take $\lambda_j\to0$, $\bar{\lambda}_j\to0$; $\mu_j\to0$, $\bar{\mu}_j\to0$ (j=1,2,…..,n).So the velocities of the lower and upper fluids are:

$$w_1 = \sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2 P}{(2n+1)\pi\mu_L \alpha_1^{\ 2}}\{1-\frac{\sinh\alpha_1(1+\frac{x}{a})-\sinh\alpha_1 \frac{x}{a}}{\sinh\alpha_1}\}+$$

$$\frac{V_0 \sinh\alpha_1(1+\frac{x}{a})}{\sinh\alpha_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ,-a \le x \le 0 \qquad ...(35)$$

$$w_2 = \sum_{n=0}^{\infty}[\frac{(-1)^n 4a^2 P}{(2n+1)\pi\mu_U \overline{\alpha}_1^{\ 2}}\{1-\frac{\sinh\overline{\alpha}_1(1+\frac{x}{a})-\sinh\overline{\alpha}_1 \frac{x}{a}}{\sinh\overline{\alpha}_1}\}+$$

$$\frac{V_0 \sinh\overline{\alpha}_1(1+\frac{x}{a}}{\sinh\overline{\alpha}_1}]\times\cos(2n+1)\frac{\pi y}{2b}\cos\omega t \quad ,0 \le x \le a \qquad ...(36)$$

$$\alpha_1^{\ 2} = \left\{m^2 - \frac{1}{\nu_1} + \frac{\sigma_1 B_0^{\ 2}}{\mu_L}\right\}a^2 \text{ and } \overline{\alpha}_1^{\ 2} = \left\{m^2 - \frac{1}{\nu_2} + \frac{\sigma_2 B_0^{\ 2}}{\mu_U}\right\}a^2$$

## 7.NUMERICAL CALCULATIONS AND DISCUSSIONS

The analytical results for velocity as exhibited in the earlier articles has been numerically computed for different times as in the following tables and graphs. For this we take $\lambda_1=0.04$, $\mu_1=0.02$, $\mu_L=0.06$, $\nu_1=0.04$, a=0.5, b=0.25, x=-0.1(lower fluid),y=0.2, $M_1^2=\sigma_1 B_0^2 a^2/\mu_L= \sigma_1 B_0^2 a^2/\mu_L=M_2^2=4$, $V_0=5$, $\omega=1$, $\overline{\lambda}_1=0.06$, $\overline{\mu}_1=0.03$, $\mu_U=0.05$, $\nu_2=0.03$, x=0.1(upper fluid).

In the following four tables the velocities of two immiscible fluids in case of visco-elastic Oldroyd fluids, Maxwell fluids, Rivlin-Ericksen fluid and ordinary viscous fluid are presented, respectively.

**OLDROYD FLUIDS** **MAXWELL FLUIDS**

| T | OW1 | OW2 |
|---|---|---|
| 0 | 68.29756134 | -144.2888114 |
| 1 | -52.60354025 | 111.132845 |
| 2 | -125.1411895 | 264.3794762 |
| 3 | -82.62460627 | 174.5568362 |
| 4 | 35.85665895 | -75.75255395 |
| 5 | 121.3714773 | -256.4153954 |
| 6 | 95.29791914 | -201.3311048 |
| 7 | -18.39210637 | 38.85607503 |
| 8 | -115.1725141 | 243.3191587 |
| 9 | -106.0638435 | 224.07573 |
| 10 | 0.559435665 | -1.181891499 |
| 11 | 106.6683723 | -225.3528874 |
| 12 | 114.7068993 | -242.3354779 |
| 13 | 17.28443215 | -36.5159476 |
| 14 | -96.02926225 | 202.8761765 |
| 15 | -121.0540958 | 255.7448795 |
| 16 | -34.78235194 | 73.48291975 |
| 17 | 83.46812589 | -176.3388976 |
| 18 | 124.9783937 | -264.0355457 |
| 19 | 51.58410272 | -108.9791308 |
| 20 | -69.23637441 | 146.2721944 |
| 21 | -126.4012482 | 267.0415386 |
| 22 | -67.35339733 | 142.2941237 |
| 23 | 53.61885644 | -113.2778523 |
| 24 | 125.2941809 | -264.7026933 |
| 25 | 81.77461324 | -172.7610989 |
| 26 | -36.92815668 | 78.01625315 |
| 27 | -121.6793497 | 257.0658218 |
| 28 | -94.55910971 | 199.7702594 |
| 29 | 19.49833962 | -41.1931582 |

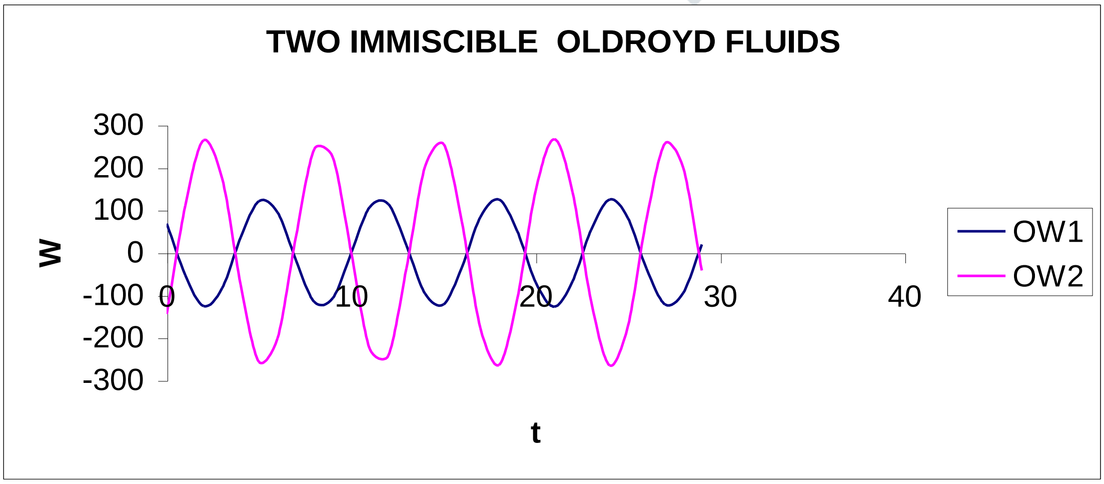
| T | MW1 | MW2 |
|---|---|---|
| 0 | 71.21281646 | -149.9614453 |
| 1 | -54.84890212 | 115.5019706 |
| 2 | -130.482793 | 274.7734074 |
| 3 | -86.15140579 | 181.4194406 |
| 4 | 37.38718664 | -78.73072324 |
| 5 | 126.5521721 | -266.4962232 |
| 6 | 99.36567415 | -209.2463246 |
| 7 | -19.17716636 | 40.38367988 |
| 8 | -120.0886086 | 252.8851153 |
| 9 | -110.5911379 | 232.885142 |
| 10 | 0.583314961 | -1.228356902 |
| 11 | 111.2214707 | -234.2125101 |
| 12 | 119.6031192 | -251.8627616 |
| 13 | 18.02221149 | -37.95155165 |
| 14 | -100.1282344 | 210.8521399 |
| 15 | -126.2212433 | 265.7993464 |
| 16 | -36.26702325 | 76.37185964 |
| 17 | 87.03093072 | -183.2715627 |
| 18 | 130.3130483 | -274.4159555 |
| 19 | 53.78595029 | -113.2635843 |
| 20 | -72.19170241 | 152.0228039 |
| 21 | -131.7966368 | 277.5401273 |
| 22 | -70.22835118 | 147.8883376 |
| 23 | 55.90755669 | -117.7313077 |
| 24 | 130.6423148 | -275.1093316 |
| 25 | 85.26513114 | -179.5531048 |
| 26 | -38.50442084 | 81.0834185 |
| 27 | -126.8731859 | 267.1722208 |
| 28 | -98.59532892 | 207.6241154 |
| 29 | 20.33061875 | -42.81264417 |

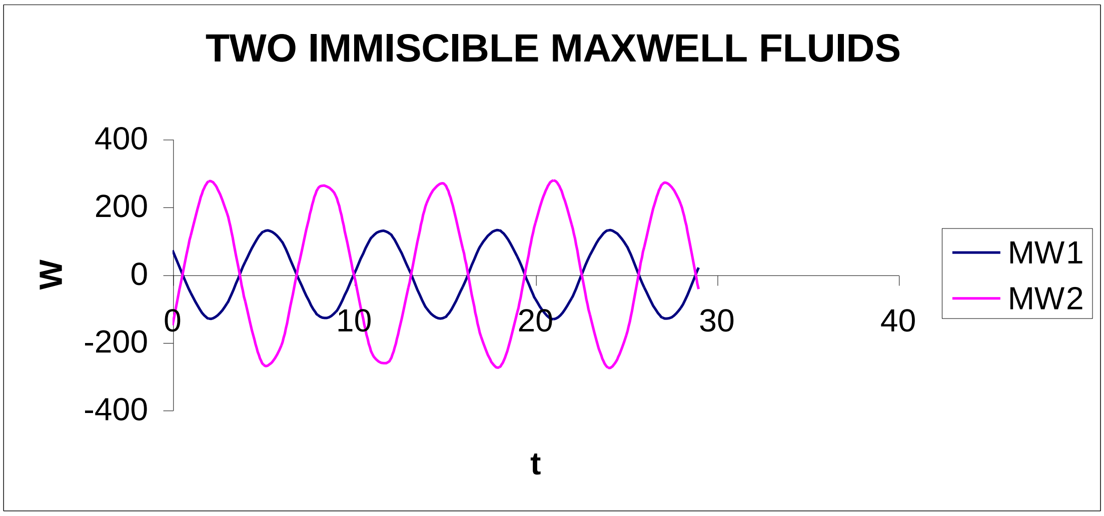
**RIVLIN-ERICKSEN FLUIDS** **ORDINARY VISCOUS FLUIDS**

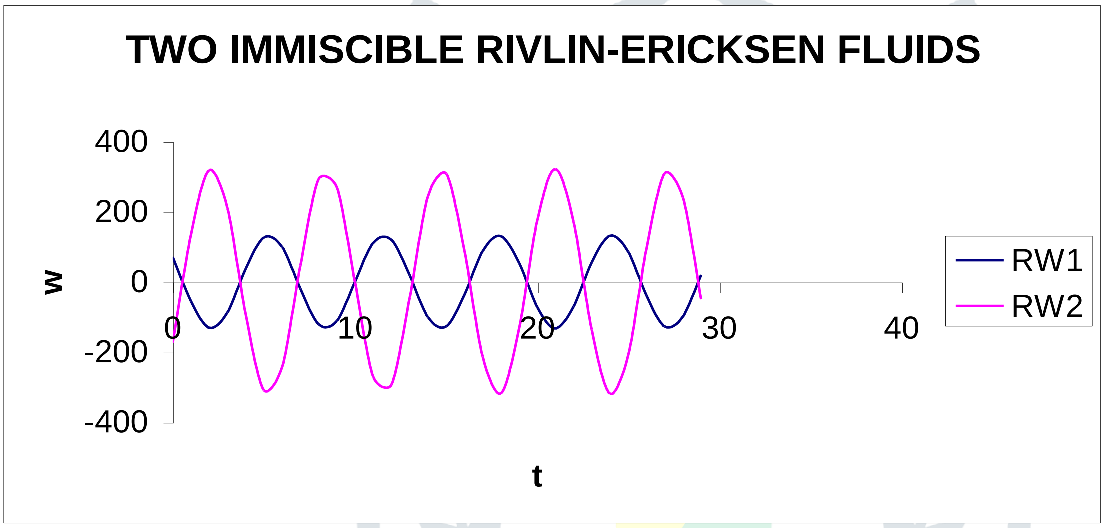
| T | RW1 | RW2 |
|---|---|---|
| 0 | 71.46032894 | -173.8952165 |
| 1 | -55.03953899 | 133.936027 |
| 2 | -130.9363086 | 318.627105 |
| 3 | -86.45083993 | 210.3738921 |
| 4 | 37.51713228 | -91.29610707 |
| 5 | 126.9920261 | -309.0288864 |
| 6 | 99.71103677 | -242.6419328 |
| 7 | -19.24381992 | 46.82889488 |
| 8 | -120.5059973 | 293.2454525 |
| 9 | -110.9755165 | 270.0534935 |
| 10 | 0.585342373 | -1.424402046 |
| 11 | 111.6080402 | -271.5927089 |
| 12 | 120.0188206 | -292.0599317 |
| 13 | 18.08485081 | -44.00860021 |
| 14 | -100.4762474 | 244.5040354 |
| 15 | -126.6599471 | 308.2207884 |
| 16 | -36.39307557 | 88.56077003 |
| 17 | 87.3334218 | -212.5216119 |
| 18 | 130.7659739 | -318.212604 |
| 19 | 53.97289269 | -131.3403954 |
| 20 | -72.44261718 | 176.2855669 |
| 21 | -132.2547189 | 321.8353921 |
| 22 | -70.47244199 | 171.4912419 |
| 23 | 56.10187309 | -136.5211652 |
| 24 | 131.0963848 | -319.0166426 |
| 25 | 85.56148488 | -208.2096901 |
| 26 | -38.63824963 | 94.02429131 |
| 27 | -127.3141556 | 309.8127729 |
| 28 | -98.93801407 | 240.7608198 |
| 29 | 20.40128134 | -49.64552064 |

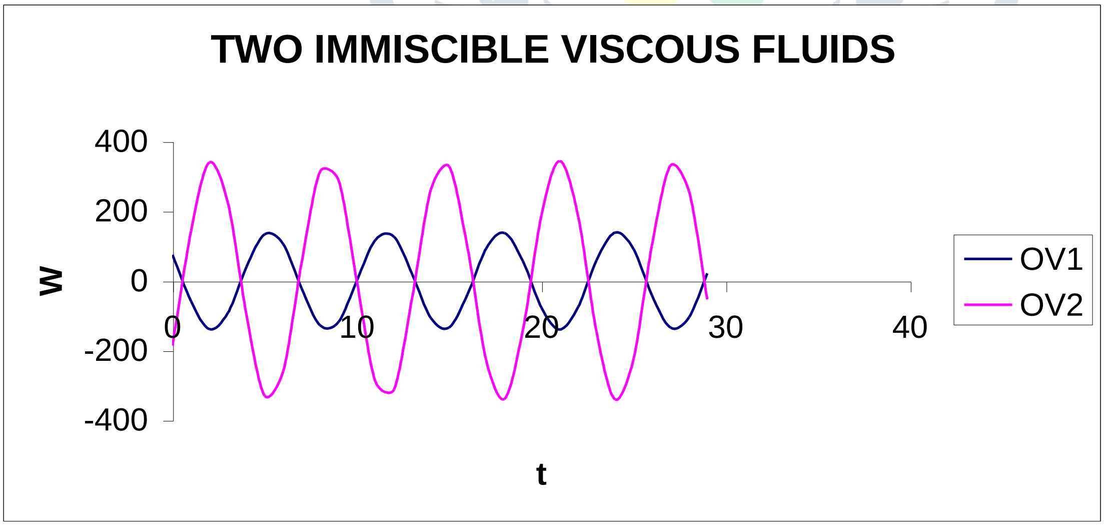
| T | OV1 | OV2 |
|---|---|---|
| 0 | 75.40837192 | -185.6154542 |
| 1 | -58.08036554 | 142.9630842 |
| 2 | -138.1702828 | 340.1020223 |
| 3 | -91.22707923 | 224.5527295 |
| 4 | 39.58988024 | -97.44930719 |
| 5 | 134.0080864 | -329.8569003 |
| 6 | 105.2198759 | -258.9955805 |
| 7 | -20.30700322 | 49.98508161 |
| 8 | -127.1637173 | 313.0096902 |
| 9 | -117.1066961 | 288.2546331 |
| 10 | 0.617681391 | -1.520404287 |
| 11 | 117.7741655 | -289.897589 |
| 12 | 126.6496249 | -311.7442673 |
| 13 | 19.08400333 | -46.97470394 |
| 14 | -106.0273629 | 260.9831856 |
| 15 | -133.6576607 | 328.9943379 |
| 16 | -38.4037216 | 94.52961315 |
| 17 | 92.15842202 | -226.845202 |
| 18 | 137.9905374 | -339.6595845 |
| 19 | 56.95478912 | -140.1925115 |
| 20 | -76.44492966 | 188.1669101 |
| 21 | -139.5615327 | 343.5265423 |
| 22 | -74.36590615 | 183.0494558 |
| 23 | 59.20139152 | -145.7224562 |
| 24 | 138.3392028 | -340.517814 |
| 25 | 90.28858906 | -222.242664 |
| 26 | -40.77293712 | 100.3613664 |
| 27 | -134.3480129 | 330.6936193 |
| 28 | -104.4041452 | 256.9876837 |
| 29 | 21.52841211 | -52.99154309 |

From the above tables and the following graphs it is quite clear that in the cases of two immiscible visco-elastic Maxwell, Oldroyd and Rivlin -Ericksen fluids as well as viscous fluids, the velocities are periodic in nature for both the fluids. This feature of investigation of the titled problem is perfectly in conformity with the usual concept of magneto-hydrodynamics. In case of purely viscous fluid or visco-elastic fluids the velocity profile has a maximum value in absence of the magnetic field.



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