ON α-SEPARATION AXIOMS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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Abstract : The aim of this paper is to introduce some new types of α-separation axioms (α-T_i space, for $i = 1,2,3$) in intuitionistic fuzzy topological spaces by using the concept of an intuitionistic fuzzy α-open(resp. intuitionistic fuzzy αclosed)sets. Moreover the topological property and relationships between theses separation axioms are investigated.

Keywords - intuitionistic fuzzy topology, intuitionistic fuzzy α-separation axioms

I. INTRODUCTION

After defining the concept of intuitionistic fuzzy set by Atanassov [1] and intuitionistic fuzzy topological spaces by Coker [5], some authors studied the concept of separation axioms in intuitionistic fuzzy topological spaces. Bayhan and Coker [2] gave some characterizations of T_1 and T_2 separation axioms in intuitionistic topological spaces, they gave interrelations between several types of separation axioms and some couter examples. Gallego, Lupianez [7] defined new notions of Hausdorffness in the intuitionistic fuzzy topological spaces.

In this paper, by using the concept of fuzzy α-open(α-closed) sets, we introduce some new types of α-separation axioms (αT_i-space, for i=0,1,2) in intuitionistic fuzzy topological spaces, and we study the topological property and hereditary property of these spaces. Relaionships between these separation axioms are investigated. Some counter examples are given to show that the inverse of those relations are not true in general.

II. Preliminaries

Throughout this paper by (X,T) or simply by X we mean an intuitionistic fuzzy topological space (Ifts, for short)

Definition 2.1 *[1] Let X be a nonempty fixed set and I be the closed interval in* $[0,1]$. *An intuitionistic fuzzy set (IFS) A is an object of the following form $A = \{< x, \mu_A(x), \nu_A(x) >; x \in X\}$ where the mappings $\mu_A(x): X \to I$ and $\nu_A(x): X \to I$ denote the degree of membership (namely) $\mu_A(x)$ and the degree of non membership (namely) $\nu_A(x)$ for each element $x \in X$ to the set A respectively, and* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ *for each $x \in X$.*

Definition 2.2 *[1] Let A and B are intuitionistic fuzzy sets of the form $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) >: x \in X\}$. Then*

1. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;
2. $\overline{A}(or A^c) = \{< x, \nu_A(x), \mu_A(x) >: x \in X\}$;
3. $A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) >: x \in X\}$;
4. $A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) >: x \in X\}$;

We will use the notation $A = \{< x, \mu_A, \nu_A >: x \in X\}$ instead of $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$.

Definition 2.3 *[1]* $\underline{0} = \{< x, 0,1 >; x \in X\}$ *and* $\underline{1} = \{< x, 1,0 >; x \in X\}$. *Let $\alpha, \beta \in [0,1]$ such that $\alpha + \beta \leq 1$. An intuitionistic fuzzy point $(IFP)_{p_{(\alpha,\beta)}}$ is intuitionistic fuzzy set defined by* $p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta) if x = p \\ (0,1) otherwise \end{cases}$

In IFP $p_{\alpha,\beta}$ is said to belong to an IFS $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle: x \in X\}$ denoted by $p_{(\alpha,\beta)} \in A$ (or $p \subseteq A$), if $\alpha \leq \mu_A(x)$ and $\beta \geq \gamma_A(x)$.

Proposition 2.1 *[1] An intuitionistic fuzzy set A in X is the union of all intuitionistic fuzzy points belonging to A*

Definition 2.4 *[4] An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family T of intuitionistic fuzzy sets in X satisfying the following axioms:*

1 $\underline{0}, \underline{1} \in T$;
2. $G_1 \cap G_2 \in T$, for any $G_1, G_2 \in T$;
3. $\cup G_i \in T$ for any arbitrary family $\{G_i; i \in J\} \subseteq T$.

Definition 2.5 *[4] Let X and Y are two non-empty sets and $f: (X,T) \to (Y,\sigma)$ be a function. If $B = \{< y, \mu_B(y), \nu_B(y) >; y \in Y\}$ is an IFS in Y, then the pre-image of B under f is denoted and defined by $f^{-1}(B) = \{< x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x)) >; x \in X\}$. Since $\mu_B(x), \nu_B(x)$ are fuzzy sets, we explain that $f^{-1}(\mu_B(x)) = \mu_B(x)(f(x)), f^{-1}(\nu_B(x)) = \nu_B(x)(f(x))$.*

Definition 2.6 *[4] Let (X,T) be an IFTS and $A = \{< x, \mu_A, \nu_A >; x \in X\}$ be an IFS in X. Then the intuitionistic fuzzy closure and intuitionistic fuzzy interior of A are defined by*
1. $cl(A) = \cap \{C: C \text{is} \text{an} \text{IFCS} \text{in} X \text{and} C \supseteq A\}$;
2. $int(A) = \cup \{D: D \text{is} \text{an} \text{IFOS} \text{in} X \text{and} D \subseteq A\}$;

It can be also shown that $cl(A)$ is an IFCS, $int(A)$ is an IFOS in X and A is and IFCS in X if and only if $cl(A) = A$; A is an IFOS in X if and only if $int(A) = A$

Proposition 2.2 *[4] Let (X,T) be an IFTS and A, B be intuitionistic fuzzy sets in X. Then the following properties hold:*
1. $cl(\overline{A}) = \overline{(int(A))}, int(\overline{A}) = \overline{(cl(A))}$;
2. $int(A) \subseteq A \subseteq cl(A)$.

Definition 2.7 *[6] An IFS A of an IFTS (X,T) is an*
1. intuitionistic fuzzy α-open set (IFαOS, in short) if $A \subseteq int(cl(int(A)))$
2. intuitionistic fuzzy α-closed set (IFαCS, in short) if $cl(int(cl(A))) \subseteq A$

Definition 2.8 *[6] Let (X,T) be an IFTS and $A = \{< x, \mu_A, \nu_A >; x \in X\}$ be an IFS in X. Then the intuitionistic fuzzy α-closure and intuitionistic fuzzy α-interior of A are defined by*
1. $\alpha cl(A) = \cap \{C: C \text{is} \text{an} \text{IF} CS in X \text{and} C \supseteq A\}$;
2. $\alpha int(A) = \cup \{D: D \text{is} \text{an} \text{IF} OS in X \text{and} D \subseteq A\}$;

Definition 2.9 *[6] A mapping $f: (X,T) \to (Y,S)$ from an intuitionistic fuzzy topological space (X,T) to another intuitionistic fuzzy topological space (Y,S) is said to be intuitionistic fuzzy α-continuous if $f^{-1}(A)$ is an intuitionistic fuzzy α-open set in X for each intuitionistic fuzzy open set A in Y*

Definition 2.10 *[5] Let (X,T) be an IFTS and $Y \subseteq X$. Then $(Y, T|Y)$ is called a subspace of (X,T) where $T|Y = \{G|Y = (\mu_G|Y, \nu_G|Y): G \in T\}$.*

III α-T_0, α-T_1 and α-T_2 separation axioms in intuitionistic fuzzy topological spaces

Definition 3.1 *An intuitionistic fuzzy topological space (X, T) is said to be intuitionistic fuzzy α-T_0 (briefly, IFα-T_0) if for every pair of intuitionistic fuzzy points $p = x_{(\alpha,\beta)}, q = y_{(\gamma,\eta)}$ with different supports, there exists an intuitionistic fuzzy α-open set M such that either $(p \subseteq M, q \nsubseteq M)$ or $(q \subseteq M, p \nsubseteq M)$*

Definition 3.2 *A mapping $f: (X,T) \to (Y,S)$ from an intuitionistic fuzzy topological space (X,T) to another intuitionistic fuzzy topological space (Y,S) is said to be*
1. intuitionistic fuzzy $\alpha^{\star}$-continuous if $f^{-1}(A)$ is an intuitionistic fuzzy α-open set in X for each intuitionistic fuzzy α-open set A in Y
2. intuitionistic fuzzy $\alpha^{\star\star}$-continuous if $f^{-1}(A)$ is an intuitionistic fuzzy open set in X for each intuitionistic fuzzy α-open set A in Y

Theorem 3.1 *An intuitionistic fuzzy topological space (X, T) is IFα- T_0 if and only if any two crisp intuitionistic fuzzy points with different supports have disjoint intuitionistic fuzzy α-closure.*

Proof. Let (X, T) be an intuitionistic fuzzy α-T_0 and $p = x_{(\alpha,\beta)}, q = y_{(\gamma,\eta)}$ be two crisp intuitionistic fuzzy points with supports x, y respectively, where $x \neq y$. Since (X, T) is $IF\alpha$-T_0, there exists an intuitionistic fuzzy α-open set M such that either. $(p \subseteq M, q \nsubseteq M)$ or $(q \subseteq M, p \nsubseteq M)$. If $p \subseteq M, q \nsubseteq M$, this implies that $q \subseteq I\alpha cl(q), I\alpha cl(q) \nsubseteq M$, since $p \nsubseteq M^c, p \nsubseteq (I\alpha cl(q))^c$. But $p \subseteq I\alpha cl(p)$ .Therefore $I\alpha cl(p) \neq I\alpha cl(q)$.

Conversely, Let p and q be any two intuitionistic fuzzy points with different supports x, y, respectively. Let p_1, q_1 be intuitionistic fuzzy points such that $p_1(x) = q_1(y) = 1$. By hypothesis $I\alpha cl(p_1) \neq I\alpha cl(q_1)$, but $p \leq p_1$ implies that $p^c \geq p_1^c \geq (I\alpha cl(p_1))^c$. Thus $(I\alpha cl(p_1))^c$ is an intuitionistic fuzzy α - open set such that $q \nsubseteq (I\alpha cl(p_1), P \subseteq I\alpha cl(p_1)$. Hence (X, T) is $IF\alpha$ T_0 .

Theorem 3.2 *Let f be an injective, intuitionistic fuzzy α*-continuous mapping from an intuitionistic fuzzy topological space (X, T) into fuzzy topological space (Y, S). If (Y, S) is an intuitionistic fuzzy α-T_0 space, then so is (X, T).*

Proof. Let $p = x_{(\alpha,\beta)}$ and $q = y_{(\gamma,\eta)}$ are intuitionistic fuzzy points with different supports in X, then f(p) and f(q) are two intuitionistic fuzzy points with different supports in Y. Since (Y, S) is an intuitionistic fuzzy $\alpha\ T_0$ -space, then there exists an intuitionistic fuzzy α- open set M such that $f(p) \subseteq M, f(q) \nsubseteq M$, or $f(q) \subseteq M, f(p) \nsubseteq M$. Consider the part. If $f(p) \subseteq M, f(q) \nsubseteq M$ .Its follows that, $p \subseteq f^{-1}(M), q \nsubseteq f^{-1}(M)$, where $f^{-1}(M)$ is an intuitionistic fuzzy α-open set in X. Hence (X, T) is a intuitionistic fuzzy α- T_0 space.

Theorem 3.3 *If $f: (X,T) \to (Y,S)$ is an injective, intuitionistic fuzzy α-continuous mapping and (Y, S) is an intuitionistic fuzzy -T_0 space, then (X, T) is intuitionistic fuzzy α- T_0 space.*

Proof. Let $p = x_{(\alpha,\beta)}$ and $q = y_{(\gamma,\eta)}$ be an intuitionistic fuzzy points with different supports in X. Then $f(p), f(q)$ are two intuitionistic fuzzy points with different supports in Y. Since (Y, S) is an intuitionistic fuzzy T_0 - space, then there exists a intuitionistic fuzzy open set M such that $f(p) \subseteq M, f(q) \nsubseteq M$ or $f(q) \subseteq M, f(p) \nsubseteq M$. Consider the part $f(q) \subseteq M, f(p) \nsubseteq M$. It follows that $q \subseteq f^{-1}(M), p \nsubseteq f^{-1}(M)$. Where $f^{-1}(M)$ is an intuitionistic fuzzy α-open set in X. Hence (X,T) is an intuitionistic fuzzy α -T_0 space.

Theorem 3.4 *If $f: (X,T) \to (Y,S)$ is an injective, intuitionistic fuzzy α**-continuous mapping and (Y, S) is an intuitionistic fuzzy α -T_0 space, then (X, T) is an intuitionistic fuzzy T_0 space.*

Proof. Let $p = x_{(\alpha,\beta)}$ and $q = y_{(\gamma,\eta)}$ be an intuitionistic fuzzy points with different supports in X. Then $f(p)$ and $f(q)$ are two intuitionistic fuzzy points with different supports in Y. Since (Y, S) is an intuitionistic fuzzy α- T_0 space, then there exists an intuitionistic fuzzy α-open set M such that $f(p) \subseteq M, f(q) \nsubseteq M$ or $f(q) \subseteq M, f(p) \nsubseteq M$. Consider the part $f(q) \subseteq M, f(p) \nsubseteq M$. It follows that $q \subseteq f^{-1}(M), p \nsubseteq f^{-1}(M)$, where $f^{-1}(M)$ is an intuitionistic fuzzy open set in X. Hence (X,T) is a fuzzy T_0 -space.

Definition 3.3 *An intuitionistic fuzzy topological space (X,T) is said to be a intuitionistic fuzzy α-T_1 (briefly, $IF\alpha$-T_1) if for every pair of intuitionistic fuzzy points $p = x_{(\alpha,\beta)}, q = y_{(\gamma,\eta)}$ with $x \neq y$, there exist a intuitionistic fuzzy α-open sets M and N such that $p \subseteq M, q \nsubseteq M$, and $q \subseteq N, p \nsubseteq N$.*

Theorem 3.5 *An intuitionistic fuzzy topological space (X, T) is an $IF\alpha$-T_1 if and only if every crisp intuitionistic fuzzy point is an intuitionistic fuzzy α-closed set.*

Proof. Let (X,T) be $IF\alpha$ - T_1 and $p_0 = x_{0(\alpha,\beta)}$ be an intuitionistic crisp fuzzy points with support x_0. Now, for any intuitionistic fuzzy point $p = x_{(\gamma,\eta)}$ with support x in X such that $x \neq x_0$, there exist an intuitionistic fuzzy α-open sets M and N such that $p_0 \subseteq M, p \nsubseteq M$ and $p \subseteq N, p_0 \nsubseteq N$. Since $p \nsubseteq N$,by proposition (2.5) every intuitionistic fuzzy set is considered as the union of all intuitionistic fuzzy points which are contained in it , we obtain in particular $p_0^c = \cup_{(p \subseteq p_0^c)} p$ from $p_0^c(x_0)1 - p_0(x_0) = 0$. We deduce that $p_0^c = \cup_{(p \subseteq p_0^c)} N$ and thus p_0^c is an intuitionistic fuzzy α-open set. Then p_0 is an intuitionistic fuzzy α-closed set.

Conversely, let $p_1 = x_{1(\alpha_1,\beta_1)}$ and $p_2 = x_{2(\alpha_2,\beta_2)}$ are intuitionistic fuzzy points with different supports x_1 and x_2. Also let$q_1 = x_{1(\gamma_1,\eta_1)}$ and $q_2 = x_{2(\gamma_2,\eta_2)}$ be an intuitionistic fuzzy points with different supports x_1 and x_2, respectively and such that $q_1(x_1) = q_2(x_2) = 1$. The intuitionistic fuzzy sets q_1^c and q_2^c are intuitionistic fuzzy α-open and satisfy the conditions $p_1 \subseteq q_2^c, p_2 \nsubseteq q_2^c$ and $p_2 \subseteq q_1^c, p_1 \nsubseteq q_1^c$ .Hence the space (X, T) is $IF\alpha$-T_1.

Remark 3.1 *Every intuitionistic fuzzy α-T_1 space is obviously fuzzy α-T_0 space. But the converse need not be true.*

Example 3.1 *Let $X = \{a, b\}$ and A, B be an intuitionistic fuzzy sets on X defined as follows $A = \langle X: ((\frac{a}{0.1}, \frac{b}{0.1}), (\frac{a}{0.3}, \frac{b}{0.3}))\rangle$; $B = \langle X: ((\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.1}, \frac{b}{0.1}))\rangle$. Let $T = \{\underline{0}, \underline{1}, A, B\}$. Then the space (X, T) is an intuitionistic fuzzy α-T_0 space but not intuitionistic fuzzy α-T_1.*

Theorem 3.6 *Let f be an injective, intuitionistic fuzzy α*-continuous mapping from an intuitionistic fuzzy topological space (X,T) into intuitionistic fuzzy topological space (Y, S). If (Y, S) is a intuitionistic fuzzy α- T_1 space, then so is X.*

Proof. Let $p = x_{(\alpha,\beta)}$ and $q = y_{(\gamma,\eta)}$ are intuitionistic fuzzy points with different supports in X, then $f(p)$ and $f(q)$ are two intuitionistic fuzzy points with different supports in Y. Since (Y, S) is an intuitionistic fuzzy αT_1-space, then there exists an intuitionistic fuzzy α- open sets M and N such that $f(p) \subseteq M, f(q) \nsubseteq M$ and $f(q) \subseteq N f(p) \nsubseteq N$. Its follows that, $p \subseteq f^{-1}(M), q \nsubseteq f^{-1}(M)$ and $q \subseteq f^{-1}(N), p \nsubseteq f^{-1}(N)$, where $f^{-1}(M)$ and $f^{-1}(N)$ are intuitionistic fuzzy α-open sets in X. Hence (X, T) is a intuitionistic fuzzy αT_1-space.

Theorem 3.7 *If $f:(X,T)\to(Y,S)$ is an injective, intuitionistic fuzzy α -continuous mapping and (Y, S) is an intuitionistic fuzzy T_1 space, then (X, T) is intuitionistic fuzzy αT_1- space.*

Proof. Similar to that of theorem 3.3.

Theorem 3.8 *If $f:(X,T)\to(Y,S)$ is an injective, intuitionistic fuzzy α**-continuous mapping and (Y, S) is an intuitionistic fuzzy αT_1 -space, then (X, T) is intuitionistic fuzzy T_1-space.*

Proof. Similar to that of theorem 3.4.

Definition 3.4 *A fuzzy topological space (X, T) is said to be a intuitionistic fuzzy stronger-αT_1 (briefly, IFα- T_s) if every intuitionistic fuzzy point is an intuitionistic fuzzy α-closed set.*

Definition 3.5 *An IFTS (X, T) is said to be an intuitionistic Fuzzy α- Hausdorff (or in short IFα-T_2) if for every pair of intuitionistic fuzzy poins $p = x_{(\alpha,\beta)}$ and $q = y_{(\gamma,\eta)}$ with different supports, there exist two intuitionistic fuzzy α-open sets M and N such that $p\subseteq M, q\nsubseteq M, q\subseteq N, p\nsubseteq N$, and $M\nsubseteq N$.*

Example 3.2 *Let $X=\{a,b\}$ and $T=\{\underline{0},\underline{1},A,B\}$ where $A=\langle X:((\frac{a}{1.0},\frac{b}{0.0}),(\frac{a}{0.0},\frac{b}{1.0}))\rangle$; $B=\langle X:((\frac{a}{1.0},\frac{b}{0.0}),(\frac{a}{0.0},\frac{b}{1.0}))\rangle$. Then (X,T) is an intuitionistic fuzzy α-T_0 ,α-T_1, α-T_2 spaces.*

Proposition 3.1 *Every subspace of IFα-T_2 space is IFα-T_2.*

Proof. Let (X,T) be a IFαT_2-space and Y be a subspace of X, where $T_Y=\{G_Y=\{\langle x,\mu_{G|Y}(x),\nu_{G|Y}(x)\rangle, x\in Y, G\in T\}$ and $G=\langle x,\mu_G(x),\nu_G(x)\rangle$ Let $p=x_{\alpha,\beta}$ and $q=y_{\gamma,\eta}$ be two distinct IFP in Y, that is they have distinct supports. Then, clearly $x_{\alpha,\beta}$ and $q=y_{\gamma,\eta}$ are also distinct IFP's in X and as X is IFα-T_2, therefore there exist two intuitionistic fuzzy α-open sets M and N such that $p\subseteq M, q\nsubseteq M, q\subseteq N, p\nsubseteq N$ and $M\nsubseteq N$. Thus, there exist $M_Y, N_Y\in T_Y$ such that $p\subseteq M_Y, q\subseteq N_Y$ and $M_Y\nsubseteq N_Y$.

Proposition 3.2 *Let (X,T) be an IFαT_2-space, if (X,T) is T_2 then $(X,T^\star)$ is a fuzzy Hausdorff fts (where, $T^\star = \{\mu_G(x): G\in T\}$)*

Proof. For any two fuzzy points x_r, y_s with distinct supports and $0<r,s<1$, we have that $p=x(r,1-r), q=y(s,1-s)$ are two distinct IFP's. Then there exists IFαOs $M=\langle x,\mu_M(X),\nu_M(x)\rangle$ and $N=\langle x,\mu_N(X),\nu_N(x)\rangle$ containing p and q respectively such that $M\nsubseteq N$. This implies that $r<\mu_M(x), s<\nu_N(y)$ and $x_r\in M, y_s\in N$ which are fuzzy α-open sets with $M\subseteq N^c$.

Theorem 3.9 *A fuzzy topological space (X,T) is a Fuzzy α-T_2-space if for every pair of intuitionistic fuzzy points $p = x_{(\alpha,\beta)}$ and $q=y_{(\gamma,\eta)}$ with different supports ,there exists an intuitionistic fuzzy α-open set U such that $p\subseteq U\subseteq I\alpha cl(U)\subseteq q^c$*

Proof. Let $p=x_{\alpha,\beta}$ and $q=y_{\gamma,\beta}$ are intuitionistic fuzzy points with different supports. Since (X,T) is an intuitionistic fuzzy α-T_2 space, then there exist two intuitionistic fuzzy α-open sets M and N such that $p\subseteq M, q\nsubseteq M, q\subseteq N$ and $M\nsubseteq N$. It follows that $p\subseteq M\subseteq I\alpha cl(M), q\nsubseteq I\alpha cl(M)$. Then $I\alpha int(I\alpha cl(M))\subseteq I\alpha cl(M)$. Let $U=I\alpha int(I\alpha cl(M))$ is an intuitionistic fuzzy α-open set. Hence $p\subseteq U\subseteq I\alpha cl(U)\subseteq q^c$

Theorem 3.10 *Let f be an injective, fuzzy α*-continuous mapping from an intuitionistic fuzzy topological space (X,T) into intuitionistic fuzzy topological space (Y,S). If (Y,S) is an intuitionistic fuzzy α- T_2 space, then so is X.*

Proof. Let $p=x_{(\alpha,\beta)}$ and $q=y_{(\gamma,\eta)}$ are intuitionistic fuzzy points with different supports in X, then $f(p)$ and $f(q)$ are two intuitionistic fuzzy points with different supports in Y. Since (Y,S) is an intuitionistic fuzzy αT_2- space, then there exists an intuitionistic fuzzy α-open sets M and N in (Y,S) such that $f(p)\subseteq M, f(q)\nsubseteq M, f(q)\subseteq N, f(p)\nsubseteq N$ and $M\nsubseteq N$. Its follows that, $p\subseteq f^{-1}(M), q\nsubseteq f^{-1}(M), q\subseteq f^{-1}(N), p\nsubseteq f^{-1}(N)$ and $f^{-1}(M)\subseteq f^{-1}(N)$, where $f^{-1}(M)$ and $f^{-1}(N)$ are intuitionistic fuzzy α-open sets in X. Hence (X,T) is a intuitionistic fuzzy αT_2-space.

Theorem 3.11 *If $f:(X,T)\to(Y,S)$ is an injective, intuitionistic fuzzy α-continuous mapping and (Y,S) is an intuitionistic fuzzy T_2- space, then (X,T) is intuitionistic fuzzy αT_2 -space.*

Proof. Similar to that of theorem 3.3 .

Theorem 3.12 *If $f:(X,T)\to(Y,S)$ is an injective, intuitionistic fuzzy α**- continuous mapping and (Y,S) is intuitionistic fuzzy α- T_2 space, then (X,T) is intuitionistic fuzzy T_2 -space.*

Proof. Similar to that of theorem 3.4

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