ROUGH TOPOLOGY ON TWO EQUIVALENCE RELATIONS

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Abstract : In this paper a new rough topology is introduced based on two equivalence relations. Some of its properties in terms of Lower and upper approximations are obtained.

Keywords—Equivalence relations, Rough topology, Neighbourhood , Boundary Regions

I. INTRODUCTION

Nirmala Rebecca Paul introduced the Rough topology in [4] using single equivalence relation. In this paper new rough topologies based on two equivalence relations are introduced and some of the properties are discussed. Moreover, new rough topology is approached via graph theory.

II. PRELIMINARIES

Definition 2.1 *Let U be a non-empty finite set of objects called the universe, R be an equivalence relation on U named as the indiscernibility relation. Let X⊆U. Here R(x) denotes the equivalence class determined by x.*

(i) The Lower approximation of X with respect to R is defined and denoted by $L_R(X) = \bigcup_{x\epsilon U}\{x|R(x) \subseteq X\}$
(ii) The Upper approximation of X with respect to R is defined and denoted by $U_R(X) = \bigcup_{x\epsilon U}\{x|R(x) \cap X \neq \emptyset\}$
(iii) The Boundary region of X with respect to R is defined and denoted by $B_R(X) = U_R(X)-L_R(X)$

Definition 2.2[1] *Let U be the universe. Let M, N and O be any three equivalence relations on U named as indiscernibility relations. Let X⊆U. Here M(x), N(x) and O(x) denote the equivalence classes determined by x.*

(i) The Multi-lower approximation of X with respect to M, N and O is defined and denoted by $L_{M+N+O}(X)=\bigcup_{x\epsilon U}\{x|M(x) \subseteq X \text{ or } N(x) \subseteq X \text{ or } O(x) \subseteq X\}$

(ii) The Multi-upper approximation of X with respect to M, N and O is defined and denoted by $U_{M+N+O}(X) = \bigcup_{x\epsilon U}\left\{x \middle| \begin{matrix} M(x)\cap X \neq \emptyset \text{ and } N(x)\cap X \neq \emptyset \text{ and} \\ O(x)\cap X \neq \emptyset \end{matrix}\right\}$

(iii) The Multi-boundary region of X with respect to M, N and O is defined and denoted by $B_{M+N+O}(X) = U_{M+N+O}(X) - L_{M+N+O}(X)$.

Definition 2.3 [2] Let U be the universe. Let R and S be any two equivalence relations on U named as indiscernibility relations. Let X⊆U. Here R(x) and S(x) denote the equivalence classes determined by x.

(i)The Multi star lower approximation of X with respect to R is defined and denoted by $L_{R*S}(X)=\bigcup_{x\epsilon U}\{x|R(x) \subseteq X \text{ and } S(x) \subseteq X\}$

(ii) The Multi star upper approximation of X with respect to R and S is defined and denoted by $U_{R*S}(X) = \bigcup_{x\epsilon U}\{x|R(x)\cap X \neq \emptyset \text{ or } S(x)\cap X \neq \emptyset\}$

iii) The Multi-boundary region of X with respect to R and S is defined and denoted by $B_{R*S}(X)=U_{R*S}(X)-L_{R*S}(X)$.

Definition 2.4[4] *Let U be the universe, Let R be an equivalence relation on U then $\tau_R(X) = \{U, \emptyset, B_R(X)\}$ forms a topology called rough topology.*

III. NEW ROUGH TOPOLOGY ON TWO EQUIVALENCE RELATIONS

Definition 3.1 Let U be the universe. Let R and S be any two equivalence relations on U named as indiscernibility relations. Let X⊆U.

(i) The Lower approximation of X with respect to R and S is defined and denoted by $L_{R\oplus S}(X) = \bigcup_{x\epsilon U}\{x|R(x) \subseteq X \text{ or } S(x) \subseteq X\}$ where R(x) and S(x) denote the equivalence classes determined by x.

(ii) The Upper approximation of X with respect to R and S is defined and denoted by $U_{R\oplus S}(X) = \bigcup_{x\epsilon U}\{x|R(x)\cap X \neq \emptyset \text{ or } S(x)\cap X \neq \emptyset\}$where R(x) and S(x) denote the equivalence classes determined by x.

(iii) The boundary region of X with respect to R and S is defined and denoted by $B_{R\oplus S}(X)=U_{R\oplus S}(X)-L_{R\oplus S}(X)$.

Definition 3.2 *Let U be the universe. Let R and S be equivalence relations on U and $\tau_{R\oplus S}(X) = \{U, \emptyset, B_{R\oplus S}(X)\}$ where X⊆U and $\tau_R(X)$ satisfies the following axioms.*

(i) U and $\emptyset \epsilon \tau_{R\oplus S}(X)$. (ii) The union of the elements of any sub collection of $\tau_{R\oplus S}(X)$ is in $\tau_{R\oplus S}(X)$.
(iii) The intersection of the elements of any finite sub collection of $\tau_{R\oplus S}(X)$ is in $\tau_{R\oplus S}(X)$.

That is $\tau_{R\oplus S}(X)$ forms a topology called new rough topology on U with respect to two equivalence relations R and S. Also $\tau_{R+S}(X), \tau_{R*S}(X)$ form rough topologies on U with respect to two equivalence relations R and S where

$\tau_{R+S}(X) = \{U, \emptyset, B_{R+S}(X)\}$; $\tau_{R*S}(X) = \{U, \emptyset, B_{R*S}(X)\}$. $\tau_{R+S}(X)$ and $\tau_{R*S}(X)$ are called new multi topology and new multi star topology on two equivalence relations.

Example 3.3 Let U= {a, b, c, d, e} Let X = {a, b, c, d}. Let U/R={{a},{b, c}, {d}, {e}}, U/S = {{a, b, c}, {d, e}}
$L_{R\oplus S}(X)$ = {a, b, c, d}; $U_{R\oplus S}(X)$ ={a, b, c, d, e}; $B_{R\oplus S}(X)$ = {e}

Lemma 3.4 *For any subset X of U,*

(i) $L_{R\oplus S}(X) = L_R(X) \cup L_S(X)$ (ii) $U_{R\oplus S}(X) = U_R(X) \cup U_S(X)$ (iii) $B_{R\oplus S}(X) \subseteq B_R(X) \cup B_S(X)$ (iv) $L_{R\oplus S}(X) \subseteq X$ (v) $X \subseteq U_{R\oplus S}(X)$

Proof

(i) $L_{R\oplus S}(X) = \cup_{x\epsilon U}\{x|R(x) \subseteq X \text{ or } S(x) \subseteq X\}$

Let $x \in L_{R\oplus S}(X)$

$x \in L_{R\oplus S}(X)$ iff[$R(x) \subseteq X$ or $S(x) \subseteq X$] iff [$x \in L_R(X)$ or $x \in L_s(X)$] iff $x \in L_R(X) \cup L_s(X)$; Therefore $L_{R\oplus S}(X) = L_R(X) \cup L_S(X)$

(ii) $U_{R\oplus S}(X) = U_R(X) \cup U_S(X)$

Let $x \in U_{R\oplus S}(X)$

$x \in U_{R\oplus S}(X)$ iff $R(x) \cap X \neq \emptyset$ or $x \in S(x)\,) \cap X \neq \emptyset$ iff $x \in U_R(X)$ or $x \in U_s(X)$ iff $x \in U_R(X) \cup U_s(X)$.

Therefore $U_{R\oplus S}(X) = U_R(X) \cup U_S(X)$

(iii) $B_{R\oplus S}(X) \subseteq B_R(X) \cup B_S(X)$

Let $x \in B_{R\oplus S}(X) \Longrightarrow x \in U_{R\oplus S}(X) \cap [L_{R\oplus S}(X)]^c \Longrightarrow x \in \{U_R(X) \cup U_S(X)\} \cap [L_{R\oplus S}(X)]^c$

$\Longrightarrow x \in \{U_R(X) \cap [L_{R\oplus S}(X)]^c\} \cup U_s(X) \cap [L_{R\oplus S}(X)]^c\}$

$\Longrightarrow x \in \{U_R(X) \cap [L_R(X)]^c \cap [L_s(X)]^c\} \cup \{U_s(X) \cap [L_R(X)]^c \cap [L_s(X)]^c\} \subseteq [U_R(X) \cap [L_R(X)]^c] \cup [U_s(X) \cap [L_s(X)]^c]$

$\Longrightarrow x \in B_R(X) \cup B_S(X)$. Therefore $B_{R\oplus S}(X) \subseteq B_R(X) \cup B_S(X)$

(iv) $L_{R\oplus S}(X) \subseteq X$ and $X \subseteq U_{R\oplus S}(X)$ (by Definition 3.1)

Remark 3.5 $B_{R\oplus S}(X) \neq B_R(X) \cup B_S(X)$ {Equality does not hold}

Example 3.6 Let U = {a, b, c}; U/R = {{a},{b},{c}} and U/S = {{a, b}{c}} Let X = {a, c}

$L_R(X) = \{a, c\}$; $U_R(X) = \{a, c\}$; $B_R(X) = \{\emptyset\}$; $L_S(X) = \{c\}$; $U_S(X) = \{a, b, c\}$; $B_S(X) = \{a, b\}$; $B_R(X) \cup B_S(X)$={ a, b }

$L_{R\oplus S}(X) = \{a, c\}$; $U_{R\oplus S}(X) = \{a, b, c\}$; $B_{R\oplus S}(X) = \{b\} \neq B_R(X) \cup B_S(X)$

Lemma 3.7 *(i)* $L_{R\oplus S}(\emptyset) = \emptyset$ (ii) $U_{R\oplus S}(\emptyset) = \emptyset$ *(iii)* $L_{R\oplus S}(U) = U$ (iv) $U_{R\oplus S}(U) = U$ *(v)* $L_{R\oplus S}(X) = L_{S\oplus R}(X)$ *(vi)* $U_{R\oplus S}(X) = U_{S\oplus R}(X)$

Proof

(i) $L_{R\oplus S}(\emptyset) = \cup_{x\epsilon U}\{x|R(x) \subseteq \emptyset \text{ or } S(x) \subseteq \emptyset\} = \emptyset$ *(ii)* $U_{R\oplus S}(\emptyset) = \cup_{x\epsilon U}\{x|R(x) \cap \emptyset \neq \emptyset \text{ or } S(x) \cap \emptyset \neq \emptyset\} = \emptyset$

(iii) $L_{R\oplus S}(U) = \cup_{x\epsilon U}\{x|R(x) \subseteq U \text{ or } S(x) \subseteq U\} = U$ *(iv)* $U_{R\oplus S}(U) = \cup_{x\epsilon U}\{x|R(x) \cap U \neq \emptyset \text{ or } S(x) \cap U \neq \emptyset\} = U$

(v) $L_{R\oplus S}(X) = L_{S\oplus R}(X)$ *(by Definition 3.1);* *(vi)* $\cup_{x\epsilon U}\{x|R(x) \cap \emptyset \neq \emptyset \text{ or } S(x) \cap \emptyset \neq \emptyset\} = \emptyset$

Note 3.8

1. $L_{R\oplus S}(X^c) \neq [U_{R\oplus S}(X)]^c$ *2.* $U_{R\oplus S}(X^c) \neq [L_{R\oplus S}(X)]^c$.

Example 3.9 Let U={a, b, c, d},U/R ={{a},{b},{c},{d}},U/S = {{a, b},{c, d}}. Let X = {a, b, c}.Therefore X^c ={d}; $U_{R\oplus S}(X)$={a, b, c, d}; $[U_{R\oplus S}(X)]^c = \emptyset$; $L_{R\oplus S}(X^c)$={d}$\neq$[$U_{R\oplus S}(X)]^c$ Therefore $L_{R\oplus S}(X^c) \neq [U_{R\oplus S}(X)]^c$

Example 3.10 Let U={a,b,c,d};U/R={{a},{b},{c},{d}}; U/S={{a,b},{c,d}}. Let X={a,b,c}.Therefore X^c={d} $U_{R\oplus S}(X^c)$={d,c}$\neq[L_{R\oplus S}(X)]^c$ where $L_{R\oplus S}(X)$={a,b,c}; $[U_{R\oplus S}(X)]^c$={d}

Theorem 3.11 For any Subset X of U

(i) . $L_{R*S}(X) \subseteq L_{R\oplus S}(X)$ *(ii)* $U_{R*S}(X) = U_{R\oplus S}(X)$ *(iii)* $B_{R\oplus S}(X) \subseteq B_{R*S}(X)$

Proof

(i) Let $x \in L_{R*S}(X) \Longrightarrow R(x) \subseteq X$ and $S(x) \subseteq X \Longrightarrow x \in L_R(X) \cap L_S(X) \subseteq L_R(X) \cup L_S(X) = L_{R\oplus S}(X)$. Therefore $L_{R*S}(X) \subseteq L_{R\oplus S}(X)$

(ii) $U_{R*S}(X) = \cup_{x\epsilon U}\{x|R(x) \cap X \neq \emptyset \text{ or} S(x) \cap X \neq \emptyset\} = U_{R\oplus S}(X)$.Therefore $U_{R*S}(X) = U_{R\oplus S}(X)$

(iii) $L_{R*S}(X) \subseteq L_{R\oplus S}(X)$ {by (i)}

$[L_{R\oplus S}(X)]^c \subseteq [L_{R*S}(X)]^c$ { by Property of Sets}; $B_{R\oplus S}(X) = U_{R\oplus S}(X) - L_{R\oplus S}(X)$

$= U_{R\oplus S}(X) \cap [L_{R\oplus S}(X)]^c \subseteq U_{R*S}(X) \cap [L_{R*S}(X))]^c = U_{R*S}(X) - L_{R*S}(X) = B_{R*S}(X)$. Therefore $B_{R\oplus S}(X) \subseteq B_{R*S}(X)$

Corollary 3.12 *The topology $\tau_{R\oplus S}(X)$ is coarser than $\tau_{R*S}(X)$*

Proof follows from the theorem 3.11 (iii)

Theorem 3.13 For any Subset X of U,

(i) $L_{R+S}(X) = L_{R\oplus S}(X)$; *(ii)* $U_{R+S}(X) \subseteq U_{R\oplus S}(X)$ *(iii)* $B_{R+S}(X) \subseteq B_{R\oplus S}(X)$

Proof

(i) $L_{R+S}(X) = \cup_{x\epsilon U}\{x | R(x) \subseteq X \text{ or } S(x) \subseteq X\} = L_{R\oplus S}(X)$. Therefore $L_{R+S}(X) = L_{R\oplus S}(X)$

(ii) $U_{R+S}(X) = \cup_{x\epsilon U}\{x | R(x) \cap X \neq \emptyset \text{ and } S(x) \cap X \neq \emptyset\} \subseteq \cup_{x\epsilon U}\{x | R(x) \cap X \neq \emptyset \text{ or } S(x) \cap X \neq \emptyset\} = U_{R\oplus S}(X)$

Therefore $U_{R+S}(X) \subseteq U_{R\oplus S}(X)$

(iii) $L_{R+S}(X) = L_{R\oplus S}(X)$; $U_{R+S}(X) \subseteq U_{R\oplus S}(X)$; $B_{R+S}(X) = U_{R+S}(X) - L_{R+S}(X) = U_{R+S}(X) \cap [L_{R+S}(X)]^c \subseteq U_{R\oplus S}(X) \cap [L_{R\oplus S}(X)]^c$ $= U_{R\oplus S}(X) - L_{R\oplus S}(X) = B_{R\oplus S}(X)$. Therefore $B_{R+S}(X) \subseteq B_{R\oplus S}(X)$

Corollary 3.14 *The topology $\tau_{R\oplus S}(X)$ is finer than $\tau_{R+S}(X)$*

Proof follows from the theorem 3.13(iii)

Theorem 3.15 For any Subset X and Y of U,

(i) $L_{R\oplus S}(X \cap Y) \subseteq L_{R\oplus S}(X) \cap L_{R\oplus S}(Y)$ *(ii)* $U_{R\oplus S}(X) \cup U_{R\oplus S}(Y) \subseteq U_{R\oplus S}(X \cup Y)$

Proof

(i) $L_{R\oplus S}(X \cap Y) = \cup_{x\epsilon U}\{x | R(x) \subseteq X \cap Y \text{ or } S(x) \subseteq X \cap Y\}$

Let $x \in L_{R\oplus S}(X \cap Y) \Rightarrow R(x) \subseteq X \cap Y$ or $S(x) \subseteq X \cap Y \Rightarrow R(x) \subseteq X$ or $S(x) \subseteq X$ and $R(x) \subseteq Y$ or $S(x) \subseteq Y$

$\Rightarrow x \in L_{R\oplus S}(X)$ and $x \in L_{R\oplus S}(Y)$ $\Rightarrow x \in L_{R\oplus S}(X) \cap L_{R\oplus S}(Y)$. Therefore $L_{R\oplus S}(X \cap Y) \subseteq L_{R\oplus S}(X) \cap L_{R\oplus S}(Y)$

(ii) Let $x \in U_{R\oplus S}(X) \cup U_{R\oplus S}(Y)$

$\Rightarrow R(x) \cap X \neq \emptyset$ or $R(x) \cap Y \neq \emptyset$ or $S(x) \cap X \neq \emptyset$ or $S(x) \cap Y \neq \emptyset$

$\Rightarrow x \in \cup_{x\epsilon U}\{x | R(x) \cap (X \cup Y) \neq \emptyset \text{ or } S(x) \cap (X \cup Y) \neq \emptyset\} = U_{R\oplus S}(X \cup Y)$

$U_{R\oplus S}(X) \cup U_{R\oplus S}(Y) \subseteq U_{R\oplus S}(X \cup Y)$

Theorem 3.16 For any Subsets X and Y of U

(i) $X \subseteq Y \Rightarrow L_{R\oplus S}(X) \subseteq L_{R\oplus S}(Y)$ *(ii)* $X \subseteq Y \Rightarrow U_{R\oplus S}(X) \subseteq U_{R\oplus S}(Y)$

Proof

(i) Let $X \subseteq Y$ Let $x \in L_{R\oplus S}(X) \Rightarrow R(x) \subseteq X$ or $S(x) \subseteq X \Rightarrow R(x) \subseteq X \subseteq Y$ or $S(x) \subseteq X \subseteq Y \Rightarrow R(x) \subseteq Y$ or $S(x) \subseteq Y \Rightarrow x \in L_{R\oplus S}(Y)$

$\Rightarrow L_{R\oplus S}(X) \subseteq L_{R\oplus S}(Y)$. Therefore $X \subseteq Y \Rightarrow L_{R\oplus S}(X) \subseteq L_{R\oplus S}(Y)$

(ii) Let $X \subseteq Y$ Let $x \in U_{R\oplus S}(X) \Rightarrow R(x) \cap X \neq \emptyset$ or $S(x) \cap X \neq \emptyset \Rightarrow R(x) \cap Y \neq \emptyset$ or $S(x) \cap Y \neq \emptyset$ since $X \subseteq Y$

$\Rightarrow x \in U_{R\oplus S}(Y) \Rightarrow U_{R\oplus S}(X) \subseteq U_{R\oplus S}(Y)$. Therefore $X \subseteq Y \Rightarrow U_{R\oplus S}(X) \subseteq U_{R\oplus S}(Y)$

IV. GRAPH THEORETIC APPROACH

Definition 4.1: Let $G_1 = G(V, E_1)$ and $G_2 = G(V, E_2)$ be two graphs with same vertex set V(G) and $v \in V(G)$. The neighbourhood of v with respect to G_1 is $N_1(v) = \{v\} \cup \{u \in V(G): uv \in E_1(G)\}$.The neighbourhood of v with respect to G_2 is $N_2(v) = \{v\} \cup \{u \in V(G): uv \in E_2(G)\}$

Definition 4.2: Let $G_1 = G(V, E_1)$ and $G_2 = G(V, E_2)$ be two graphs and H be a graph with a vertex set $V(H) \subseteq V(G)$. Then

(i) The Lower approximation $L: P(V(G)) \rightarrow P(V(G))$ is $L_{N_1\oplus N_2}(V(H)) = \cup_{v\epsilon V(G)}\{v: N_1(v) \subseteq V(H) \text{ or } N_2(v) \subseteq V(H)\}$;

(ii) The upper approximation $U: P(V(G)) \rightarrow P(V(G))$ is $U_{N_1\oplus N_2}(V(H)) = \cup_{v\epsilon V(G)}\{v: N_1(v) \cap V(H) \neq \emptyset \text{ or } N_2(v) \cap V(H) \neq \emptyset\}$;

(iii) The boundary region is $B_{N_1\oplus N_2}(V(H)) = U_{N_1\oplus N_2}(V(H)) - L_{N_1\oplus N_2}(V(H))$.

Definition4.3: Let G_1 and G_2. be a graphs. Let $N_1(v)$ be a neighbourhood of v with respect to G_1 and $N_2(v)$ be a neighbourhood of v with respect to G_2.Let H be a subgraph with a vertex set $V(H) \subseteq V(G)$.

$\tau_{N_1\oplus N_2}(V(H)) = \{V(G), \emptyset, B_{N_1\oplus N_2}(V(H))\}$ forms a topology on V(G) called new rough topology on V(G) with respect to V(H). $(V(G), \tau_{N_1\oplus N_2}(V(H)))$ forms a new rough topological space induced by graphs G_1 and G_2.

Example 4.4

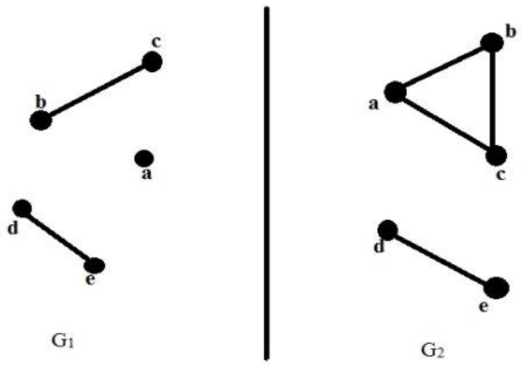

Figure 4.5

The following table shows rough topology for possible graphs H for the figure 4.4 such that V(H) ⊆V(G).

V(H)	$\tau_{N_1 \oplus N_2}(V(H))$
{a}	{V(G), ∅,{b,c}}
{b}	{V(G), ∅,{b,c,a}}
{c}	{V(G), ∅,{b,c,a}}
{d}	{V(G), ∅,{d,e}}
{e}	{V(G), ∅, {d,e}}
{a, b}	{V(G), ∅,{b,c}}
{a, c}	{V(G), ∅,{b,c}}
{a, d}	{V(G), ∅,{b,c,d,e}}
{a, e}	{V(G), ∅,{b,c,d,e}}
{b, c}	{V(G), ∅,{a}}
{b, d}	{V(G), ∅}
{b, e}	{V(G), ∅}
{c, d}	{(V(G), ∅}
{c, e}	{V(G), ∅}
{d, e}	{V(G), ∅}
{a, b, c}	{V(G), ∅}
{a, b, d}	{V(G), ∅,{b,c,d,e}}
{a, b, e}	{V(G), ∅,{b,c,d,e}}
{b, c, d}	{V(G), ∅,{a,d,e}}
{b, c, e}	{V(G), ∅,{a,d,e}}
{c, d, e}	{V(G), ∅,{a,b,c}}
{a, c, e}	{V(G), ∅,{b,c,d,e}}
{a, e, d}	{V(G), ∅,{b,c}}
{e, b, d}	{V(G), ∅,{a,b,c}}

{a, c, d}	{V(G), ∅,{b,c,d,e}}
{a, b, c, d}	{V(G), ∅,{e}}
{a, b, d, e}	{V(G). ∅,{b,c}}
{a, b, c, e}	{V(G), ∅,{d,e}}
{a, e, c, d}	{V(G), ∅,{b,c}}
{b, c, d, e}	{V(G), ∅,{a}}
{a, b, c, d, e}	{V(G), ∅}
∅	{V(G), ∅}

Table 4.6

Theorem 4.7 Let G_1=G(V,E_1) and G_2=G(V, E_2)be two graphs.Let H and K be graphs such that V(H) ⊆ V(G) and V(K) ⊆ V(G).

(i) If V(H) ⊆ V(K) then $L_{N_1 \oplus N_2}$(V(H)) ⊆ $L_{N_1 \oplus N_2}$(V(K)) and $U_{N_1 \oplus N_2}$(V(H)) ⊆ $U_{N_1 \oplus N_2}$(V(K))

(ii) $L_{N_1 \oplus N_2}$(V(H))∪ $L_{N_1 \oplus N_2}$ (V(K))⊆ $L_{N_1 \oplus N_2}$(V(H) ∪ V(K))

(iii) $U_{N_1 \oplus N_2}$(V(H) ∩ V(K)) ⊆ $U_{N_1 \oplus N_2}$(V(H))∩ $U_{N_1 \oplus N_2}$ (V(K))

Proof

(i) Let V(H) ⊆ V(K) and Let v ∈ $L_{N_1 \oplus N_2}$(V(H) ⟹ N_1(v) ⊆ V(H) or N_2(v) ⊆ V(H)

⟹ N_1(v) ⊆ V(K) or N_2(v) ⊆ V(K) (Since V(H) ⊆ V(K))

⟹ v ∈ $L_{N_1 \oplus N_2}$(V(K)).

Therefore $L_{N_1 \oplus N_2}$(V(H)) ⊆ $L_{N_1 \oplus N_2}$(V(K)) if V(H) ⊆ V(K). Let v ∈ $U_{N_1 \oplus N_2}$(V(H)) ⟹ N_1(v)∩V(H)≠ ∅ or N_2(v)∩V(H)≠ ∅

⟹ N_1(v)∩V(K)≠ ∅ or N_2(v)∩V(K)≠ ∅ (since v(H) ⊆ V(K))

⟹ v ∈ $U_{N_1 \oplus N_2}$(V(K)). Therefore $U_{N_1 \oplus N_2}$(V(H)) ⊆ $U_{N_1 \oplus N_2}$(V(K))

(ii) Since V(H) ⊆V(H) ∪ V(K) and V(K) ⊆V(H) ∪ V(K)

by (i) $L_{N_1 \oplus N_2}$(V(H)) ⊆ $L_{N_1 \oplus N_2}$(V(H)∪V(K)) and $L_{N_1 \oplus N_2}$(V(K)) ⊆$L_{N_1 \oplus N_2}$(V(H)∪V(K))

$L_{N_1 \oplus N_2}$(V(H))∪ $L_{N_1 \oplus N_2}$(V(K)) ⊆$L_{N_1 \oplus N_2}$(V(H)∪V(K))

(iii) V(H)∩V(K) ⊆ V(H) and V(H)∩V(K) ⊆ V(K)

$U_{N_1 \oplus N_2}$(V(H)∩V(K)) ⊆ $U_{N_1 \oplus N_2}$(V(H)) and $U_{N_1 \oplus N_2}$(V(H)∩V(K)) ⊆ $U_{N_1 \oplus N_2}$(V(K)) {by (i)}
$U_{N_1 \oplus N_2}$(V(H)∩V(K)) ⊆ $U_{N_1 \oplus N_2}$(V(H))∩ $U_{N_1 \oplus N_2}$(V(K))

Note 4.8: Equality does not hold in (ii) and (iii) of theorem 4.7

Consider the example 4.4 Let V(H)={a, b, e} and Let V(K)= {d}; $L_{N_1 \oplus N_2}$(V(H))∪ $L_{N_1 \oplus N_2}$(V(K))= {a}

$L_{N_1 \oplus N_2}$(V(H)∪V(K)) = {a, d, e} ≠ $L_{N_1 \oplus N_2}$(V(H))∪ $L_{N_1 \oplus N_2}$(V(K)). Let V(H) = {a, d, e} and V(K)= {b, d, e}
$U_{N_1 \oplus N_2}$(V(H)∩V(K)) = {d, e}; $U_{N_1 \oplus N_2}$V(H)∩ $U_{N_1 \oplus N_2}$V(K) = {a, b,c, d,e} ≠ $U_{N_1 \oplus N_2}$(V(H)∩V(K))

V. Conclusion

New Topology with respect to two equivalence relations are discussed. Moreover, a graph theoretic approach to new rough topology is introduced. These results can be used for further studies in Rough topology based on equivalence relations.

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