LOCATING DOMINATION NUMBER FOR SOME SPECIAL TYPES OF GRAPHS

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Abstract : Let G (V, E) be a simple, finite, undirected connected graph. A non – empty set $S \subseteq V$ of a graph G is a dominating set, if every vertex in V – S is adjacent to atleast one vertex in S. A dominating set $S \subseteq V$ is called a locating dominating set, if for any two vertices v, w ∈ V – S, $N(v) \cap S \neq N(w) \cap S$. The locating domination number $\gamma_{\ell d}$ is the minimum cardinality of a locating dominating set. In this paper, the location domination number for some special types of graphs namely like Fan, Sun graph, Jelly Fish graph, Umbrella graph and Lotus graph are studied.

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I. INTRODUCTION

Let G = (V, E) be a simple graph of order n. For v ∈ V(G), the neighborhood $N_G(v)$ (or simply N(v)) of v is the set of all vertices adjacent to v in G. The concept of domination in graphs was introduced by Ore [5]. A non – empty set S ⊆ V(G) of a graph G is a dominating set, if every vertex in V(G) – S is adjacent to some vertex in S. A special case of dominating set S is called a locating dominating set. It was defined by D. F. Rall and P. J. Slater in [6]. A dominating set S in a graph G is called a locating dominating set in G, if for any two vertices v, w ∈ V(G) – S, $N_G(v) \cap S$, $N_G(w) \cap S$ are distinct . The location dominating number of G $\gamma_{\ell d}$ is defined as the minimum number of vertices of a locating dominating set in G. In this paper, the locating domination number for some special types of graphs like Fan, Sun graph, Jelly Fish graph, Umbrella graph, Lotus graph are studied. For graph theoretic notations [1] is referred.

II. PRIOR RESULTS The following results are obtained in [4] & [6].

Theorem 2.1[6]: For every non – trivial simple connected graph G, $1 \leq \gamma_{\ell d}(G) \leq n - 1$.

Theorem 2.2[6]:$\gamma_{\ell d}(G) = 1$ if and only if $G \cong K_2$.

Theorem 2.3 [6]:$\gamma_{\ell d}(K_n) = n - 1$, where K_n is a complete graph on n vertices.

Theorem 2.4 [6]:$\gamma_{\ell d}(K_n - e) = n - 1$, where K_n is a complete graph on n vertices and $e \in E(K_n)$

Observation 2.5 [4]: If S is an locating dominating set of G(V, E) with | S | = k, then V(G) – S contains atmost $nC_1 + nC_2 + \ldots + nC_k$ vertices.

III. MAIN RESULTS

In this section, $\gamma_{ld}(P_n)$ is found and using this result, γ_{ld} of the Sun Graph, Lotus Graph, Jelly Fish Graph and the Umbrella Graph are obtained.

Theorem 3.1 : For a path P_n on n vertices, $\gamma_{ld}(P_n) = \begin{cases} 2k & ; if\ n = 5k \\ 2k+1 & ; if\ n = 5k+1\ 0r\ 5k+2 \\ 2k+2 & ; if\ n = 5k+3\ or\ 5k+4 \end{cases}$; where k = 0, 1, 2,3,. . .

Proof: Let $V(P_n) = \{ v_1, v_2, \ldots, v_n\}$ where v_1 and v_n are pendant vertices and all the other vertices are of degree 2 and let S be a minimum locating dominating set. The set $\{v_1, v_2, v_3, \ldots, v_{5k}\}$, $5k \leq n$ can be partitioned into k sets S_i such that $|S_i| = 5$ and $\langle S_i \rangle \cong P_5$, i = 1, 2, 3, …, k. Let D_i be the set of supports of S_i and $D = \bigcup_{i=1}^{k} D_i$. In a path P_5 there are exactly two supports. Therefore, $|D_i| = 2$ for i = 1, 2, . . , k. Hence, $|D| = \sum_{i=1}^{k} |D_i| = 2k$. Now let us consider the following cases,

Case (i): n = 5k

Then S = D. Hence, $\gamma_{ld}(P_n) = 2k$.

Case (ii): n = 5k + 1 or 5k + 2

In this case we observe that $D \cup \{v_{n-2}\}$ forms a γ_{ld} – set. Hence, $\gamma_{ld}(P_n) = 2k + 1$.

Case (ii): n = 5k + 3 or 5k + 4

In this case we observe that $D \cup \{v_{n-2}, v_n\}$ forms a γ_{ld} – set. Hence, $\gamma_{ld}(P_n) = 2k + 2$.

Hence the theorem.

Corollary 3.2: For a cycle C_n on n vertices, $n \geq 3$, $\gamma_{ld}(C_n) = \begin{cases} 2k & ; if\ n = 5k \\ 2k+1 & ; if\ n = 5k+1\ 0r\ 5k+2 \\ 2k+2 & ; if\ n = 5k+3\ or\ 5k+4 \end{cases}$

Notations 3.3 :1. By attaching a pendant edge e at a vertex v of a graph G, we mean merging a vertex of e to v.

2. By attaching P_n ($n \geq 2$) to a vertex v of a graph, we mean merging a pendant vertex of P_n to v.

Definition 3.4 : Join of the two graphs

Let G_1 and G_2 be any two graphs then the graph join (or complete join) $G_1 + G_2$ of two graphs is their graph union with all the edges that connect the vertices of the graph $G_1$with the vertices of the graph G_2. It is a commutative operation (for unlabelled graphs).

Definition 3.5 : Fan

The fan F_n on n vertices is the join of the two graphs, one is the path P_{n-1} on $n - 1$ vertices and the other is the complete graph K_1 on one vertex. That is, $F_n \cong P_{n-1} + K_1$.

Theorem 3.6 : Let F_n be a fan on n vertices. Then $\gamma_{ld}(F_n) = \gamma_{ld}(P_{n-1})$, $n \geq 6$.

Proof: Let $F_n \cong P_{n-1} + K_1$ and $V(K_1) = v_1$. Let S be a minimum locating dominating set. By adding the v_1 to P_{n-1} the γ_{ld} – set will not be affected since, $deg(v_1) = n - 1$. Also, $v_1 \in V - S$ ensures that $\gamma_{ld}(F_n) = \gamma_{ld}(P_{n-1})$. Hence the theorem follows.

Remark 3.6: $\gamma_{ld}(F_n) = 3$, for n = 3, 4, 5.

Definition 3.7: Sun Graph

The sun graph $S(n, P_1, P_2)$ is the graph obtained from the cycle C_n, n-even with vertex set $\{v_1, v_2, \ldots v_n\}$ in the following way

(i) To each vertex v_i (odd suffix) attach P_2 and

(ii) To each vertex v_i (even suffix) attach P_3.

Example 3.8 : Sungraph $S(6, P_2, P_3)$ is given in Figure 3.8.1.

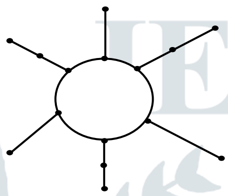

Figure 3.8.1. S(6, P₂, P₃)

Theorem 3.9: Let G be a Sun graph $S(n, P_2, P_3)$ $(n \geq 4)$. Then, $\gamma_{ld}(G) = n$.

Proof : Let S be a γ_{ld} – set of G. Then, to dominate the pendant vertices, S contains the supports of G. Therefore, S contains the vertices of odd suffix of C_n and the vertices of degree two of P_3. Since, $|C_n|$ is even, the number of vertices of odd suffix is $\frac{n}{2}$. Similarly, the number of vertices of degree two of P_3 is also $\frac{n}{2}$. Hence, $|S| = \frac{n}{2} + \frac{n}{2} = n$. This completes the proof of the theorem.

Theorem 3.10 : Let G be a graph with $\delta(G) = 1$ and let v be a support of G such that v is adjacent to exactly one pendant vertex. Let G′ be the graph obtained from G by attaching k pendant edges at v. Then, $\gamma_{ld}(G') = \gamma_{ld}(G) + k$.

Proof : Let $u \in V(G)$ such that deg(u) = 1 and let v be adjacent to u. Let S be a locating dominating set of G .

Case (i): $v \in S$

Then $u \in V - S$ and $N(u) \cap S = \{ v \}$. Let $v_1, v_2, \ldots, v_k$ be the pendant vertices, ($v_i \neq u$, i = 1, 2, . . ., k) attached at v in G′. Then, S cannot be a locating dominating set of G′, since $N(v_1) \cap S = N(v_2) \cap S = \ldots = N(v_k) \cap S = \{v\}$ in G′. Also, $N(u) \cap S = \{v\}$. Therefore, $S' = S \cup \{v_1, v_2, \ldots, v_k\}$ is a locating dominating set of G′ .

Case (ii): $v \notin S$

Then $u \in S$. In this case, $N(v_1) \cap S = N(v_2) \cap S = \ldots = N(v_k) \cap S = \phi$ in G′. Therefore, $S' = S \cup \{v_1, v_2, \ldots, v_k\}$ is a locating dominating set of G′.

From case (i) and case(ii), $\gamma_{ld}(G') \leq |S| + k$. That is, $\gamma_{ld}(G') \leq \gamma_{ld}(G) + k$. Since S is a minimum locating dominating set of G, $S \cup \{v_1, v_2, \ldots, v_k\}$ is a minimum locating dominating set of G′. Therefore, $\gamma_{ld}(G') = \gamma_{ld}(G) + k$. Hence the theorem

Definition 3.11: Jelly Fish Graph

Jelly Fish graph J(m, n), $m \geq 0$, $n \geq 0$ be the graph obtained from $K_4 - e$, by attaching m and n number of pendant edges each at vertices of degree 2 of $K_4 - e$.

Example 3.12: Jelly fish graph J(5, 6) is given in Figure3.12.1.

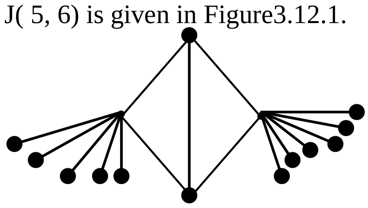

Figure 3.12.1. J(5, 6)

Theorem 3.13 : $\gamma_{ld}(J(m, n)) = m + n + 1$, for integers m , $n \geq 0$.

Proof: We know that, $\gamma_{ld}(K_4 - e) = 3$. Let G′ be the graph obtained from G by attaching exactly one pendant edge at the vertices (say u, v) of degree 2 in $K_4 -$ e. Then, $\gamma_{ld}(G') = 3$. That is, the locating domination number will not be affected by the attachment of these edges. The Jelly Fish graph J(m, n) is obtained from G′ by adding m – 1 and n – 1 pendant edges at the vertices u and v respectively. By Theorem 3.10, the locating domination number increases by (m – 1) + (n – 1). Therefore, $\gamma_{ld}(J(m, n)) = \gamma_{ld}(G')+(m-1)+(n-1) = 3 + m - 1 + n - 1$. $\gamma_{ld}(J(m, n)) = m + n + 1$. Hence the theorem.

Definition 3.14: Umbrella Graph

For $m \geq 6$, $n \geq 4$ the Umbrella graph U (m, n) is obtained from a Fan F_{m+1} by attaching a path P_n at the vertex of degree m of F_{m+1}.

Example 3.15 : Umbrella Graph U(8, 4) is given in figure 3.14.1.

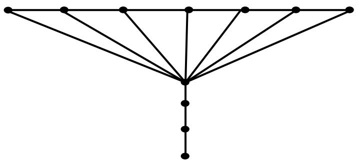

Figure 3.14.1. U (8, 4)

Theorem 3.16: $\gamma_{ld}(U(m, n)) = \gamma_{ld}(F_{m+1}) + \gamma_{ld}(P_{n-1})$, $m \geq 0$, $n \geq 0$.

Proof : The Umbrella graph is obtained by merging the vertex of degree m of the fan F_{m+1} with a pendant vertex of the path P_{n-1}. Let S, S_1 and S_2 be the minimum locating dominating set for the umbrella graph U(m,n), the fan F_{m+1} and the path P_{n-1} respectively. Then, $|S| = |S_1| + |S_2|$ since, the merged vertex belong to V – S.

Therefore, $\gamma_{ld}(U(m, n)) = \gamma_{ld}(F_{m+1}) + \gamma_{ld}(P_{n-1})$. This completes the proof of the theorem.

Definition 3.17: Lotus graph

For any integer m > 1, and $n \geq 1$, the Lotus graph L (m, n, u, v) whose vertex and edge sets are defined as V (L (m, n, u, v)) = $\{u_1, u_2 .., u_m, v_1, v_2, \ldots v_n, u, v\}$ where n = m-1.

E (L(m, n, u, v)) = $\begin{cases} (u, v) \\ (u_i, u) ; i = 1 \text{ and } m \\ (v_i, u) ; i = 1, 2, \ldots m-1 \\ (u_i, v_i) ; i = 1, 2, \ldots m-1 \\ (u_{i+1}, v_i) ; i = 1, 2, \ldots m\text{-}1 \end{cases}$

Example 3.18:

Lotus graph L(6, 5, u, v) is given in figure 3.18.1.

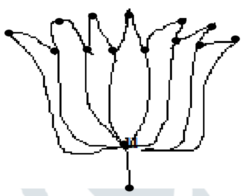

Figure 3.18.1. L (6, 5, u, v)

Theorem 3.19: Let G be a Lotus graph L(m, n, u, v), where n = m – 1, then $\gamma_{ld}(G) = m - \lfloor (m-1)/5 \rfloor$ $m > 1$, $n \geq 1$.

Proof : Let V (G) = $\{u_1, u_2, \ldots u_m, v_1, v_2, \ldots v_n, u, v\}$, where n = m – 1

and E (G) = $\begin{cases} (u, v) \\ (u_i, u) ; i = 1, 2 \ldots, m \\ (vi, u) ; i = 1, 2, \ldots, m-1 \\ (u_i, v_i) ; i = 1, 2, \ldots, m-1 \\ (u_{i+1}, v_i) ; i = 1, 2, \ldots, m\text{-}1 \end{cases}$

Let S be a locating dominating set of G.

Then $S = \{u, y_{1+5i}, y_{5j}, x_{3+5k}, x_{4+5k'} / i = 0, 1, 2 \ldots \lfloor (m-2)/5 \rfloor, j = 1, 2 \ldots \lfloor (m-1)/5 \rfloor, k = 0, 1, 2, \ldots \lfloor (m-3)/5 \rfloor, k' = 0, 1, 2, \ldots, \lfloor (m-2)/5 \rfloor\}$. Therefore, $\gamma_{ld}(G) \leq m - \lfloor (m-1)/5 \rfloor$.

It can be easily proved that $\gamma_{ld}(G) \not< m - \lfloor (m-1)/5 \rfloor$.

Therefore, $\gamma_{ld}(G) = m - \lfloor (m-1)/5 \rfloor$. Hence the theorem.

IV. CONCLUSION

Thus the locating domination number for some special types of graphs like Fan, Sun graph , Jelly Fish graph , Umbrella graph, Lotus graph are studied. Finding the locating domination number for Helm graph, Flower graph and Jahangir graph are some of the open problems.

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