THE DOMINATION NUMBER TOWARDS QUOTIENT OF AN EUCLIDIAN DIVISION ALGORITHM OF DEVISOR 3 USING A CIRCULAR-ARC GRAPHS

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Abstract

A circular-arc graph is the intersection graph of family arcs on a circle. Many projects involving graphs even pure graph theory itself, involve algorithms. Most of real life problems when transformed into graph problems exhibit some special inequality properties. This has given rise to special classes of graphs such as circle graphs, circular-arc graphs. In this chapter, we present algorithms for finding the comparison of neighborhood set degrees of a circular-arc graph G.

Keywords

Circular-arc family, circular-arc graph, minimum dominating set, domination number, division algorithm

Introduction:

Graph theory is a relevantly new are of mathematics first studied by the super fames mathematician Leonhard Euler in 1735. Since then it has blossomed in to powerful tool used in nearly every branch of science and is currently an active area of mathematics reach.

Now a days graph is really important in different fields. Probably, more important than we think. Graphs are among the most ubiquitous models of both natural and human-made structures. Graph theory is a fascinating subject. Today graph theory is one of the most flourishing branches modern mathematics[3].

A connected dominating set is used as a backbone for communications and vertices that are not in this set communicate by passing message through neighbors that are in the set. Among the various applications of the theory of domination and the distance, the most often discussed is a communication network. This network consists of communication links all distance between affined set of sites. Circular-arc graphs[1,9] are rich in combinatorial structures and have found applications in several disciplines such as Biology, Ecology, Psychology, Traffic control, Genetics, Computer sciences and particularly useful in cyclic [2,4] scheduling and computer storage allocation problems etc. Suppose communication network does not work due to link failure, then the problem is what is the fewest number of communication links such that at least one additional transmitter would be required in order that communication with all sites is possible.

A Euclidean graph is a graph in which the vertices represent points in the plane, and the edges are assigned lengths equal to the Euclidean distance between those points. By Euclidean minimum spanning tree is the minimum spanning tree of a Euclidean complete graph. To my understanding a graph is Euclidean if each edges connecting two vertices represents the distance between those two vertices, where the vertices are points or nods in a plane.

Preliminaries:

Graph theory is useful in biology and conservation efforts where a vertex can represent regions where certain species exits (or habitats) and the edges represent migration paths, or movement between the regions. This information is important when looking at breeding patterns or tracking the spread of disease or parasites or how changes to the movement can affect other species. In mathematics, graphs are useful in geometry and certain part of topology such as knot theory. Algebraic graph theory has close links with group theory [7]. A graph structure can be extended by assigning a weight to each edge of the graph. Graphs with weights or weighted graphs are used to represent structures in which pair wise connections have some numerical values. Recently graph theory has also found its application in ontology. Ontology is an organizational system designed to categorize and help explain the relationships between various concepts of science in the same area of knowledge and research.

Among the numerous applications of the domination theory in graphs, the most often discussed is a communication network. This network consists of communication links between a fixed set of sites. The problem is to select a smallest set of sites at which the transmitters are placed so that every other site in the network is joined by a direct communication link to the site which has a transmitter. In other words the problem is to find a minimum dominating set in the graph corresponding to this network [10].

Let $A = \{A_1,A_2,A_3 \ldots\ldots\ldots A_n\}$ be a family of n arcs on a circle C. Each end point of the arc is assigned a positive integer called a coordinate. The end point of each arc is located on the circumference of C in the ascending order of the values of the coordinates in the clock wise direction. For convenience, each arc is A_i $i=1,2,3,\ldots\ldots n$ is represented as (h_i,t_i). Where h_i is the head point and t_i is the tail point respectively that is starting and ending points of the arc when it is traversed in clock wise manner, starting with an end of any arc [8] in A. Without loss of generality, we will assume that the following conditions are satisfied by circular arc graph.

(i) No two arcs share a common end point.

(ii) No single arc in A covers the entire circle C by itself otherwise the shortest path result becomes trivial and in this case the distance between any two arcs is either 1 or 2 units.

(iii) $\bigcup_{i=1}^{n} c_i = c$, otherwise the result becomes one or trivial graph.

(iv) The end points of the arcs in A are already given and sorted according to the order in which they are visited during clockwise traversal along C by starting at arc a_1.

(v) The arcs are sorted in increasing values of h_i's that is $h_i > h_j$ for $i > j$.

(vi) The family of arc A is said to be canonical if h_i's and t_i's for i = 1,2,3,…..n are all distinct integers between 1 and 2n and the point 1 is the head of the arc a_1.

Theorem 1: Let A = {A_2,A_5,A_8,…………..A_p} be a finite circular arc family and let G be a circular arc graph corresponding to circular arc family A such that every arc A_i, i ≠ p is intersect next arc only and suppose p =3x+y where y = 0,1,2 and x is any integer. If y = 0 then the domination number$\gamma(G) = x$ and the minimum dominating set D = {A_2,A_5,A_8……….A_{p-1}}

Proof: Let set D = {A_2,A_5,A_8,…………..A_p} be any finite circular arc family and G be a circular arc family and G be a circular arc graph G corresponding to circular arc family A. Such that every arc A_i, i ≠ p is intersect the next interval only like a tree. Suppose p = 3x+y where y =0,1,2 and x is any integer. Our aim to show that if y = 0 then the domination number$\gamma(G) = x$ and the minimum dominating set D = {A_2,A_5,A_8……….A_{p-1}}. Know we consider the division algorithm p = 3x+y where y =0,1,2 and x any integer. Since p is the number of circular arc families of A.

If y = 0 then p =3x+ 0 = 3x, when x is any integer. By induction method we have if x = 1then

p = 3 × 1 =3 this means that there are three arc in A that is {A_1,A_2,A_3} by the hypothesis every arc intersect the next arc only. That is A_1 intersect A_2 and A_2 intersect A_3 as well as A_1

Hence we get A_2 dominates A_1 andA_3. Therefore minimum dominating set D = {A_2}.

Since $\gamma(G) = 1 = x$

Therefore $\gamma(G) = x$

If x = 2 then p = 3 × 2 = 6

Hence the circular arc family A = {A_1,A_2,A_3,……A_6}

By the hypothesis A_2 dominates A_1 and A_3

A_5 dominates A_4 and A_6

Hence the minimum dominating set D = $\{A_2,A_5\}$

And $\gamma(G) = 2 = x$

Therefore $\gamma(G) = x$

We proceed in the same way we get x = l then p = 3x = 3l

Hence the circular arc family A = $\{A_1,A_2,A_3,\ldots\ldots A_{3l}\}$

By the hypothesis A_2 dominating A_1 and A_3

A_5 dominating A_4 and A_6

A_8 dominating A_7 and A_9

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A_{3l-1} dominating A_{3l-2} and A_{3l}

Therefore the minimum dominating set D = $\{A_2,A_5,A_8\ldots\ldots\ldots A_{3l-1}\}$

Therefore the cardinality dominating set $|D|$ = l =x

In the same way we can use arithmetical progression like 2,5,8…….3l-1 arc in A.P

The number of terms in A.P $t_p = a+ (p-1)d$

Which implies that 3l-1 = 2+(p-1)3

3l-1 = 2+3p-3

3l-1 = 3p-1

3l = 3p

P = l = x

Therefore the cardinality of dominating set D = l = x

Hence the domination number $\gamma(G) = x$ and minimum dominating set D = {A_2,A_5,A_8………A_{3l-1}}

That isD = {A_2,A_5,A_8………A_{p-1}}

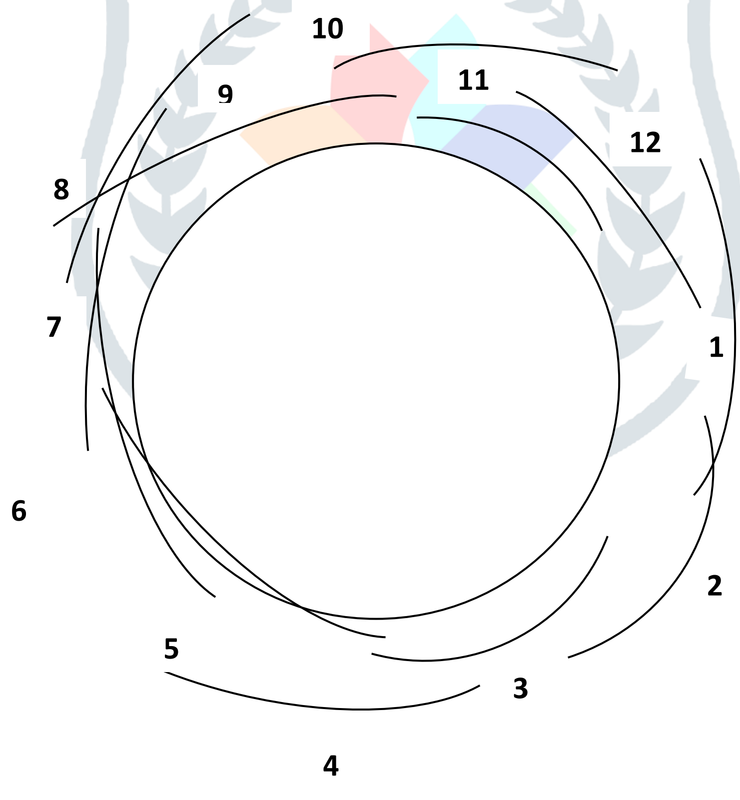
Since p = 3x = 3l

Therefore if y = 0 then the minimum dominating set D = {A_2,A_5,A_8………A_{p-1}} and $\gamma(G) = x$

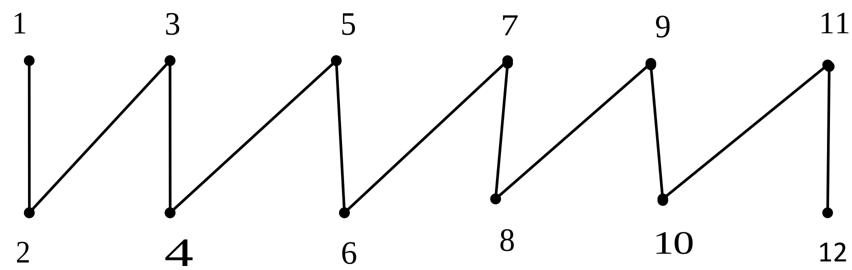
Therefore the theorem is hold.

Illustration 1:

Let A = {A_1,A_2,A_3………A_{12}} be a circular arc family and let G(V,E) be an circular arc graph corresponding to an circular arc family A is as family.



Circular-arc family G



Circular-arc graph G

Here p = 12= 3 × 4+0

This is of the form p = 3x+y and then x = 4, y = 0

If y =0 then minimum dominating set D = {2,5,8,11}

Therefore 4 is a cardinality of the minimum dominating set of D from the theorem1.

And the domination number $\gamma(G) = 4$

That is $\gamma(G) = 4 =$ x Since x = 4

Therefore $\gamma(G) = x$

Hence theorem is verified at y = 0

Theorem 2: Let A = {A_2,A_5,A_8,…………..A_p} be a finite circular arc family and let G be a circular arc graph corresponding to circular arc family A such that every arc A_i, i ≠ p is intersect next arc only and suppose p =3x+y where y = 0,1,2 and x is any integer. If y = 1 then the domination number $\gamma(G) = x + 1$ and minimum dominating set D = {A_2,A_5,A_8,…………..A_p} where A_p is the last interval of A.

Proof: Let A = {A_2,A_5,A_8,…………..A_p} be any finite circular arc family and G be a circular arc graph G corresponding to circular arc family A. Such that every arc A_i, i ≠ p is intersect the next interval only like a tree.

Suppose p = 3x+y where y =0,1,2 and x is any integer. Our aim to show that if y = 1 then the dominate number $\gamma(G) = x$ and the minimum dominating set D = {A_2,A_5,A_8,…………..A_p}. Know we consider the division algorithm p = 3x+y where y =0,1,2 and x any integer. Since p is the number of circular arc families of A.

If y = 1, then p = 3x+1 where x is any integer by induction method we have if x = 1. Then p = 3×1+1 = 4. This means that there are four arcs in A. That is {A_1,A_2,A_3,A_4} by the hypothesis every arc intersect the next arc only. That is A_1 intersect A_2 and A_2 intersect A_3 and A_3 intersect A_4 well as A_1

Hence we get A_2 dominates A_1 and $A_3$and A_4. Therefore minimum dominating set D = {A_2,A_4}.

Since$\gamma(G) = 2 = 1 + 1 = x + 1$, since x = 1

Therefore $\gamma(G) = x$+1

Suppose x = 2 then p = 3 × 2+1 = 6+1 =7

Hence the circular arc family A = {A_1,A_2,A_3,……A_7}

By the hypothesis A_2 dominates A_1 and A_3

A_5 dominates A_4 and A_6

And A_7 is pendent interval

Hence the minimum dominating set D = {A_2,A_5,A_7}

And $\gamma(G) = 3 = 2 + 1 = x + 1$

Therefore $\gamma(G) = x$+1

We proceeds in the same way, this is also proved for x = l then p = 3l+1

Hence the circular arc family A = {A_1,A_2,A_3,……,A_{3l-1},A_{3l},A_{3l+1}}

By the hypothesis A_2 dominating A_1 and A_3

A_5 dominating A_4 and A_6

A_8 dominating A_7 and A_9

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A_{3l-1} dominating A_{3l-2} and A_{3l}

A_{3l+1} is a pendent interval.

Therefore the minimum dominating set D = {$A_2, A_5, A_8 \ldots\ldots\ldots A_{3l-1}, A_{3l+1}$}

D = {$A_2, A_5, A_8 \ldots\ldots\ldots A_{3l-1}$} ∪{$A_{3l+1}$}

D = $D_1 \cup D_2$ where D_1 = {$A_2, A_5, A_8 \ldots\ldots\ldots A_{3l-1}$}, D_2 = {A_{3l+1}}

Therefore the cardinality dominating set $|D| = |D_1| + |D_2|$…………………(1)

Here $|D_1|$ = l and $|D_2|$ =1

(Since 2,5,8…….3l-1 arc in A.P

The number of terms in A.P t_p = a+ (p-1) d

Which implies that 3l-1 = 2+ (p-1)3

3l-1 = 2+3p-3

3l-1 = 3p-1

3l = 3p

P = l

Therefore the cardinality of dominating set D = l = x)

From $|D| = |D_1| + |D_2|$

Therefore $\gamma(G) = l + 1 = x + 1$, since x = l, $|D_2|$ =1

Hence the domination number $\gamma(G) = x + 1$ and minimum dominating set D = {$A_2, A_5, A_8 \ldots\ldots\ldots A_{3l-1}, A_{3l+1}$}

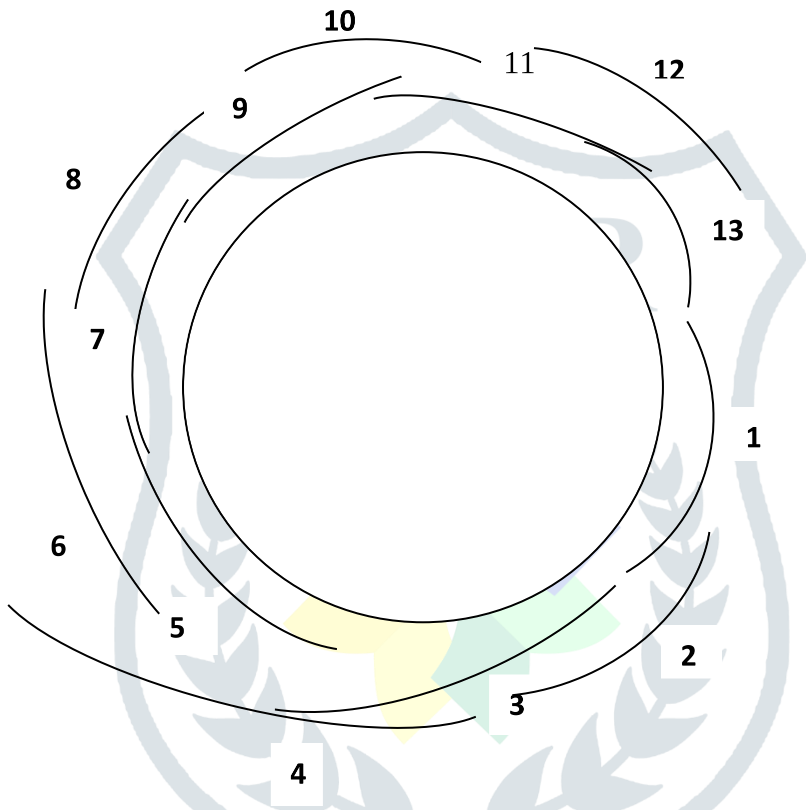
That is D = {$A_2, A_5, A_8 \ldots\ldots\ldots A_{p-2}, A_p$}

Since p = 3l+1

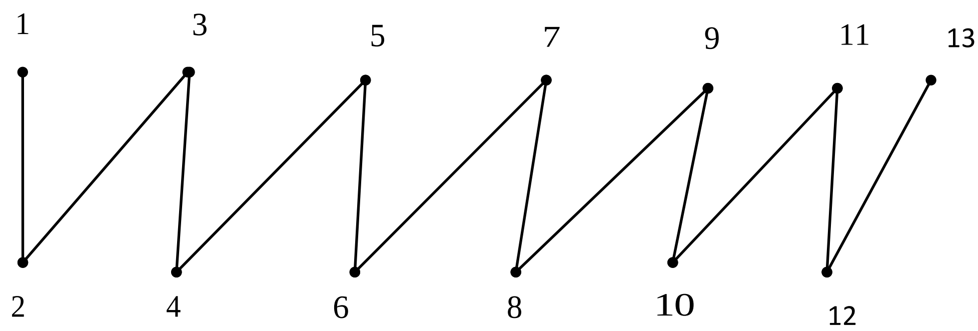
Therefore if y = 1 then the minimum dominating set D = {$A_2, A_5, A_8 \ldots\ldots\ldots A_{p-2}, A_p$} and $\gamma(G) = x$+1

Therefore the theorem is hold.

Illustration 2: Let $A = \{A_1, A_2, A_3 \ldots\ldots\ldots A_{13}\}$ be a circular arc family and let G(V,E) be an circular arc graph corresponding to an circular arc family A is as family.



Circular-arc family G



Circular-arc graph G

Here p = 13 = 3 × 4+ 1

This is of the form P = 3x+y and thenx = 4 and y = 1

If y =1 then minimum dominating set D = {2,5,8,11,13}

Therefore 5 is a cardinality of the minimum dominating set of D from the theorem2.

And the domination number $\gamma(G) = 5$

That is $\gamma(G) = 4 + 1 = x + 1$

Therefore $\gamma(G) = x$+1

Hence theorem is verified at y = 1

Theorem 3: Let G be a finite circular arc graph corresponding to circular arc family A_i, i ≠ p is intersect the next interval only and suppose p = 3x+y where y =0,1,2 and x is any integer. If y = 2. Then the domination number $\gamma(G) = x + 1$ and minimum dominating set D, either D = {A_2,A_5,A_8,…………..A_{p-1}} or D = {A_2,A_5,A_8,…………..A_p}

Proof: Let A = {A_2,A_5,A_8,…………..A_p} be any finite circular arc family and G be a circular arc graph G corresponding to circular arc family A. Such that every arc A_i, i ≠ p is intersect the next interval only like a tree.

Suppose p = 3x+y where y =0,1,2 and x is any integer. Our aim to show that if y = 2 then the dominate number $\gamma(G) = x$ and the minimum dominating set D = {A_2,A_5,A_8,…………..A_p}. Know we consider the division algorithm p = 3x+y where y =0,1,2 and x any integer. Since p is the number of circular arc families of A.

If y = 2, then p = 3x+1 where x is any integer by induction method we have if x = 1. Then p = 3×1+2 = 5. This means that there are five arcs in A. That is $\{A_1,A_2,A_3,A_4,A_5\}$ by the hypothesis every arc intersect the next arc only. That is A_1 intersect A_2 and A_2 intersect A_3 and A_3 intersect A_4 and A_5well as A_1

Hence we get A_2 dominates A_1 and A_3 and A_4 and $A_5$are mutually dominates to each other. Therefore minimum dominating set D = $\{A_2,A_4\}$ or $\{A_2,A_5\}$.

And$\gamma(G) = 2 = 1 + 1 = x + 1$, since x = 1

Therefore $\gamma(G) = x$+1

Suppose x = 2 then p = 3 × 2+2 = 6+2 = 8

Hence the circular arc family A = $\{A_1,A_2,A_3,\ldots\ldots A_8\}$

By the hypothesis A_2 dominates A_1 and A_3

A_5 dominates A_4 and A_6

And A_7, A_8 are mutually dominates to each other

Hence the minimum dominating set D = $\{A_2,A_5,A_7\}$ or D = $\{A_2,A_5,A_8\}$

And $\gamma(G) = 3 = 2 + 1 = x + 1$

Since x = 2

Therefore $\gamma(G) = x$+1

We proceeds in the same way, this is also proved for x = l then p = 3l+2

Hence the circular arc family A = $\{A_1,A_2,A_3,\ldots\ldots,A_{3l-2},A_{3l-1},A_{3l},A_{3l+1},A_{3l+2}\}$

By the hypothesis A_2 dominating A_1 and A_3

A_5 dominating A_4 and A_6

A_8 dominating A_7 and A_9

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A_{3l-1} dominating A_{3l-2} and A_{3l}

A_{3l+1}, A_{3l+2} are mutually dominates to each other

Therefore the minimum dominating set D = {A_2,A_5,A_8……….A_{3l-1},A_{3l+1}}

(or) D = {A_2,A_5,A_8……….A_{3l-1},A_{3l+2}}

Suppose D = {A_2,A_5,A_8……….A_{3l-1},A_{3l+1}}

D = {A_2,A_5,A_8……….A_{3l-1}} ∪{A_{3l+1}}

D = D_1∪ D_2 where D_1 = {A_2,A_5,A_8……….A_{3l-1}}, D_2 = {A_{3l+1}}

Therefore $|D|$ = $|D_1|$ + $|D_2|$

$\gamma(G) = l + 1 = x + 1$, since x = l,

Hence the domination number $\gamma(G) = x + 1$ and minimum dominating set D = {A_2,A_5,A_8……….A_{3l-1},A_{3l+2}}

D = {A_2,A_5,A_8……….A_{3l-1}} ∪{A_{3l+2}}

$|D|$ = $|D_1|$ + $|D_2|$

$\gamma(G) = l + 1 = x + 1$, since x = l,

Hence domination number $\gamma(G) = x$+1,

minimum dominating set D = {A_2,A_5,A_8……….A_{3l-1},A_{3l+1}}

(or) D = {A_2,A_5,A_8……….A_{3l-1},A_{3l+2}}

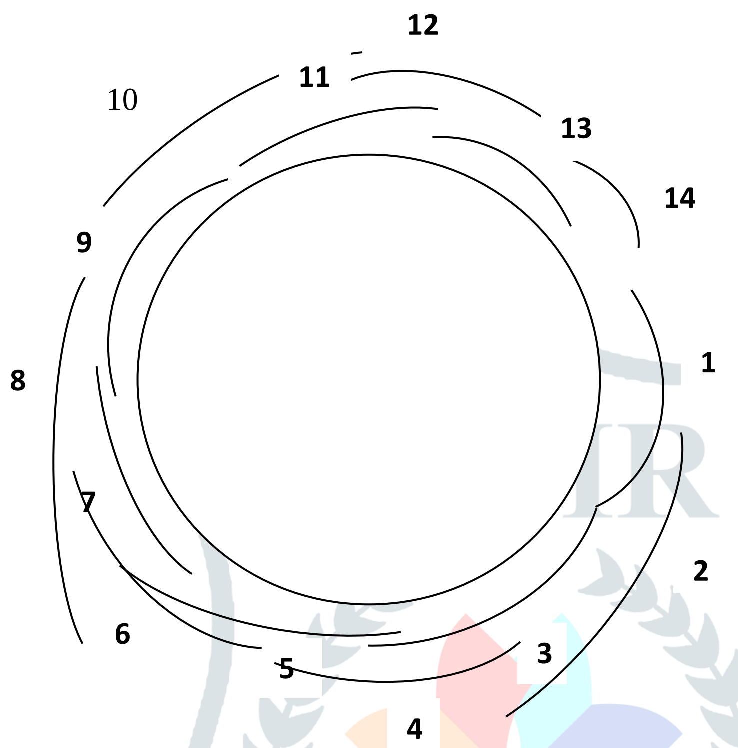
That is D = {A_2,A_5,A_8……….A_{p-2},A_{p-1}}, Since p = 3l+2

D = {A_2,A_5,A_8……….A_{p-2},A_p} and $\gamma(G) = x$+1

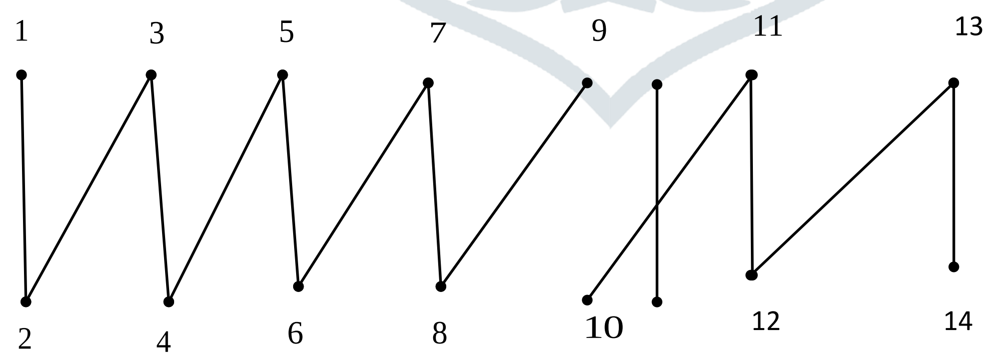
Therefore if y = 1 then the minimum dominating set D = {A_2,A_5,A_8……….A_{p-1}} or D = {A_2,A_5,A_8……….A_p}, and $\gamma(G) = x$+1

Therefore the theorem is proved.

Illustration 3: Let A = {$A_1, A_2, A_3 \ldots\ldots\ldots A_{14}$} be a circular arc family and let G(V,E) be an circular arc graph corresponding to an circular arc family A is as family.



Circular-arc family (C.G)



Circular-arc graph G

Here p = 14 = 3 × 4+ 2

This is of the form P = 3x+y and then x = 4 and y = 2

If y = 2 then minimum dominating set either D = {2,5,8,11,13} or D = {2,5,8,11,14}

Therefore 5 is a cardinality of the minimum dominating set of D from the theorem 3.

And the domination number $\gamma(G) = 5$

That is$\gamma(G) = 4 + 1 = x + 1$, since x = 4

Therefore $\gamma(G) = x$+1

Hence theorem is verified at y = 2

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