A FUZZY PRODUCTION INVENTORY MODEL WITH THREE WASTAGE ITEMS UNDER TWO CONSTRAINTS USING REGULAR WEIGHTED POINT TECHNIQUE

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*Abstract :*In this paper, a fuzzy production inventory model with three wastage items and two warehouse under two constraints have been considered. In this model the deterioration rate and demand rate considered as random variable and the production rate depends directly on demand rate. The lotus petal fuzzy number is defined and its properties are given. The parameters involved in this model are represented by lotus petal fuzzy number. The expected average total cost is defuzzified by the regular weighted point technique. The analytical expressions for expected inventory level in temporary warehouse, maximum inventory level, maximum wastage levels and average total cost are derived for the proposed model by using nonlinear programming technique. A numerical example is presented to illustrate the results.

*Keywords***:** lotus petal fuzzy number, regular weighted point technique, temporary warehouse cost constraint, machine time constraint.

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I. INTRODUCTION

Inventory problems are common in manufacturing, service and business operations in general. Some inventory models were formulated in a static environment where the demand is assumed to be constant and steady over a finite planning horizon. Many items of inventory such as electronic products, fashionable clothes, tasty food products etc.,

Most of the existing inventory models in the literature assume that items can be stored indefinitely to meet the future demands. However, certain types of commodities either deteriorate or become obsolete in the course of time and hence are unstable. Therefore, if the rate of deterioration is not sufficiently low, its impact on modelling of such an inventory model cannot be ignored. In this connections, inventory problems for deteriorating items have been studied extensively by many researchers [3],[6],[7] and [9] from time to time. Research in this area started with the work of Whitin [10], who considered fashion goods deteriorating at the end of prescribed storage period. Goyal and Giri [5] gave recent trends of modelling in deteriorating item inventory. The inventory model for variable demand and production is given in [1]. A production inventory model for variable demand and production is explained in [2]. Samantha and Ajanta roy work based on realistic production lot-size inventory model for deteriorating items is given in [8].

In conventional inventory models, uncertainties are treated as randomness and are handled by probability theory. Furthermore, when addressing real world problems, frequently the parameters are imprecise numerical quantities. However, in certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory introduced by Zadeh [11] is applicable. A. FarithaAsma and E.C. Henry Amirtharajanalized multi objective inventory model of deteriorating items with two constraints using fuzzy optimization technique [4].

In the real situation, at the time of production the deterioration rate and demand rate are varied. So that the deterioration rate and demand rate are considered as a random variable which follows rayleigh distribution and bounded pareto distribution respectively.

In the existing inventory models, they used only rented warehouse and own warehouse. When using the rented warehouse the transportation cost, rental and maintenance cost are all high. Here, the model newly constructed with permanent warehouse and temporary warehouse.

As warehouse is a key factor in production inventory model, so these days researcher are paying more rent in stored the goods. The temporary warehouse used for reducing the rented amount and maintenance cost.

This paper is organized as follows:

In section 2, assumptions and notations for the fuzzy production inventory model under consideration are given. The mathematical formulation for the proposed model is explained in crisp environment under section 3. In section 4, lotus petal fuzzy number is defined and its properties are given. To defuzzify the model, the regular weighted point of lotus petal fuzzy number is determined in section 5. In section 6, the mathematical model is explained in fuzzy environment. In section 7, an application of this model is given in both crisp and fuzzy environment.

II. ASSUMPTION AND NOTATIONS:

The following assumptions and notations are used throughout this paper:

Assumptions :

1. Demand rate for inventory (sugar and wastage items) are taken as a random variable which follows bounded pareto distribution.
2. Production rate for inventory (sugar) is demand dependent that is p = cd, $0 < c < n$, n is a finite number.
3. Deterioration rate for inventory (sugar and wastage items) are taken as a random variable which follows reyleigh distribution.
4. First, the goods (sugar) are stored in permanent warehouse then the remaining goods (sugar) are stored in temporary warehouse.
5. The goods (sugar) of permanent warehouse are consumed only after consuming the goods (sugar) kept in temporary warehouse.
6. The permanent warehouse has a fixed capacity of I_P units.
7. The inventory (sugar and wastage items) level depleted due to demand and deterioration.

8. Shortages are not allowed.

Notations:

$I_1(t)$ – the level of inventory at time t, $0 \le t \le t_1$.

$I_2(t)$ – the level of inventory in temporary warehouse at time t, $t_1 \le t \le t_2$.

$I_3(t)$ – the level of inventory in permanent warehouse at time t, $t_1 \le t \le t_6$.

$W_i(t)$ - the levels of wastage items at time t , $0 \le t \le t_1$, (i = 1,2,3).

$W_{11}(t)$ – the level of wastage item (Bagasse) at time t , $t_1 \le t \le t_5$.

$W_{12}(t)$ – the level of wastage item (Mud) at time t , $t_1 \le t \le t_4$.

$W_{13}(t)$ – the level of wastage item (Molasses) at time t , $t_1 \le t \le t_3$.

I_P – fixed capacity for permanent warehouse at time t_1.

I_m – expected maximum inventory (sugar) level at time t_1. (Decision variable)

I_{wi} - expected maximum wastage levels at time t_1, (i = 1,2,3). (Decision variables)

I_T– expected maximum inventory (sugar) level of temporary warehouse at time t_1. (Decision variable)

p ,d , θ – production rate, demand rate, deterioration rate for inventory (sugar).

θ_i , d_i – deterioration rate , demand rate for wastage items, (i = 1,2,3).

t_1 - machine time available for product

TC – expected average total cost.

$\tilde{S}_c$ - fuzzy setup cost per cycle

$\tilde{m}$ - fuzzy production time per cycle.

$\tilde{h}_c, \tilde{d}_c$ - fuzzy holding cost, fuzzy deteriorating cost for inventory (sugar) per unit per unit time.

$\tilde{h}_{wi}, \tilde{d}_{wi}$ - fuzzy holding cost, deteriorating cost for wastage items per unit per unit time.

$\tilde{T}_{wc}$ - fuzzy cost for 1 square feet of temporary warehouse.

$\tilde{T}_{ws}$ - fuzzy total space for temporary warehouse,

$\tilde{T}_{twc}$ - fuzzy total temporary warehouse cost.

$\tilde{T}_m$ - fuzzy machine working time consumed per month.

III. MATHEMATICAL FORMULATION:

The proposed inventory model is formulated to minimize the average total cost, which includes setup cost, holding cost, deterioration cost and temporary warehouse setup cost. The rate of change of the inventory during the following periods are governed by the following differential equations
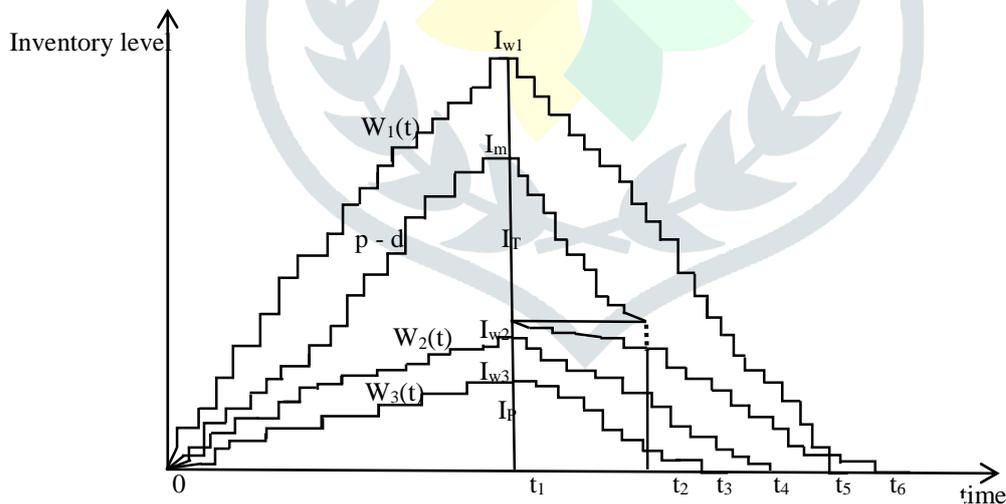
Fig1: Graphical representation of the state of inventory system

$$\frac{dI_1(t)}{dt} + \theta\, I_1(t) = p - d \qquad 0 \le t \le t_1 \qquad \text{----- (3.1)}$$

with boundary condition $I_1(t) = 0$

$$\frac{dI_2(t)}{dt} + \theta\, I_2(t) = -d \qquad t_1 \le t \le t_2 \qquad \text{----- (3.2)}$$

with boundary conditions $I_2(t_2) = 0$ and $I_2(t_1) = I_T$

$$\frac{dI_3(t)}{dt} + \theta\, I_3(t) = 0 \qquad t_1 \le t \le t_2 \qquad \text{----- (3.3)}$$

with boundary conditions $I_3(t_1) = I_P$

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -d \qquad\qquad t_2 \leq t \leq t_6 \qquad\qquad\qquad\text{----- (3.4)}$$

with boundary condition $I_3(t_6) = 0$

$$\frac{d W_i(t)}{dt} + \theta_i W_i(t) = 0 \qquad ,(i = 1,2,3) \qquad\qquad 0 \leq t \leq t_1 \text{----- (3.5)}$$

with boundary conditions, $W_i(t) = 0$

$$\frac{d W_{11}(t)}{dt} + \theta_1 W_{11}(t) = -d_1 \qquad\qquad t_1 \leq t \leq t_5 \text{----- (3.6)}$$

with boundary conditions $W_{11}(t_1) = I_{w1}$ and $W_{11}(t_5) = 0$

$$\frac{d W_{12}(t)}{dt} + \theta_2 W_{12}(t) = -d_2 \qquad\qquad t_1 \leq t \leq t_4 \text{----- (3.7)}$$

with boundary conditions $W_{12}(t_1) = I_{w2}$ and $W_{12}(t_4) = 0$

$$\frac{d W_{13}(t)}{dt} + \theta_3 W_{13}(t) = -d_3 \qquad\qquad t_1 \leq t \leq t_3 \qquad\qquad \text{----- (3.8)}$$

with boundary conditions $W_{13}(t_1) = I_{w3}$ and $W_{13}(t_3) = 0$

From (3.1) $\qquad I_1(t) = \frac{p-d}{\theta}\left(1 - e^{-\theta t}\right)$----- (3.9)

From (3.2) $\qquad I_T = \frac{d}{\theta}\left(e^{\theta(t_2 - t_1)} - 1\right)$---- (3.10)

From (3.3) $I_3(t) = I_p e^{\theta(t_1 - t)}$ ---- (3.11)

From (3.4) $\quad I_3(t) = \frac{d}{\theta}\left(e^{\theta(t_6 - t)} - 1\right)$ ---- (3.12)

From (3.5) $\qquad W_i(t) = I_{wi}\, e^{\theta_i(t_1 - t)}$, (i = 1,2,3) $\qquad\qquad$ ---- (3.13)

From (3.6) $\qquad I_{w1} = \frac{d_1}{\theta_1}\left(e^{\theta_1(t_5 - t_1)} - 1\right)$ ---- (3.14)

Similarly, $\qquad I_{w2} = \frac{d_2}{\theta_2}\left(e^{\theta_2(t_4 - t_1)} - 1\right)$ ---- (3.15)

$$I_{w3} = \frac{d_3}{\theta_3}\left(e^{\theta_3(t_3 - t_1)} - 1\right) \qquad\qquad \text{---- (3.16)}$$

The maximum inventory level per cycle is $I_m = I_T + I_P$ $\qquad\qquad$ ---- (3.17)

Expected demand rate for inventory (sugar) is

$$E(d) = \frac{L^\alpha}{1 - \left(\frac{L}{H}\right)^\alpha}\left(\frac{\alpha}{\alpha - 1}\right)\left(\frac{1}{L^{\alpha-1}} - \frac{1}{H^{\alpha-1}}\right)$$

Expected deterioration rate for inventory (sugar) is $E(\theta) = \sigma\sqrt{\frac{\pi}{2}}$

Expected demand rate for wastage items are

$$E(d_i) = \frac{L_i^{\alpha_i}}{1 - \left(\frac{L_i}{H_i}\right)^{\alpha_i}}\left(\frac{\alpha_i}{\alpha_i - 1}\right)\left(\frac{1}{L_i^{\alpha_i-1}} - \frac{1}{H_i^{\alpha_i-1}}\right), \text{ (i = 1,2,3)}$$

Expected deterioration rate for wastage items are $E(\theta_i) = \sigma_i\sqrt{\frac{\pi_i}{2}}$ (i = 1,2,3)

$$\textit{Expected average total } \mathrm{cost} = \frac{1}{t_6}\begin{bmatrix} \mathrm{setup\ cost} + E(holding\ \mathrm{cost}) \\ + E(\det erioration\ \mathrm{cost}) + temporary\ warehouse\ setup\ \mathrm{cost} \end{bmatrix}$$

where,

Setup cost = S_c

Holding cost = Holding cost for inventory (sugar) + Holding cost for wastage items

$$= \left[h_c \left\{ \int_0^{t_1} I_1(t)\,dt + \int_{t_1}^{t_2} (I_2(t)+I_3(t))\,dt + \int_{t_2}^{t_4} I_3(t)\,dt \right\} \right]$$

$$+ \left[h_{w1} \left\{ \int_0^{t_1} W_1(t)\,dt + \int_{t_1}^{t_5} W_{11}(t)\,dt \right\} + h_{w2} \left\{ \int_0^{t_1} W_2(t)\,dt + \int_{t_1}^{t_4} W_{12}(t)\,dt \right\} + h_{w3} \left\{ \int_0^{t_1} W_3(t)\,dt + \int_{t_1}^{t_3} W_{13}(t)\,dt \right\} \right]$$

on simplification

$$= \left[h_c \left\{ \frac{p-d}{\theta^2}\left(e^{-\theta t_1}-1\right) + \frac{d}{\theta^2}\left(e^{\theta(t_2-t_1)}+e^{\theta(t_4-t_2)}-2+\theta t_1-\theta t_4\right) + I_p\left(\frac{1-e^{\theta(t_1-t_2)}}{\theta}\right) \right\} \right]$$

$$+ \left[\begin{array}{l} h_{w1}\dfrac{d_1}{\theta_1}\left\{\left(\dfrac{e^{\theta_1 t_5}}{\theta_1}-\dfrac{e^{\theta_1 t_1}}{\theta_1}-t_5+t_1\right)\right\} + h_{w2}\dfrac{d_2}{\theta_2}\left\{\left(\dfrac{e^{\theta_2 t_4}}{\theta_2}-\dfrac{e^{\theta_2 t_1}}{\theta_2}-t_4+t_1\right)\right\} \\[2ex] + h_{w3}\dfrac{d_3}{\theta_3}\left\{\left(\dfrac{e^{\theta_3 t_3}}{\theta_3}-\dfrac{e^{\theta_3 t_1}}{\theta_3}-t_3+t_1\right)\right\} \end{array} \right]$$

Deterioration cost = deterioration cost for inventory (sugar)+ deterioration cost for wastage items

$$= \left[d_c\left\{\int_0^{t_1} I_1(t)\,dt - \int_{t_1}^{t_4} d\,dt\right\} \right] + \left[d_{w1}\left\{\int_0^{t_1} W_1(t)\,dt - \int_{t_1}^{t_5} d_1\,dt + d_{w2}\left\{\int_0^{t_1} W_2(t)\,dt - \int_{t_1}^{t_4} d_2\,dt\right\} + d_{w3}\left\{\int_0^{t_1} W_3(t)\,dt - \int_{t_1}^{t_3} d_3\,dt\right\}\right\} \right]$$

on simplification

$$= \left[d_c\left\{\frac{p-d}{\theta^2}\left(e^{-\theta t_1}+\theta t_1-1\right)-d(t_4-t_1)\right\} \right]$$

$$+ \left[\begin{array}{l} d_{w1}\left\{\dfrac{d_1}{\theta_1^2}\left(1+e^{-\theta_1 t_5}-e^{\theta_1 t_1}-e^{\theta_1(t_5-t_1)}\right)-d_1(t_5-t_1)\right\} + d_{w2}\left\{\dfrac{d_2}{\theta_2^2}\left(1+e^{-\theta_2 t_4}-e^{\theta_2 t_1}-e^{\theta_2(t_4-t_1)}\right)-d_2(t_4-t_1)\right\} \\[2ex] + d_{w3}\left\{\dfrac{d_3}{\theta_3^2}\left(1+e^{-\theta_3 t_3}-e^{\theta_3 t_1}-e^{\theta_3(t_3-t_1)}\right)-d_3(t_3-t_1)\right\} \end{array} \right]$$

Temporary warehouse cost $=T_{wc} \times T_{ws}$

$$Min\, E(TC(t_1,t_2,t_3,t_4,t_5,t_6)) = \frac{1}{t_6}\left[\begin{array}{l} S_c + h_c\left\{ \begin{array}{l} \dfrac{c\,E(d)-E(d)}{E(\theta)^2}\left(e^{-E(\theta)t_1}-1\right) + \dfrac{E(d)}{E(\theta)^2}\left(e^{E(\theta)(t_2-t_1)}+e^{E(\theta)(t_6-t_2)}-2+E(\theta)t_1-E(\theta)t_6\right) \\[2ex] + I_p\left(\dfrac{1-e^{E(\theta)(t_1-t_2)}}{E(\theta)}\right) \end{array} \right\} \\[4ex] + h_{w1}\dfrac{E(d_1)}{E(\theta_1)}\left\{\left(\dfrac{e^{E(\theta_1)t_5}}{E(\theta_1)}-\dfrac{e^{E(\theta_1)t_1}}{E(\theta_1)}-t_5+t_1\right)\right\} + h_{w2}\dfrac{E(d_2)}{E(\theta_2)}\left\{\left(\dfrac{e^{E(\theta_2)t_4}}{E(\theta_2)}-\dfrac{e^{E(\theta_2)t_1}}{E(\theta_2)}-t_4+t_1\right)\right\} \\[3ex] + h_{w3}\dfrac{E(d_3)}{E(\theta_3)}\left\{\left(\dfrac{e^{E(\theta_3)t_3}}{E(\theta_3)}-\dfrac{e^{E(\theta_3)t_1}}{E(\theta_3)}-t_3+t_1\right)\right\} \\[3ex] + d_c\left\{\dfrac{c\,E(d)-E(d)}{E(\theta)^2}\left(e^{-E(\theta)t_1}+E(\theta)t_1-1\right)-E(d)(t_6-t_1)\right\} \\[3ex] + d_{w1}\left\{\dfrac{E(d_1)}{E(\theta_1)^2}\left(1+e^{-E(\theta_1)t_5}-e^{E(\theta_1)t_1}-e^{E(\theta_1)(t_5-t_1)}\right)-E(d_1)(t_5-t_1)\right\} \\[3ex] + d_{w2}\left\{\dfrac{E(d_2)}{E(\theta_2)^2}\left(1+e^{-E(\theta_2)t_4}-e^{E(\theta_2)t_1}-e^{E(\theta_2)(t_4-t_1)}\right)-E(d_2)(t_4-t_1)\right\} \\[3ex] + d_{w3}\left\{\dfrac{E(d_3)}{E(\theta_3)^2}\left(1+e^{-E(\theta_3)t_3}-e^{E(\theta_3)t_1}-e^{E(\theta_3)(t_3-t_1)}\right)-E(d_3)(t_3-t_1)\right\} + T_{wc}\,T_{ws} \end{array} \right]$$

subject to : $T_{wc}\,T_{ws} \le T_{twc}$, $T_m\,m \le t_1$ ----- (3.18)

Using Lagrange multipliers method, by using Kuhn-Tucker necessary condition. Solve the differential equations using MATLAB software, the values of t_1^*, t_2^*, t_3^*, t_4^*, t_5^* and t_6^* are obtained. Substituting t_1^*, t_2^*, t_3^*, t_4^*, t_5^* and t_6^* in (3.10), (3.14), (3.15), (3.16), (3.17) and (3.18), the optimum values I_T^*, I_{w1}^*, I_{w2}^*, I_{w3}^*, I_m^* and TC^* are obtained.

IV. LOTUS PETAL FUZZY NUMBER AND ITS PROPERTIES:

Definition: Lotus petal fuzzy number

A Lotus petal fuzzy number $\widetilde{A}$ described as a normalized convex fuzzy subset on the real line R whose membership function $\mu_{\widetilde{A}}(x)$ is defined as follows:

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{1}{2} & at \quad x=a,c \\[2mm] \dfrac{1}{2}\left[1+\sqrt{\dfrac{x-a}{b-a}}\right] & at \quad a \le x \le b \\[2mm] \dfrac{1}{2}\left[1+\sqrt{\dfrac{c-x}{c-b}}\right] & at \quad b \le x \le c \\[2mm] \dfrac{1}{2}\left[\left(1-\dfrac{c-x}{c-b}\right)^2\right] & at \quad c \ge x \ge b \\[2mm] \dfrac{1}{2}\left[\left(1-\dfrac{x-a}{b-a}\right)^2\right] & at \quad b \ge x \ge a \\[2mm] 0 \,\&\, 1 & at \quad x=b \end{cases}$$

Fig 2: Lotus Petal Fuzzy Number

This type of fuzzy number be denoted as $\widetilde{A}$ = [a, b, c], whose membership function $\mu_{\widetilde{A}}(x)$ satisfies the following conditions:

1. $\mu_{\widetilde{A}}(x)$ is a continuous mapping from R to the closed interval [0,1]

2. $\mu_{\widetilde{A}}(x)$ is a convex function.

3. $\mu_{\widetilde{A}}(x) = 0$ & 1 at x=b.

4. $\mu_{\widetilde{A}}(x) = \dfrac{1}{2}$ at x= a & c.

5. $\mu_{\widetilde{A}}(x)$ is strictly decreasing as well as increasing and continuous on [a,b] and [b,c].

Properties:
1. Left and right opposite angles are equal.
2. The horizontal and vertical diagonal bisect each other and meet at 90°.

V. REGULAR WEIGHTED POINT OF LOTUS PETAL FUZZY NUMBER:

For the Lotus petal fuzzy number $\widetilde{A}$ =[a, b, c] , the α -cut is $\widetilde{A}_\alpha = [L_{\widetilde{A}}(\alpha), R_{\widetilde{A}}(\alpha)]$ and the regular weighted point for $\widetilde{A}$ is given by,

$$r_w(\widetilde{A}) = \frac{\displaystyle\int_0^1 \frac{L_{\widetilde{A}}(\alpha)+R_{\widetilde{A}}(\alpha)}{2} f_1(\alpha)\,d\alpha}{\displaystyle\int_0^1 f(\alpha)\,d\alpha}$$

$$= \int_0^1 \left[L_{\widetilde{A}}(\alpha)+R_{\widetilde{A}}(\alpha)\right] f_1(\alpha)\,d\alpha$$

where,

$$f(\alpha) = \begin{cases} (1-2\alpha) & when \quad \alpha \in \left[0,\dfrac{1}{2}\right] \\[2mm] (2\alpha-1) & when \quad \alpha \in \left[\dfrac{1}{2},1\right] \end{cases}$$

$$f_1(\alpha) = \begin{cases} \omega(1-2\alpha) & when \quad \alpha \in \left[0,\dfrac{1}{2}\right] \\[2mm] (1+\omega)(2\alpha-1) & when \quad \alpha \in \left[\dfrac{1}{2},1\right] \end{cases}, \; 0 < \omega < 1.$$

The regular weighted point of a lotus petal fuzzy number is of the form

$$r_w(\tilde{A}) = \omega \left(\frac{31a + 58b + 31c}{120} \right) + \left(\frac{a + 2b + c}{8} \right).$$

VI. INVENTORY MODEL IN FUZZY ENVIRONMENT:

The proposed inventory model in fuzzy environment is

$$Min\, E(TC(t_1, t_2, t_3, t_4, t_5, t_6))$$

$$
\begin{aligned}
&\left[\tilde{S}_c + \tilde{h}_c \left\{ \frac{c E(d) - E(d)}{E(\theta)^2}\left(e^{-E(\theta)t_1} - 1\right) + \frac{E(d)}{E(\theta)^2}\left(e^{E(\theta)(t_2 - t_1)} + e^{E(\theta)(t_6 - t_2)} - 2 + E(\theta)t_1 - E(\theta)t_6\right) \right. \right. \\
&\left. \left. + I_p\left(\frac{1 - e^{E(\theta)(t_1 - t_2)}}{E(\theta)}\right) \right\} \right. \\
&\left. + \tilde{h}_{w1}\frac{E(d_1)}{E(\theta_1)}\left\{\left(\frac{e^{E(\theta_1)t_5}}{E(\theta_1)} - \frac{e^{E(\theta_1)t_1}}{E(\theta_1)} - t_5 + t_1\right)\right\} + \tilde{h}_{w2}\frac{E(d_2)}{E(\theta_2)}\left\{\left(\frac{e^{E(\theta_2)t_4}}{E(\theta_2)} - \frac{e^{E(\theta_2)t_1}}{E(\theta_2)} - t_4 + t_1\right)\right\} \right. \\
&\left. + \tilde{h}_{w3}\frac{E(d_3)}{E(\theta_3)}\left\{\left(\frac{e^{E(\theta_3)t_3}}{E(\theta_3)} - \frac{e^{E(\theta_3)t_1}}{E(\theta_3)} - t_3 + t_1\right)\right\} \right. \\
&= \frac{1}{t_6} \left[+ \tilde{d}_c\left\{\frac{c E(d) - E(d)}{E(\theta)^2}\left(e^{-E(\theta)t_1} + E(\theta)t_1 - 1\right) - E(d)\left(t_6 - t_1\right)\right\} \right. \\
&\left. + \tilde{d}_{w1}\left\{\frac{E(d_1)}{E\theta_1)^2}\left(1 + e^{-E(\theta_1)t_5} - e^{E(\theta_1)t_1} - e^{E(\theta_1)(t_5 - t_1)}\right) - E(d_1)\left(t_5 - t_1\right)\right\} \right. \\
&\left. + \tilde{d}_{w2}\left\{\frac{E(d_2)}{E(\theta_2)^2}\left(1 + e^{-E(\theta_2)t_4} - e^{E(\theta_2)t_1} - e^{E(\theta_2)(t_4 - t_1)}\right) - E(d_2)\left(t_4 - t_1\right)\right\} \right. \\
&\left. + \tilde{d}_{w3}\left\{\frac{E(d_3)}{E(\theta_3)^2}\left(1 + e^{-E(\theta_3)t_3} - e^{E(\theta_3)t_1} - e^{E(\theta_3)(t_3 - t_1)}\right) - E(d_3)\left(t_3 - t_1\right)\right\} + \tilde{T}_{wc}\tilde{T}_{ws} \right]
\end{aligned}
$$

Subject to: $\tilde{T}_{wc}\,\tilde{T}_{ws} \le \tilde{T}_{twc}$, $\tilde{T}_m\,\tilde{m} \le t_1$ ----- (6.1)

where ~ represents for fuzzification of the parameters.

$\tilde{S}_c = (S_{c1}, S_{c2}\, S_{c3})$,

$\tilde{h}_c = (h_{c1}, h_{c2}, h_{c3})$,

$\tilde{d}_c = (d_{c1}, d_{c2}, d_{c3})$,

$\tilde{h}_{w1} = (h_{w11}, h_{w12}, h_{w13})$,

$\tilde{h}_{w2} = (h_{w21}, h_{w22}, h_{w23})$,

$\tilde{h}_{w3} = (h_{w31}, h_{w32}, h_{w33})$,

$\tilde{d}_{w1} = (d_{w11}, d_{w12}, d_{w13})$,

$\tilde{d}_{w2} = (d_{w21}, d_{w22}, d_{w23})$,

$\tilde{d}_{w3} = (d_{w31}, d_{w32}, d_{w33})$,

$\tilde{T}_{wc} = (T_{wc1}, T_{wc2}\, T_{wc3})$

$\tilde{T}_{ws} = (T_{ws1}, T_{ws2}, T_{ws3})$,

$\tilde{T}_{twc} = (T_{twc1}, T_{twc2}, T_{twc3})$,

$\tilde{T}_m = (T_{m1}, T_{m2}, T_{m3})$,

$\tilde{m} = (m_1, m_2, m_3)$.

Now using the technique, regular weighted point of lotus petal fuzzy number, the above model is defuzzified as follow:

$Min\ E(TC(t_1,t_2,t_3,t_4,t_5,t_6))$

$$
\begin{aligned}
=\frac{1}{t_6}\Bigg[& r_w(S_c)+r_w(h_c)\left\{\frac{cE(d)-E(d)}{E(\theta)^2}\left(e^{-E(\theta)t_1}-1\right)+\frac{E(d)}{E(\theta)^2}\left(e^{E(\theta)(t_2-t_1)}+e^{E(\theta)(t_6-t_2)}-2+E(\theta)t_1-E(\theta)t_6\right)+I_p\left(\frac{1-e^{E(\theta)(t_1-t_2)}}{E(\theta)}\right)\right\}\\
& +r_w(h_{w1})\frac{E(d_1)}{E(\theta_1)}\left\{\left(\frac{e^{E(\theta_1)t_5}}{E(\theta_1)}-\frac{e^{E(\theta_1)t_1}}{E(\theta_1)}-t_5+t_1\right)\right\}+r_w(h_{w2})\frac{E(d_2)}{E(\theta_2)}\left\{\left(\frac{e^{E(\theta_2)t_4}}{E(\theta_2)}-\frac{e^{E(\theta_2)t_1}}{E(\theta_2)}-t_4+t_1\right)\right\}\\
& +r_w(h_{w3})\frac{E(d_3)}{E(\theta_3)}\left\{\left(\frac{e^{E(\theta_3)t_3}}{E(\theta_3)}-\frac{e^{E(\theta_3)t_1}}{E(\theta_3)}-t_3+t_1\right)\right\}\\
& +r_w(d_c)\left\{\frac{cE(d)-E(d)}{E(\theta)^2}\left(e^{-E(\theta)t_1}+E(\theta)t_1-1\right)-E(d)\left(t_6-t_1\right)\right\}\\
& +r_w(d_{w1})\left\{\frac{E(d_1)}{E\theta_1)^2}\left(1+e^{-E(\theta_1)t_5}-e^{E(\theta_1)t_1}-e^{E(\theta_1)(t_5-t_1)}\right)-E(d_1)\left(t_5-t_1\right)\right\}\\
& +r_w(d_{w2})\left\{\frac{E(d_2)}{E(\theta_2)^2}\left(1+e^{-E(\theta_2)t_4}-e^{E(\theta_2)t_1}-e^{E(\theta_2)(t_4-t_1)}\right)-E(d_2)\left(t_4-t_1\right)\right\}\\
& +r_w(d_{w3})\left\{\frac{E(d_3)}{E(\theta_3)^2}\left(1+e^{-E(\theta_3)t_3}-e^{E(\theta_3)t_1}-e^{E(\theta_3)(t_3-t_1)}\right)-E(d_3)\left(t_3-t_1\right)+r_w(T_{wc})\,r_w(T_{ws})\right\}
\Bigg]
\end{aligned}
$$

Subject to : $r_w(T_{wc})\ r_w(T_{ws})\le r_w(T_{twc}),\ r_w(T_m)\ r_w(m)\le t_1$ ----- (6.2)

Using Lagrange multipliers method, by using Kuhn-Tucker necessary condition. Solve the above differential equations using MATLAB software, the values of t_1^*, t_2^*, t_3^*, t_4^*, t_5^* and t_6^* are obtained. Substituting t_1^*, t_2^*, t_3^*, t_4^*, t_5^* and t_6^* in (3.10), (3.14), (3.15), (3.16), (3.17) and (6.2), the optimum values I_T^*, I_{w1}^*, I_{w2}^*, I_{w3}^*, I_m^* and TC^* are obtained.

VII. NUMERICAL ANALISIS:

During the time period of production, came from the sugar and it has been the three wastages are sold out. Holding cost and deterioration costs assessed on the average inventory level amount to his and does rupees per unit per unit time respectively. The setup cost and temporary warehouse cost are 1000000 and 200 rupees per cycle. The total temporary warehouse cost available is 500000 rupees. Production time per cycle is m and machine working time consumed per month is 24days. The permanent warehouse has a fixed capacity 10000 Ton.

a. Develop a mathematical program to minimize the average total cost, subject to temporary warehouse cost and machine time.
b. Use I_p= 10000 Ton and the following data find the optimal values and average total cost.

L = 100, H = 1000, α = 0.5, c = 2, σ = 5, π = 3.14,

L_1 = 50, H_1 = 500, α_1 = 0.3, σ_1 = 4, π_1 = 3.14,

L_2 = 40, H_2 = 400, α_2 = 0.2, σ_2 = 3, π_2 = 3.14,

L_3 = 30, H_3 = 300, α_3 = 0.1, σ_3 = 2, π_3 = 3.14,

The following costs are taken in rupees.

$\tilde{S}_c$ = (1000000, 2000000, 3000000) ,

$\tilde{h}_c$ = (5000, 6000, 7000), $\tilde{d}_c$ = (500, 600, 700) ,

$\tilde{h}_{w1}$ = (4000, 5000, 6000), $\tilde{h}_{w2}$ = (2000, 3000, 4000), $\tilde{h}_{w3}$ = (1000, 2000, 3000),

$\tilde{d}_{w1}$ = (400, 500, 600), $\tilde{d}_{w2}$ = (200, 300, 400), $\tilde{d}_{w3}$ = (100, 200, 300) ,

$\tilde{T}_{wc}$ = (200, 250, 300), $\tilde{T}_{ws}$ = (2000, 3000, 4000) in sqft,

$\tilde{T}_{twc}$ = (500000, 900000, 1300000),

$\tilde{T}_m$ = (24, 27, 30) in days, $\tilde{m}$ = (4.5, 5, 5.5) in months.

Using MATLAB software, the optimum values I_T^*, I_m^*, I_{w1}^*, I_{w2}^*, I_{w3}^* and TC^* are found out in both crisp and fuzzy environment and the results are tabulated.

<div align="center">Comparison table 7.1</div>

Model	t_1^* /Yrs	t_2^* /Yrs	t_3^* /Yrs	t_4^* /Yrs	t_5^* /Yrs	t_6^* /Yrs	I_T^* /Ton	I_m^* /Ton	I_{w1}^* /Ton	I_{w2}^* /Ton	I_{w3}^* /Ton	TC^* Rs
Crisp 1	0.57	1.5	1.6	1.7	2.0	2.76	17, 068	27, 068	44,536	2,638	548	29,68,60,000
Crisp 2	0.549	1.467	1.59	1.69	1.95	2.76	15,828	25, 828	38,506	2,751	564	31,23,50,000
Crisp 3	0.49	1.46	1.6	1.7	1.9	2.7	21,943	31, 943	40,284	3,577	680	30,04,30,000
Fuzzy	0.5	1.51	1.7	1.8	1.94	2.85	28, 207	38, 207	46,826	5,033	863	28,57,30,000

Observation:

From the above table, it should be noted that compared to crisp model, the fuzzy model is very effective method, because of the time consuming in fuzzy analysis and the optimal results are obtained easily.

i. The average total cost is obtained in fuzzy model is less than the crisp model.

ii. The optimal values I_T^*, I_m^*, I_{w1}^* I_{w2}^*, I_{w3}^* in fuzzy model are higher than the crisp model.

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