TWO DIMENSIONAL BOUNDARY LAYER ALONG A PERMEABLE WALL IN SOURCE FLOW

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ABSTRACT : In this paper we discuss the study two dimensional boundary layer of an incompressible flow in source flow. The momentum integral and kinetic energy integral equation to solve boundary layer equation in present paper with sufficient accuracy. The momentum integral equation, the kinetic energy integral equation and wall compatibility condition in non-dimensional form have been derived.

Keywords : kinetic energy, permeable wall, source flow.

NOTATIONS

x = radial distance from the source along the wall.
a = Representative length
t^* = Momentum thickness parameter
$\frac{a}{x} =$ distance of the edge of wall from the source.
$\frac{x}{a} =$ non-dimensional radial distance.
u = velocity in the boundary layer in x-direction.
v = velocity in y-direction
$U(x)$ = potential flow velocity.
U_0 = entrance velocity at the edge of the wall.
V_s = Normal velocity at the surface
$U(\bar{x}) = \frac{U(x)}{U_0}$

SOURCE FLOW

(Solution with the aid of Pohlhausen's Profile P_4)

In the case of laminar incompressible flow in Source flow, the potential velocity is represented by $U(x) = U_0 \frac{a}{x}$.

Let $\bar{x} = \frac{x}{a}$, the non-dimensional radial distance.

$$\bar{U} = \frac{U}{U_0} = \frac{U_0 \frac{a}{x}}{U_0} = \frac{a}{x} = \frac{1}{\bar{x}} \tag{1}$$

$$t^* = \left(\frac{\theta}{a}\right)^2 = \frac{U_0 a}{\upsilon} \tag{2}$$

$$\Delta = \frac{\theta^2}{\upsilon}\frac{dU}{dx} = t^* \frac{d\bar{U}}{dx} = -\frac{t^*}{\bar{x}^2} \tag{3}$$

$$\bar{V}_s = \frac{V_s}{U_0}\sqrt{\frac{U_0 a}{\upsilon}} \tag{4}$$

$$\lambda = \frac{V_s \theta}{\upsilon} = t^{*\frac{1}{2}} \bar{v}_s \tag{5}$$

$$\frac{dt^*}{d\bar{x}} = f\left(\bar{x}, H_\varepsilon, t^*\right) = 2\bar{x}^2 \left\{ I + \frac{(2+H)t^*}{\bar{x}^2} + t^{*\frac{1}{2}} \bar{V}_s \right\} \tag{6}$$

$$\frac{dH_\varepsilon}{d\bar{x}} = g\left(\bar{x}, H_\varepsilon, t^*\right) = \frac{\bar{x}}{t^*}\left[2D - H_\varepsilon\left\{I + \frac{(H-1)t^*}{\bar{x}^2} + t^{*\frac{1}{2}}\bar{V}_s\right\} + t^{*\frac{1}{2}}\bar{V}_s\right] \quad (7)$$

and $m = \frac{t^*}{\bar{x}^2} + t^{*\frac{1}{2}}\bar{V}_s$ (8)

VALUE AT THE STARTING POINT

The boundary layer starts to develop from the leading edge $\bar{x} = a$ of the wall. The point $\bar{x} = \frac{x}{a} = 1$ is taken as the starting point $t^* = 0$ and hence $\lambda = t^{*\frac{1}{2}}\bar{V}_s = 0$ for all value of the suction velocity.

At the starting point the boundary layer parameters are

$\bar{x} = 1, \bar{U} = 1, t^* = 0, \lambda = 0, \wedge = 0$

Hence $m = 0$, from equation (8)

$H_\varepsilon = 1.571$

$H = 2.554$

$I = 0.235$

$D = 0.1745.$

Numerical Solution of The Momentum and The Kinetic Energy Integral Equation

The Runga-Kutta method for the two ordinary first order simultaneous differential equation is

$$K_1 = f\left(\bar{x}_0, t_0^*, H_{\varepsilon 0}\right)\Delta\bar{x}$$

$$K_2 = f\left(\bar{x}_0 + \frac{\Delta\bar{x}}{2}, t_0^* + \frac{K_1}{2}, H_{\varepsilon 0} + \frac{\ell_1}{2}\right)\Delta\bar{x}$$

$$K_3 = f\left(\bar{x}_0 + \frac{\Delta\bar{x}}{2}, t_0^* + \frac{K_2}{2}, H_{\varepsilon 0} + \frac{\ell_2}{2}\right)\Delta\bar{x}$$

$$K_4 = f\left(\bar{x}_0 + \Delta\bar{x}, t_0^* + K_3, H_{\varepsilon 0} + \ell_3\right)\Delta\bar{x}$$

$$\Delta t^* = \frac{1}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right)$$

for f and g are computed and calculation proceeds step upto the point of separation.

SOURCE FLOW

(SOLUTION WITH THE AID OF SCHLICHTING'S PROFILE) Along A Solid Wall.

For Schlichting profile the compatibility condition becomes

$$(K+1)\left(\frac{\theta}{\delta}\right)^2\left[1 + \left(1 - \frac{\Pi}{6}\right)K\right]\frac{\theta}{\delta}\lambda - \wedge = 0$$

becomes

$$(K+1)\left(\frac{\theta}{\delta}\right)^2 + \bar{V}_s t^{*\frac{1}{2}}\frac{\theta}{\delta}\left[1 + \left(1 - \frac{\Pi}{6}\right)K\right] + \frac{t^*}{\bar{x}^2} = 0 \quad (9)$$

At the starting point the boundary layer parameters are

$\bar{x} = 1, \bar{U} = 1, t^* = 0, \lambda = 0, \wedge = 0$

Hence by equation (9), K = -1 then corresponding to K = -1, the values of the boundary layer parameter are

$\frac{\theta}{\delta} = 0.4098, \ell = 0.2145, H = 2.6600, H_\varepsilon = 1.5532, D = 0.1685.$

The momentum integral equation (6) and kinetic energy integral equation (7) have been solved with the aid of wall compatibility condition (9) by Ranga-Kutta method for procedure of numerical integration has been motion (6). The point of separation for solid wall $(\bar{V}_s = 0)$ by this method is $\bar{X}_s = 1.2060$ which is very close to the known-result of Pohlhausen's results of numerical calculation have been entered for $\bar{V}_s = 0.$

RESULTS

Calculation have been made for three different constant value of $\bar{V}_s = 0, -0.2, -0.3$ when solved with the aid of Pohlhausen's profile P_4. For, $\bar{V}_s = 0$ the equation reduce to the equations for the solid wall problem. The point of separation for $\bar{v}_s = 0$ is found to be at $\bar{x} = 1.1612$ which is in close agreement with the value $\bar{x} = 1.2130$ obtained by Pohlhausen.

DISCUSSION OF THE RESULTS

Calculation have been made for

(i) $\bar{V}_s = 0, -0.2, -0.3$ when solved with the aid of Pohlhausen's profile P_4 and

(ii) $\bar{V}_s = 0, -0.5,$ when solved with the aid of Schlichting's profile.

The results of calculations are shown in the following comparison table.

Comparison Table

Methods	Suction parameter $\bar{v}_s$	Points of separation $\bar{x}$
1. Pohlhausen	0.0	1.2130
2. Present method with the aid of Pohlhausen's profile	0.0	1.1612
	-0.2	1.1947
	-0.3	1.2115
3. Present method with the aid of Schlichting's profile	0.0	1.2060
	-0.5	1.3260

From above we observe that with increasing rate of suction parameter $\bar{V}_s$ the point of separation moves further down stream.

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