HYDRODYNAMIC FLOW NEAR A TIME VARYING ACCELERATED POROUS PLATE IN ROTATING SYSTEM

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ABSTRACT : In the paper we discuss an initial value investigation has been made of the motion of an incompressible viscous fluid over a porous plate with uniform suction. Both the plate and the fluid are in state of solid body rotation with constant angular velocity about *z*-axix. Normal to plate and the plate as assumed to accelerated with a given velocity. A solution describing the general feature of the unsteady Hydrodynamic boundary layer flow in a rotating system with suction has been obtained. The result obtained have been compared with previous investigation of the topic.

Keywords: Hydrodynamic, incompressible, viscous fluid, accelerated, porous plate, rotating system.

INTRODUCTION

The flow of viscous incompressible and electrically conducting fluid past and impulsively moving infinite plate in the presence of on external magnetic field has been investigated by Rassow and Kukutani. Further Gupta, Nanda and Asundram, Mahaptra, Mishra extended the problem. Gupta has studied an exect solution for the flow past a plate with uniform suction in a rotating reference from.

MATHEMATICAL FORMULATION

The Navier–Strokes equation and the equation of continuity for the unsteady motion of viscous fluid in a rotating reference frame are

$$\frac{\partial \vec{u}}{\partial t'} + (\nabla \cdot \vec{u})\vec{u} + 2\Omega \vec{K} x \vec{u} = \frac{1}{\rho}\nabla p + v\nabla^2 \vec{u} \qquad (1)$$

$$\text{div}\vec{u} = 0 \qquad (2)$$

where $\vec{u} = (u, v, \omega)$ is the velocity vector , $\vec{k}$ the unit vector along z-axis, p the pressure including centrifugal term, ρ the density of fluid, ν the kinematic viscosity. The velocity is assumed to be dependent on z and t so that

$$\vec{u}(z,t) = [u(z,t), v(z,t), \omega(z,t)] \qquad (3)$$

It follows from equation (2), together with uniform suction that $W = -W_0$ is constant. Clearly $W_0 > 0$ for suction and $W_0 < 0$ for blouling.

In absence of pressure gradient, the equation of motion (1) can be written as

$$\frac{\partial u}{\partial t'} - W_0 \frac{\partial u}{\partial z} - 2\Omega \upsilon = \upsilon \frac{\partial^2 u}{\partial z^2} \qquad (4)$$

$$\frac{\partial v}{\partial t} - W_0 \frac{\partial \upsilon}{\partial z} + 2\Omega \upsilon = \upsilon \frac{\partial^2 v}{\partial z^2} \qquad (5)$$

Let us take $q = u + iv$ we take

$$\frac{\partial q}{\partial t} - W_0 \frac{\partial q}{\partial z} + 2\Omega i q = \upsilon \frac{\partial^2 q}{\partial z^2} \qquad (6)$$

The boundary conditions for the present problem are

$$q(z,t) = 0 \text{ for all } z = 0 \text{ and } t \le 0 \qquad (7)$$

$$q(z,t) = \lambda e^{t\omega t} t^n, \omega = \omega_0 \text{ for } z = 0 \text{ and } t > 0 \qquad (8)$$

and

$$q \to 0 \text{ or infinite as } z \to \infty \text{ for } t > 0 \qquad (9)$$

where λ is constant.

SOLUTION OF THE PROBLEM

It is convenient to introduce non-dimensional variables and non-dimensional parameters respectively in the forms

$$\eta = \frac{z\lambda}{\upsilon}, t = \Omega t', U = \frac{q}{\lambda} \qquad (10)$$

and

$$R = \frac{\omega_0}{\upsilon}, N = \frac{2\Omega\upsilon}{\lambda^2}, \sigma = \frac{\omega}{\Omega} \qquad (11)$$

then equation (6) and the boundary and initial condition (7) to (9) becomes

$$\frac{\partial^2 U}{\partial \eta^2} + R\frac{\partial u}{\partial \eta} - iNU = \frac{N}{2}\frac{\partial u}{\partial t} \qquad (12)$$

$$U = 0 \text{ everywhere for } t \le 0 \qquad (13)$$

In order to solve the initial value problem we introduce the Laplace-transformation from

$$\bar{U}(\eta,P)=\int_0^\infty e^{-pt}\cdot U(\eta,t)dt \tag{14}$$

The Laplace-transformation equation (12) and boundary condition (13) and (14) are given by

$$\frac{\partial^2\bar{U}}{\partial\eta^2}+R\frac{\partial\bar{U}}{\partial\eta}-\left(iN+\frac{P}{2}N\right)\bar{U}=0 \tag{15}$$

$$\bar{U}(\eta,P)=\frac{\lambda\Gamma(m+1)}{(p-i\sigma)^{(m+1)}}\ \text{at}\ \eta=0 \tag{16}$$

and $$\bar{U}(\eta,P)=0\ \text{or infinite as}\ \eta\to\infty \tag{17}$$

Taking $\frac{d\bar{U}}{d\eta}\equiv D,$ equation (15) can be written as

$$\left[D^2+RD-\left(iN+P\frac{N}{2}\right)\right]\bar{U}=0$$

$$D=\frac{-R\pm\sqrt{R^2+4\left(iN+\frac{P}{2}N\right)}}{2}=\frac{-R}{2}\pm\sqrt{\frac{N}{2}}\cdot\sqrt{P+\frac{R^2+4iN}{2N}}$$

The solution of (15) is written as

$$\bar{U}(\eta,P)=C_1\exp\left[\frac{-R\eta}{2}+\eta\sqrt{\frac{N}{2}}\cdot\sqrt{P+\frac{R^2+4iN}{2N}}\right]+C_2\exp\left[\frac{-R\eta}{2}-\eta\sqrt{\frac{N}{2}}\cdot\sqrt{P+\frac{R^2+4iN}{2N}}\right] \tag{18}$$

Let $$F(\alpha)=I(\eta,t,\alpha,0)=\frac{1}{2\pi i}\int_{Br_2}\exp\left[\left\{x^2-\frac{R^2+4iN}{2N}\right\}t-Qx\right]\frac{2x}{(x^2-\alpha)}dx \tag{19}$$

Differentiating w.r.t. α we get

$$I(\eta,t,\alpha,1)=\frac{dF}{d\alpha}+tF \tag{20}$$

Again differentiating (m – 1) times we have

$$I(\eta,t,\alpha,1)=\frac{1}{2}\exp(i\sigma t)\left[\exp\left(Q\sqrt{\alpha}\ \text{erfc}\left(\frac{Q+2t\sqrt{\alpha}}{2\sqrt{t}}\right)\left(\frac{Q}{2\sqrt{\alpha}}+1\right)-\exp\left(Q\sqrt{\alpha}\right)\text{erfc}\left(\frac{Q-2t\sqrt{\alpha}}{2\sqrt{t}}\right)\left(\frac{C}{2\sqrt{\alpha}}-t\right)\right)\right] \tag{21}$$

CONCLUSION AND REMARKS

The equation (18) and (19) describe the general feature of the hydrodynamic boundary layer flow in rotating unsteady system. In particular the steady state solution has been obtained when $I\sigma t\neq 0$ and $m=0$.

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