Phase space analysis to determine the one D, two D, three D and DD fermi Parameters

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Abstract:

Phase space is a consequence of Heisenberg uncertainty principle (HUP) and it has become one of the greatest invention of modern physics because study of phase space provide very deep understanding of many microscopical physical phenomenon. Since as we are aware that nature loves symmetry and when symmetry and phase space both are taken into account simultaneously then it provide deep and easiest understanding of 1 D, 2D,3D and DD fermi parameters. In this research paper we have calculated various physical properties like as fermi energy, fermi momentum, fermi wavelength, fermi temperature, fermi wavevector etc with the help of symmetry and concept of phase space.

Keywords: Heisenberg uncertainty principle ,phase space, symmetry, fermi energy etc

Introduction:

In classical mechanics, microstates of any system can be defined by position and momentum of all the particles of the system. Now if a system consist N , number of particles in the system then microstate of system can be specified by 3N position co-ordinates $q_1,q_2,q_3\ldots\ldots\ldots q_{3N}$ and 3N momentum co-ordinates $p_1,p_2,p_3\ldots\ldots\ldots p_{3N}$. obviously 6N dimension is required to completely described the system. This type of 6N dimensional space is known to be phase space. Phase point(q_j,p_j) are said to be representative point of the given system.

State of a particle in phase space is then given by specifying that its position co-ordinate lie in the interval between q and q+δq , and at the same instant its momentum co-ordinates lies in the interval between p and p+δp , it means phase space is divided into very small phase cells of size δpδq here each cell have the same size and each cell represents a different states of the particle.

According to Heisenberg uncertainty principle (HUP) $\Delta q\Delta p \sim$h here h is planck constant and having the dimension of joule-sec. Δq represents uncertainty in the measurement of position and Δp is the corresponding uncertainty in the measurement of the momentum. HUP itself ensure that phase space should be divided into subparts having the volume h of a particular phase cell. Therefore two dimensional volume of a particular phase cell will be of the order of h (planck constant). In a 2f dimensional space volume of a particular phase cell will be h^f . This result ensure that in two dimensional phase space, volume of a particular phase cell will be order of h. Similarly in four dimensional phase space and six dimensional space volume of a corresponding phase cell will be of the order of h^2 and h^3 respectively. In mathematical form we can represent volume of a phase cell

$\Delta x\Delta p_x \sim$h (joule-sec) two dimension phase cell volume

$\Delta x\Delta p_x\Delta y\Delta p_y \sim h^2$ (joule-sec)^2 four dimension phase cell volume

$\Delta x\Delta p_x\Delta y\Delta p_y\Delta z\Delta p_z \sim h^3$(joule-sec)^3 six dimension phase cell volume

From modern quantum physics we know that state of a system is described by spatial parts and spin parts. Due to this volume of phase space also depends upon spatial or position- momentum part and it also depends upon total spin states. For a particular system which consist many particles and each individual particle have spin S then for such particle magnetic spin quantum number m_s have the value lies between -s to +s , so tatal spin state is equal to $\sum_{-s}^{+s} m_s$= (2s+1)=γ values.

Fermi Energy:

It is described as the highest energy that the electrons assumes at a temperature of 0K. under the free electron model , the electrons in a metal can be considered to form a fermi gas and number density (N/V) of the conduction electron in metals range between approximately 10^{28} to 10^{29} electrons per met^3 and value of fermi energy is of the order of 2 to 10 eV . More appropriately fermi energy is lowest for cesium metal 1.53 eV and it is highest for aluminium metal 11.8eV. for white dwarf fermi energy is about to be order of 0.3 MeV. For nucleus it is of the order of 40MeV. Fermi energy in different dimensions have different dependency but in all the cases it always depends upon density and mass of the particle.

Fermi Temperature:

The fermi temperature can be assumed to be the temperature at which thermal effects becomes comparable to quantum effects associated with fermi statistics. The fermi temperature for a metal is a couple of orders of magnitude above room temperature.

In mathematical form fermi temperature is defined as $T_f = \left(\frac{\varepsilon_f}{K_B}\right)$ where ε_f is the fermi energy and K_B is the Boltzmann constant.

Fermi momentum:

Momentum of a electron corresponding to fermi energies said to be fermi momentum. In mathematical form $\varepsilon_f = \left(\frac{{p_f}^2}{2m}\right)$ where p_f is the fermi momentum

Fermi velocity can be described as $V_f = \frac{p_f}{m}$ here fermi velocity is the group velocity of a fermion at the fermi surface .

Fermi momentum $p_f = \hbar k_f$ where k_f is called fermi wave vector.

Use of phase space in classical and quantum theory was discussed in detail [1] . quantum field theory [2], Ising model [3] random walk [4] phenomenon was explained with the help of phase space and its dimensions. Energy, density of states and dimensions relation was discussed and necessary mathematics was drawn in detail, [5,6]. Path integration of a relativistic particle in D dimensional space [7,8] was discussed by different research workers and finite dimensional Hillbert space [9-13] and its construction also discussed by various workers.

$N_s = [D-1]\left[\frac{L}{2\pi}\right]^D \frac{\pi^{D/2}}{\Gamma 1+\frac{D}{2}} \left(\frac{2m}{\hbar^2}\right)^{\frac{D}{2}} E^{D/2}$ here N_s is the number of particle quantum states , E is the energy of the particle, m is the mass of the particle, L is the length of the system and D is the dimension of the system.

Density of states g(E) is defined as the number of particles quantum states per unit energy range so number g(E)dE of particles states with in the energy range E to E+dE is thus given by dN_s so that we have

$$g(E) = \frac{dNs}{dE} = \frac{D}{2}\ [D-1]\left[\frac{L}{2\pi}\right]^{D}\frac{\pi^{D/2}}{\Gamma 1+\frac{D}{2}}\left(\frac{2m}{\hbar^2}\right)^{\frac{D}{2}} E^{\frac{D-2}{2}}$$

This expression clearly shows that density of state is directly proportional to

$$g(E) \propto L^{D}$$

$$g(E) \propto E^{\frac{D-2}{2}}$$

As we know that for 1D space D=1 so we have

$$g(E) \propto L$$

$$g(E) \propto E^{\frac{-1}{2}}$$

similarly for 2D space D=2 so we have

$$g(E) \propto L^{2}$$

$g(E) \propto E^{0}$ independent upon E

and in the same way for 3D space D=3 so we have

$$g(E) \propto L^{3}$$

$$g(E) \propto E^{\frac{1}{2}}$$

now to evaluate for fermi energy we have

$$N = \int_0^{\varepsilon_F} g(E)dE$$

$$N = [D-1]\left[\frac{L}{2\pi}\right]^{D}\frac{\pi^{D/2}}{\Gamma 1+\frac{D}{2}}\left(\frac{2m}{\hbar^2}\right)^{\frac{D}{2}} \varepsilon_F{}^{D/2}$$

$$\varepsilon_F = \frac{\hbar^2}{2m}4\pi\left[\frac{\Gamma 1+\frac{D}{2}}{D-1}\rho\right]^{\frac{2}{D}}$$

From this expression for a 1D system

$$\varepsilon_F \propto \rho^2 \propto \left(\frac{N}{L}\right)^2$$

For 2D system

$$\varepsilon_F \propto \rho \propto \frac{N}{A}$$

For 3D system

$$\varepsilon_F \propto \rho^{\frac{2}{3}} \propto \left(\frac{N}{V}\right)^{\frac{2}{3}}$$

In above expressions L represents the length of the system in 1D , A represents the area of the system in 2D and V represents the volume of the system in 3D .

Now total energy of a fermi gas in the ground state at absolute temperature T = 0K is

$$E_{total} = \int_0^{\varepsilon_F} Eg(E)dE$$

$$= \frac{D(D-1)V}{(D+2)(2\pi)^D}\left(\frac{2m}{\hbar^2}\right)^{\frac{D}{2}}\frac{\pi^{D/2}}{\Gamma 1+\frac{D}{2}}\varepsilon_F^{\frac{D+2}{2}}$$

So we have $E_{total} = \left(\frac{D}{D+2}\right)N\varepsilon_F$ And average energy per particle

$E_0 = \frac{\text{Etotal}}{N} = \left(\frac{D}{D+2}\right)\varepsilon_F$

From this expression it can be concluded that

For 1D , D=1 $E_0 = \frac{\varepsilon_F}{3}$

For 2D, D=2 $E_0 = \frac{\varepsilon_F}{2}$

For 3D, D=3 $E_0 = \frac{3}{5}\varepsilon_F$

Fermi wave vector $K_F = \left[\frac{(4\pi)^{D/2}}{D-1}\Gamma 1 + \frac{D}{2} \quad \rho\right]^{\frac{1}{D}}$

For 2D , D=2 then $K_F = [4\pi\,\rho]^{\frac{1}{2}}$

For 3D, D=3 then $K_F = [3\pi^2\,\rho]^{\frac{1}{3}}$

Symmetrical mathematical analysis for fermi parameters:

1D Analysis:

N(total number of particle quantum states) = $\frac{\iint dx dp_x}{h}$ × $\sum_{-S}^{+S} m_s$

Due to spatial states due to spin states

Now taking the limit of x from 0 to L and for the momentum limit of p_x is from 0 to p_f where L is the length of the system and p_f is the fermi momentum.so we have

$N = \frac{\int_0^L \int_0^{p_f} dx dp_x}{h} \times (2S+1)$ where S is the spin of the particle

$N = \frac{L}{h}p_f \times \gamma$ where $\gamma = (2S+1)$

$p_f = \frac{Nh}{\gamma L}$ This expression shows that fermi momentum depends upon the directly proportional to density $\frac{N}{L}$ and inversely proportional to γ in the case of 1D

Fermi wavelength:

Wavelength corresponding to fermi momentum is said to be fermi wavelength. From the De Broglie hypothesis of wave particle duality we have

$$\lambda_f = \frac{h}{p_f} = \frac{h}{\frac{Nh}{\gamma L}} = \left(\frac{N}{\gamma L}\right)^{-1}$$

This expression shows that fermi wavelength depends upon inversely proportional to density $\frac{N}{L}$ and directly proportional to γ in the case of 1D.

Fermi Energy:

Energy corresponding to the fermi momentum is said to be fermi energy. So we have

$$\varepsilon_f = \frac{(p_f)^2}{2m} = \frac{1}{2m}\left[\frac{Nh}{\gamma L}\right]^2$$

This expression shows that fermi energy depends upon directly proportional to square power of density and inversely proportional to square power of γ in case of 1D.

Fermi Temperature:

Temperature at which thermal effects becomes comparable to quantum effects associate with the fermi Dirac statistics is said to be fermi temperature and mathematically we have

$$\varepsilon_f = k_B T_f \text{ or } T_f = \frac{\varepsilon_f}{k_B} = \frac{1}{2mk_B}\left[\frac{Nh}{\gamma L}\right]^2$$

This expression shows that fermi temperature depends upon directly proportional to square power of density and inversely proportional to square power of γ in case of 1D.

Fermi wavevector:

Wave vector corresponding to fermi momentum is said to be fermi wavevector. Mathematically we have

$$p_f = \hbar k_f \text{ or } k_f = \frac{p_f}{\hbar} = \frac{1}{\hbar}\frac{Nh}{\gamma L} = \frac{2\pi N}{\gamma L}$$

This expression shows that fermi wavevector is directly proportional to density and inversely proportional to γ in case of 1D.

Relation between average energy and fermi energy:

$$\overline{\varepsilon_0} = \frac{\int_0^{\varepsilon_f} \varepsilon D(\varepsilon) d\varepsilon}{\int_0^{\varepsilon_f} D(\varepsilon) d\varepsilon}$$

In 1D density of states is inversely proportional to half power of energy so we have

$$\overline{\varepsilon_0} = \frac{\int_0^{\varepsilon_f} \varepsilon A \varepsilon^{\frac{-1}{2}} d\varepsilon}{\int_0^{\varepsilon_f} A \varepsilon^{\frac{-1}{2}} d\varepsilon}$$

Here A is proportional constant

$$\overline{\varepsilon_0} = \frac{\varepsilon_f}{3}$$

This shows that average energy of the particle is equal to one third of fermi energy in 1D.

2D Analysis:

N(total number of particle quantum states) = $\frac{\iint dx dp_x \iint dy dp_y}{h^2}$ × $\sum_{-s}^{+s} m_s$

Due to spatial states due to spin states

$\iint dxdy = A(area\ of\ the\ system)$ and for $\iint dp_x\, dp_y$ construct a ring of radius having the momentum p and width dp and then take the limit of momentum from 0 to p_f where p_f is the fermi momentum.so we have

$$N = \frac{A}{h^2}\int_0^{p_f} 2\pi p dp \times (2S+1) \quad \text{where S is the spin of the particle}$$

$$N = \frac{\pi A}{h^2} {p_f}^2 \times \gamma \quad \text{where } \gamma = (2S+1)$$

$${p_f}^2 = \frac{Nh^2}{\pi A\gamma}$$

$$\text{So } p_f = h\left[\frac{N}{\pi A\gamma}\right]^{\frac{1}{2}}$$

This expression shows that fermi momentum depends upon the directly proportional to half power of density $\frac{N}{A}$ and inversely proportional to half power of γ in the case of 2D

Fermi wavelength:

Wavelength corresponding to fermi momentum is said to be fermi wavelength. From the De Broglie hypothesis of wave particle duality we have

$$\lambda_f = \frac{h}{p_f} = \frac{h}{h\left[\frac{N}{\pi A\gamma}\right]^{\frac{1}{2}}} = \left[\frac{N}{\pi A\gamma}\right]^{\frac{-1}{2}}$$

This expression shows that fermi wavelength depends upon inversely proportional to half power of density $\frac{N}{A}$ and directly proportional to half power of γ in the case of 2D.

Fermi Energy:

Energy corresponding to the fermi momentum is said to be fermi energy. So we have

$$\varepsilon_f = \frac{(p_f)^2}{2m} = \frac{1}{2m}\frac{Nh^2}{\pi A\gamma}$$

This expression shows that fermi energy depends upon directly proportional to density and inversely proportional to γ in case of 2D.

Fermi Temperature:

Temperature at which thermal effects becomes comparable to quantum effects associate with the fermi Dirac statistics is said to be fermi temperature and mathematically we have

$$\varepsilon_f = k_B T_f \text{ or } T_f = \frac{\varepsilon_f}{k_B} = \frac{1}{k_B}\frac{1}{2m}\frac{Nh^2}{\pi A\gamma}$$

This expression shows that fermi temperature depends upon directly proportional to density and inversely proportional to γ in case of 2D.

Fermi wavevector:

Wave vector corresponding to fermi momentum is said to be fermi wavevector. Mathematically we have

$$p_f = \hbar k_f \text{ or } k_f = \frac{p_f}{\hbar} = \frac{1}{\hbar} h\left[\frac{N}{\pi A\gamma}\right]^{\frac{1}{2}} = 2\pi\left[\frac{N}{\pi A\gamma}\right]^{\frac{1}{2}}$$

This expression shows that fermi wavevector is directly proportional to half power of density and inversely proportional to half power of γ in case of 2D.

Relation between average energy and fermi energy:

$$\bar{\varepsilon_0} = \frac{\int_0^{\varepsilon_f} \varepsilon D(\varepsilon) d\varepsilon}{\int_0^{\varepsilon_f} D(\varepsilon) d\varepsilon}$$

In 2D density of states is independent upon energy so we have

$$\bar{\varepsilon_0} = \frac{\int_0^{\varepsilon_f} \varepsilon A d\varepsilon}{\int_0^{\varepsilon_f} A d\varepsilon}$$

Here A is proportional constant

$$\bar{\varepsilon_0} = \frac{\varepsilon_f}{2}$$

This shows that average energy of the particle is equal to half of fermi energy in 2D.

3D Analysis:

N(total number of particle quantum states) = $\frac{\iint dx dp_x \iint dy dp_y \iint dz dp_z}{h^3}$ × $\sum_{-s}^{+s} m_s$

Due to spatial states due to spin states

$\iiint dxdydz = V(volume\ of\ the\ system)$ and for $\iiint dp_x dp_y dp_z$ construct a spherical shell having the radius of momentum p and width dp and then take the limit of momentum from 0 to p_f where p_f is the fermi momentum.so we have

N = $\frac{V}{h^3}\int_0^{p_f} 4\pi p^2 dp$ × $(2S+1)$ where S is the spin of the particle

N = $\frac{V}{h^3}\frac{4}{3}\pi {p_f}^3 \times \gamma$ where γ = $(2S+1)$

${p_f}^3 = \frac{3Nh^3}{4\pi V\gamma}$

So $p_f = h\left[\frac{3N}{4\pi V\gamma}\right]^{\frac{1}{3}}$

This expression shows that fermi momentum depends upon the directly proportional to one third power of density $\frac{N}{V}$ and inversely proportional to one third power of γ in the case of 3D

Fermi wavelength:

Wavelength corresponding to fermi momentum is said to be fermi wavelength. From the De Broglie hypothesis of wave particle duality we have

$$\lambda_f = \frac{h}{p_f} = \frac{h}{h\left[\frac{3N}{4\pi V\gamma}\right]^{\frac{1}{3}}} = \left[\frac{3N}{4\pi V\gamma}\right]^{\frac{-1}{3}}$$

This expression shows that fermi wavelength depends upon inversely proportional to one third power of density $\frac{N}{V}$ and directly proportional to one third power of γ in the case of 3D.

Fermi Energy:

Energy corresponding to the fermi momentum is said to be fermi energy. So we have

$$\varepsilon_f = \frac{(p_f)^2}{2m} = \frac{1}{2m}h^2\left[\frac{3N}{4\pi V\gamma}\right]^{\frac{2}{3}}$$

This expression shows that fermi energy depends upon directly proportional to two third power of density and inversely proportional to two third power of γ in case of 3D.

Fermi Temperature:

Temperature at which thermal effects becomes comparable to quantum effects associate with the fermi Dirac statistics is said to be fermi temperature and mathematically we have

$$\varepsilon_f = k_B T_f \text{ or } T_f = \frac{\varepsilon_f}{k_B} = \frac{1}{k_B}\frac{1}{2m}h^2\left[\frac{3N}{4\pi V\gamma}\right]^{\frac{2}{3}}$$

This expression shows that fermi temperature depends upon directly proportional to two third power of density and inversely proportional to two third power of γ in case of 3D.

Fermi wavevector:

Wave vector corresponding to fermi momentum is said to be fermi wavevector. Mathematically we have

$$p_f = \hbar k_f \text{ or } k_f = \frac{p_f}{\hbar} = \frac{1}{\hbar}h\left[\frac{3N}{4\pi V\gamma}\right]^{\frac{1}{3}} = 2\pi\left[\frac{3N}{4\pi V\gamma}\right]^{\frac{1}{3}}$$

This expression shows that fermi wavevector is directly proportional to one third power of density and inversely proportional to one third power of γ in case of 3D.

Relation between average energy and fermi energy:

$$\bar{\varepsilon_0} = \frac{\int_0^{\varepsilon_f} \varepsilon D(\varepsilon)d\varepsilon}{\int_0^{\varepsilon_f} D(\varepsilon)d\varepsilon}$$

In 3D density of states is directly proportional to half power of energy so we have

$$\bar{\varepsilon_0} = \frac{\int_0^{\varepsilon_f} \varepsilon A\varepsilon^{\frac{1}{2}}d\varepsilon}{\int_0^{\varepsilon_f} A\varepsilon^{\frac{1}{2}}d\varepsilon}$$

Here A is proportional constant

$$\bar{\varepsilon_0} = \frac{3}{5}\varepsilon_f$$

This shows that average energy of the particle is equal to three fifth part of fermi energy in 3D.

DD Analysis:

N(total number of particle quantum states) = $\frac{\iint dx dp_x \iint dy dp_y \iint dz dp_z \ldots\ldots \iint dx_D dp_D}{h^D} \times \sum_{-s}^{+s} m_s$

Due to spatial states due to spin states

On solving we get

$$N = [D-1]\left[\frac{L}{2\pi}\right]^D \frac{\pi^{D/2}}{\Gamma 1+\frac{D}{2}}\left(\frac{2m}{\hbar^2}\right)^{\frac{D}{2}} \varepsilon_F{}^{D/2} \times \gamma$$

where $\gamma = (2S+1)$, S is the spin of the particle

$$\varepsilon_F = \frac{\hbar^2}{2m} 4\pi \left[\frac{\Gamma 1+\frac{D}{2}}{D-1}\rho\right]^{\frac{2}{D}} \times \left[\frac{1}{\gamma}\right]^{\frac{2}{D}}$$

Fermi momentum:

$$\varepsilon_F = \frac{p_f{}^2}{2m} = \frac{\hbar^2}{2m} 4\pi \left[\frac{\Gamma 1+\frac{D}{2}}{D-1}\rho\right]^{\frac{2}{D}} \times \left[\frac{1}{\gamma}\right]^{\frac{2}{D}}$$

$$p_f = \hbar(4\pi)^{\frac{1}{2}} \left[\frac{\Gamma 1+\frac{D}{2}}{D-1}\rho\right]^{\frac{1}{D}} \times \left[\frac{1}{\gamma}\right]^{\frac{1}{D}}$$

This expression shows that fermi momentum depends upon the directly proportional to one upon D power of density ρ and inversely proportional to one upon D power of γ in the case of DD

Fermi wavelength:

Wavelength corresponding to fermi momentum is said to be fermi wavelength. From the De Broglie hypothesis of wave particle duality we have

$$\lambda_f = \frac{h}{p_f} = \frac{h}{\hbar(4\pi)^{\frac{1}{2}}\left[\frac{\Gamma 1+\frac{D}{2}}{D-1}\rho\right]^{\frac{1}{D}} \times \left[\frac{1}{\gamma}\right]^{\frac{1}{D}}} = 2\pi(4\pi)^{\frac{-1}{2}} \left[\frac{\Gamma 1+\frac{D}{2}}{D-1}\rho\right]^{\frac{-1}{D}} \times \left[\frac{1}{\gamma}\right]^{\frac{-1}{D}}$$

This expression shows that fermi wavelength depends upon inversely proportional to one upon D power of density ρ and directly proportional to one upon D power of γ in the case of DD.

Fermi Energy:

Energy corresponding to the fermi momentum is said to be fermi energy. So we have

$$\varepsilon_f = \frac{(p_f)^2}{2m} = \frac{\hbar^2}{2m} 4\pi \left[\frac{\Gamma 1+\frac{D}{2}}{D-1}\rho\right]^{\frac{2}{D}} \times \left[\frac{1}{\gamma}\right]^{\frac{2}{D}}$$

This expression shows that fermi energy depends upon directly proportional to two upon D power of density and inversely proportional to two upon D power of γ in case of DD.

Fermi Temperature:

Temperature at which thermal effects becomes comparable to quantum effects associate with the fermi Dirac statistics is said to be fermi temperature and mathematically we have

$$\varepsilon_f = k_B T_f \text{ or } T_f = \frac{\varepsilon_f}{k_B} = \frac{1}{k_B}\frac{\hbar^2}{2m}4\pi\left[\frac{\Gamma 1+\frac{D}{2}}{D-1}\rho\right]^{\frac{2}{D}} \times \left[\frac{1}{\gamma}\right]^{\frac{2}{D}}$$

This expression shows that fermi temperature depends upon directly proportional to two upon D power of density and inversely proportional to two upon D power of γ in case of DD.

Fermi wavevector:

Wave vector corresponding to fermi momentum is said to be fermi wavevector. Mathematically we have

$$p_f = \hbar k_f \text{ or } k_f = \frac{p_f}{\hbar} = \frac{1}{\hbar}\hbar(4\pi)^{\frac{1}{2}}\left[\frac{\Gamma 1+\frac{D}{2}}{D-1}\rho\right]^{\frac{1}{D}} \times \left[\frac{1}{\gamma}\right]^{\frac{1}{D}} = (4\pi)^{\frac{1}{2}}\left[\frac{\Gamma 1+\frac{D}{2}}{D-1}\rho\right]^{\frac{1}{D}} \times \left[\frac{1}{\gamma}\right]^{\frac{1}{D}}$$

This expression shows that fermi wavevector is directly proportional to one upon D power of density and inversely proportional to one upon D power of γ in case of DD.

Relation between average energy and fermi energy:

$$\overline{\varepsilon_0} = \frac{\int_0^{\varepsilon_f}\varepsilon D(\varepsilon)d\varepsilon}{\int_0^{\varepsilon_f}D(\varepsilon)d\varepsilon}$$

In DD density of states is directly proportional to half power of energy so we have

$$\overline{\varepsilon_0} = \frac{\int_0^{\varepsilon_f}\varepsilon A\varepsilon^{\frac{D-2}{2}}d\varepsilon}{\int_0^{\varepsilon_f}A\varepsilon^{\frac{D-2}{2}}d\varepsilon}$$

Here A is proportional constant

$$\overline{\varepsilon_0} = \frac{D}{D+2}\varepsilon_f$$

This shows that average energy of the particle is equal to $\frac{D}{D+2}$ part of fermi energy in DD.

Result and Discussion:

Symmetry always play an important role with respect to law of nature. As nature loves symmetry to make thing beautiful so here we have applied symmetry concept on phase space to evaluate the various physical fermi parameters in 1D,2D,3D and DD. Various physical properties like as fermi energy, fermi momentum, fermi wavelength, fermi temperature, fermi wavevector calculated with the help of symmetry and concept of phase space and fermi parameters dependency on density of the system and spin of the particle is shown in the table 1.1 in symmetrical form.

Table 1.1: Comparative table for fermi parameters in 1D,2D and 3D

Physical quantities	1D	2D	3D	DD
No. of states filled upto energy ε is proportional to	$\varepsilon^{\frac{1}{2}}$	$\varepsilon^{\frac{2}{2}}$	$\varepsilon^{\frac{3}{2}}$	$\varepsilon^{\frac{D}{2}}$
Density of states D(ε) is proportional to	$\varepsilon^{\frac{-1}{2}}$	$\varepsilon^{\frac{2-2}{2}}$	$\varepsilon^{\frac{1}{2}}$	$\varepsilon^{\frac{D-2}{2}}$
Density Dependency of fermi momentum	$\left[\frac{N}{L}\right]^{\frac{1}{1}}$	$\left[\frac{N}{A}\right]^{\frac{1}{2}}$	$\left[\frac{N}{V}\right]^{\frac{1}{3}}$	$\rho^{\frac{1}{D}}$
Spin Dependency of fermi momentum γ = 2S+1	$\left[\frac{1}{\gamma}\right]^{\frac{1}{1}}$	$\left[\frac{1}{\gamma}\right]^{\frac{1}{2}}$	$\left[\frac{1}{\gamma}\right]^{\frac{1}{3}}$	$\left[\frac{1}{\gamma}\right]^{\frac{1}{D}}$
Density Dependency Fermi wavelength	$\left[\frac{N}{L}\right]^{\frac{-1}{1}}$	$\left[\frac{N}{A}\right]^{\frac{-1}{2}}$	$\left[\frac{N}{V}\right]^{\frac{-1}{3}}$	$\rho^{\frac{-1}{D}}$
Spin Dependency Fermi wavelength γ = 2S+1	$[\gamma]^{\frac{1}{1}}$	$[\gamma]^{\frac{1}{2}}$	$[\gamma]^{\frac{1}{3}}$	$[\gamma]^{\frac{1}{D}}$
Density Dependency Fermi energy	$\left[\frac{N}{L}\right]^{\frac{2}{1}}$	$\left[\frac{N}{A}\right]^{\frac{2}{2}}$	$\left[\frac{N}{V}\right]^{\frac{2}{3}}$	$\rho^{\frac{2}{D}}$
Spin Dependency Fermi energy γ = 2S+1	$\left[\frac{1}{\gamma}\right]^{\frac{2}{1}}$	$\left[\frac{1}{\gamma}\right]^{\frac{2}{2}}$	$\left[\frac{1}{\gamma}\right]^{\frac{2}{3}}$	$\left[\frac{1}{\gamma}\right]^{\frac{2}{D}}$
Density Dependency Fermi temperature	$\left[\frac{N}{L}\right]^{2}$	$\left[\frac{N}{A}\right]$	$\left[\frac{N}{V}\right]^{\frac{2}{3}}$	$\rho^{\frac{2}{D}}$
Spin Dependency Fermi temperature γ = 2S+1	$\left[\frac{1}{\gamma}\right]^{\frac{2}{1}}$	$\left[\frac{1}{\gamma}\right]^{\frac{2}{2}}$	$\left[\frac{1}{\gamma}\right]^{\frac{2}{3}}$	$\left[\frac{1}{\gamma}\right]^{\frac{2}{D}}$
Density Dependency Fermi wavevector	$\left[\frac{N}{L}\right]^{\frac{1}{1}}$	$\left[\frac{N}{A}\right]^{\frac{1}{2}}$	$\left[\frac{N}{V}\right]^{\frac{1}{3}}$	$\rho^{\frac{1}{D}}$
Spin Dependency Fermi wavevector γ = 2S+1	$\left[\frac{1}{\gamma}\right]^{\frac{1}{1}}$	$\left[\frac{1}{\gamma}\right]^{\frac{1}{2}}$	$\left[\frac{1}{\gamma}\right]^{\frac{1}{3}}$	$\left[\frac{1}{\gamma}\right]^{\frac{1}{D}}$
Average energy $\overline{\varepsilon_0}$ and fermi energy ε_f relation	$\overline{\varepsilon_0} = \frac{\varepsilon_f}{3}$	$\overline{\varepsilon_0} = \frac{\varepsilon_f}{2}$	$\overline{\varepsilon_0} = \frac{3}{5}\varepsilon_f$	$\overline{\varepsilon_0} = \frac{D}{D+2}\varepsilon_f$

References:

1. Corbem H.C , physical review 74,788 (1948).
2. Bender, C.M and Boettcher, S. and Lipatov L. Phys. Rev.D 46,5557,(1992).
3. Bender, C.M and Boettcher, S. Phys. Rev.D 51,1875,(1994).
4. Bender, C.M and Boettcher, S. and Mead L.R. J. Math. Phys. 35,368,(1994)
5. Menon, V.J., and Agrawal D.C, J.Phys.A Math.Gen. 31, 1109 (1998)
6. Sami M. Al Jaber, International journal of theoretical physics, 38, 3,(1998).
7. Grosche, C., and Steiner, F. J. Math. Phys. 36, 2354, (1995).
8. Lin, D.H , J Phys A. Math. Gen. 30, 3201 (1997).
9. Kuang, L.M. and Chen, X. Phys Rev. A 50, 4228 (1994).
10. Kuang, L.M. and Chen, X. Phys Lett.. A 186, 8 (1994).

11. Miranowicz, A., Piatek, K. and Tanas, R. Phys Rev. A 50, 3423 (1994).
12. Kuang, L.M. , Wang, F.B and Zhou, Y.G.. Phys Lett.. A 183, 1 (1993).
13. Roy, B. and Roy P. J Phys A. Math. Gen. 31, 1307 (1998).
14. Griffiths, D.J introduction to quantum mechanics, prentice-hall, Englewood cliffs, New Jersey (1995).
15. Federick Reif, fundamentals of statistical and thermal physics, McGraw -Hill international editions(1985).
16. Loknathan S,. and Gambhir R.S, an introduction statistical and thermal physics, Prentice Hall of India New Delhi(2007).
17. Patharia R.K, statistical mechanics,Butterworth Heinemann (2001).