A STUDY OF (4,3)-JECTION OPERATOR AND SOME EXAMPLES

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Abstract :- In this paper we use notion of projection operator on a linear space as the Simmons, G.F."Introduction to topology and modern Analysis". In analogue to the above notions we have introduced a new type of operator called (4,3)-jection on a linear space. It is a generalization of projection operator in the sense that every projection is a (4,3)-jection but a (4,3)-jection is not necessarily a projection. We shall study (4,3)-jection on R^2 and consider different examples of (4,3)-jection.

Keyword : projection, trijection, tetrajection, (4,3)-jection.

1. Introduction :

A linear transformation on L into itself , we refer to it as a linear transformation on L. Also E is called a projection on M along N where N is linear subspace of M clearly $E^2 = E$.

In analogue to this definition a trijection operator E has been defined by Dr. P. Chandra in his Ph.D. thesis (P.U.1977) titled " Investigation into the theory of operators and linear spaces" by the relation $E^3=E$. Where E is a linear operator an a linear space L.

Clearly if E is a projection, then it is also a trijection,

For $E^2=E \Rightarrow E^3=E^2.E=E.E=E^2=E$.

Thus it follows that a projection in necessarily a trijection but a trijection not a necessarily a projection. Also Dr. R. K. Mishra in Ph.D. thesis (J.P.U. 2010) titled "Study of linear operators and related topics in functional analysis" has defined an operator E to be a tetrajection if $E^4=E$. Where E is a linear operator an a linear space L.

We define a new type of operator is called (4,3)-jection analogously. A linear operator E is called (4,3)-jection on a linear space L.

If $E^4=E^3$

Clearly $E^3 = E^2.E=E.E=E^2=E$

and $E^4 = E^2.E^2=E^2.E = E^3$

Thus it follows that a projection is necessarily a (4,3)-jection but (4,3)-jection is not necessarily a projection. This would be clear from the examples as given bellow:

2. We consider C^2, where C is the set of all complex number. Let Z be an element in C^2.
Thus Let Z =(x,y) where x,y ∊ C

Let $E(Z) = E(x,y)= (ax+by,cx+dy)$
where a,b,c,d are scalers.
$E^3(x,y)= (a_1x+b_1y,c_1x+d_1y)$ $E^4(x,y)= (a_2x+b_2y,c_2x+d_2y)$ By calculation $a_1 = a^3+2abc+bcd$ $b_1 = a^2b+abd+b^2c+bd^2$ $c_1= a^2c+bc^2+acd+cd^2$ $d_1= abc+2bcd+d^3$ And $a_2 = a^4+3a^2bc+2abcd+b^2c^2+bcd^2$ $b_2= a^3b+a^2bd+2ab^2c+abd^2+2b^2cd+bd^3$ $c_2= a^3c+2abc^2+2bc^2d+a^2cd+acd^2+cd^3$
$d_2= a^2bc+2abcd+b^2c^2+3bcd^2+d^4$
If E is a (4,3)-jection. Then
$E^4(x,y) =E^3(x,y)$ Hence $a_2=a_1$ ⇒ $a^4+3a^2bc+2abcd+b^2c^2+bcd^2= a^3+2abc+bcd$................(2.1)

$b_2=b_1 \Rightarrow a^3b+a^2bd+2ab^2c+abd^2+2b^2cd+bd^3= a^2b+abd+b^2c+bd^2$............(2.2)

$C_2=c_1 \Rightarrow a^3c+2abc^2+2bc^2d+a^2cd+cd^3= a^2c+bc^2+acd+cd^2$...............(2.3)

$d_2=d_1 \Rightarrow a^2bc+2abcd+b^2c^2+3bcd^2+d^4= abc+2bcd+d^3$..................(2.4)

2.1. Now we consider the following cases:

Case (1) : Let a= b= 0 then

From (2.1), 0 =0

From (2.2), 0=0

From (2.3), $cd^3 = cd^2 \Rightarrow cd^2(d-1)=0$

⇨ c=0 or d = 0, d=1

From (2.4), $d^4=d^3 \Longrightarrow d^3(d-1)= 0$

⇨ d=0 ,d=1

Thus c=0 or d=0 , d=1

Subcase(1.i): When c=0 , d=1

i.e a=b=c=0 , d=1

Then we get

$E(x,y)= (0,y)$

$E^2(x,y)= E(E(x,y))= E(0,y)=(0,y)$

$E^3(x,y)=E(E^2(x,y))=E(0,y) =(0,y)$

$E^4(x,y)= E(E^3(x,y))= E(0,y)= (0,y)$

Thus in this case $E^4=E^3=E^2=E$ Therefore in this case E is a projection, trijection , tetrajection as well as (4,3)-jection.

Subcase(1-ii): When $c=0$, $d=0$

i.e, $a = b = c = d = 0$

Then we get

$E(x,y)=(0,0)$

$E^2(x,y)= E(E(x,y)) = E(0,0) = (0,0)$

$E^3(x,y) = E(E^2(x,y)) = E(0,0)= (0,0)$

$E^4(x,y) = E(E^3(x,y)) = E(0,0) =(0,0)$ The zero operator.

Hence in this case E is a projection , trijection, tetrajection as well as (4,3)-jecton.

Subcase(1-iii) : When $d=0$, $c\neq0$

Then we get

$E(x,y)= (0,cx)$

$E^2(x,y) =E(E(x,y)) = E(0,cx) = (0, 0)=$ zero operator $\neq E(x,y)$

$E^3(x,y)= E(E^2(x,y)) = E(0, 0)= (0,0) \neq E(x,y)$

$E^4(x,y)= E(E^3(x,y)) = E(0, 0)= (0, 0) \neq E(x,y)$

Clearly $E^4=E^3$

Thus in this case E is neither projection nor trijection nor tetrajection but it is (4,3)-jection.

Subcase (1.iv): When $d = 1$, $c \neq0$

Then we get

$E(x,y) = (0 ,cx+y)$

$E^2(x,y) = E(E(x,y))$
$= E(0 ,cx+y)$
$= (0, cx+y)$

$E^3(x,y) = E(E^2(x,y))$
$= E(0, cx+y)$
$= (0,cx+y) =E(x,y)$

$E^4(x,y) = E(E^3(x,y))$
$= E(0 ,cx+y)$
$= (0 ,cx+y) = E(x,y)$

$\therefore \quad E^4 = E^3 = E^2 = E.$ Thus in this case E is projection, trijection, tetrajection as well as (4,3)-jection.

Case(2): Let a= c =0

from (2.1) , 0 =0

from (2.2), $bd^3 = bd^2$

$\Rightarrow bd^2(d-1) = 0$

$\Rightarrow b = 0$, $d=0$, $d=1$

from (2.3), 0 =0 from (2.4), $d^4 = d^3$ $\Rightarrow d^3(d-1) = 0$

$\Rightarrow d=0, d=1$

Thus b =0 or d =0 or d =1

Subcase(2.i): When $b \neq 0$, $d = 0$

i.e , a =c =d =0

Then we get

$E(x,y) = (by, 0)$

$E^2(x,y) = E(E(x,y))$

$= E(by, 0)$

$= (0,0) = E^2 = 0$

$E^3(x,y) = E(E^2(x,y))$

$= E(0,0)$

$= (0,0)$

$E^4(x,y) = E(E^3(x,y))$

$= E(0,0)$

$= (0,0)$

Therefore in this case E is neither projection nor trijection nor tetrajection but it is (4,3)-jection.

Subcase (2.ii) : When $b \neq 0$, $d = 1$

i.e , a =c =0

Then we get

$E(x,y) = (by, y)$

$E^2(x,y) = E(E(x,y))$

$= E(by, y)$

$= (by, y)$

$E^3(x,y) = E(E^2(x,y))$

$= E(by, y)$

$= (by, y)$

$E^4(x,y) = E(E^3(x,y))$

$= E(by ,y)$

$= (by ,y)$

$E^4= E^3 =E^2 =E.$

Thus in this case E is projection, trijection, tetrajection as well as (4,3)-jection.

Subcase (2.iii) : When $b = 0$, $d = 1$

i.e, $a = b = c = 0$

Then we get

$E(x,y) = (0 ,y)$

$E^2(x,y) = E(E(x ,y))$

$= E(0 ,y)$

$= (0,y)$

$E^3(x,y) =E(E^2(x,y))$

$= E(0 ,y)$

$= (0 ,y)$

$E^4(x,y) = E(E^3(x,y))$

$= E(0 ,y)$

$= (0,y)$ Clearly $E^4 = E^3 = E^2 = E.$

Thus in this case E is projection, trijection, tetrajection as well as (4,3)-jection.

Case (3) : Let $a= c =d = \alpha\neq 0$, then

From (2.1),

$a_2 =a_1 \Rightarrow \alpha^4+3\alpha^3b+2\alpha^3b+b^2\alpha^2+b\alpha^3= \alpha^3+2\alpha^2b+ \alpha^2b$

$\Rightarrow \alpha^4+5\alpha^3b+b^2\alpha^2 +b\alpha^3 = \alpha^3+3\alpha^2b$

$\Rightarrow \alpha^2(\alpha^2+5\alpha b+b^2+\alpha b) = \alpha^2(\alpha+3b)$

Cancelling α^2 from both the sides.

$\Rightarrow \alpha^2+6\alpha b +b^2 = \alpha+3b$ ……………………………….(2.11)

From (2.2) ,

$b_2= b_1 \Rightarrow \alpha^3b+ \alpha^3b+2\alpha^2b^2+ \alpha^3b+2\alpha^2b^2+b\alpha^3 = \alpha^2b+ \alpha^2b+b^2\alpha+b\alpha^2$

$\Rightarrow 4\alpha^3b+4\alpha^2b^2 =3\alpha^2b+b^2\alpha$

$\Rightarrow b\alpha(4\alpha^2+4\alpha b) = \alpha b(3\alpha+b)$

Cancelling αb from both the sides. $(b\neq 0)$

$\Rightarrow 4\alpha^2+4\alpha b = 3\alpha+b$…………………………………….(2.12)

From (2.3),

$c_2=c_1 \Rightarrow \alpha^4+2\alpha^3b+2b\alpha^3+\alpha^4+\alpha^4+\alpha^4 = \alpha^3+b\alpha^2+\alpha^3+\alpha^3$

$\Rightarrow 4\alpha^4+4\alpha^3b = 3\alpha^3+\alpha^2b$

$\Rightarrow \alpha^2(4\alpha^2+4\alpha b) = \alpha^2(3\alpha+b)$

Cancelling α^2 from both the sides.

$\Rightarrow 4\alpha^2+4\alpha b = 3\alpha+b$ ……………………………(2.13)

Which is same as (2.12)

From (2.14),

$d_2=d_1 \Rightarrow \alpha^3b+2\alpha^3b+b^2\alpha^2+3b\alpha^3+\alpha^4= \alpha^2b+2b\alpha^2+\alpha^3$

$\Rightarrow 6\alpha^3b+b^2\alpha^2+\alpha^4 = 3\alpha^2b+\alpha^3$

$\Rightarrow \alpha^2(6\alpha b+b^2+\alpha^2) = \alpha^2(3b+\alpha)$

Cancelling α^2 from both the sides.

$\Rightarrow 6\alpha b+b^2+\alpha^2 = 3b+\alpha$ ………………………………….(2.14)

Which is same as (2.11)

Substracting (2.12) from (2.11)

$\alpha^2+6\alpha b+b^2-(4\alpha^2+4\alpha b) = \alpha+3b-(3\alpha+b)$

$\Rightarrow b^2+2\alpha b-3\alpha^2 = 2b-2\alpha$

$\Rightarrow b^2+2\alpha b+\alpha^2-4\alpha^2= 2(b-\alpha)$

$\Rightarrow (b+\alpha)^2-(2\alpha)^2 = 2(b-\alpha)$

$\Rightarrow (b-\alpha)(b+3\alpha) = 2(b-\alpha)$

Cancelling $(b-\alpha)$ from both the sides. $(\alpha\neq b)$

$\Rightarrow b+3\alpha = 2$

$\therefore b = 2-3\alpha$

From (2.11), we get

$\alpha^2+6\alpha(2-3\alpha)+(2-3\alpha)^2 = \alpha+3(2-3\alpha)$

$\Rightarrow \alpha^2+12\alpha-18\alpha^2+4-12\alpha+9\alpha^2= \alpha+6-9\alpha$

$\Rightarrow -8\alpha^2+4 = 6-8\alpha$

$\Rightarrow 8\alpha^2-8\alpha+2 = 0$

$\Rightarrow$ $2(4\alpha^2-4\alpha+1) = 0$

$\Rightarrow$ $4\alpha^2-4\alpha+1 = 0$

$\Rightarrow$ $(2\alpha-1)^2 = 0$

$\Rightarrow$ $2\alpha - 1 = 0$

$\therefore$ $\alpha = 1/2$

Taking $\alpha = 1/2$

$\therefore$ $b = 2-3.1/2 = 1/2$

Hence $a = b = c = d = \alpha = 1/2$

$E(x,y) = (ax+by, cx+dy)$

$= (\frac{1}{2}x+\frac{1}{2}y, \frac{1}{2}x+\frac{1}{2}y) = (\frac{x+y}{2}, \frac{x+y}{2})$

We have $E^2(x,y) = E(E(x,y)) = E(\frac{x+y}{2}, \frac{x+y}{2})$

$= (\frac{x+y}{4}+\frac{x+y}{4}, \frac{x+y}{4}+\frac{x+y}{4})$

$= (\frac{x+y}{2}, \frac{x+y}{2}) = E(x,y)$, So E is projection.

$E^2 = E$

$\Rightarrow E.E^2 = E.E$

$\Rightarrow E^3 = E^2 = E$

$\Rightarrow E^4 = E^2 = E$

Hence $E^4 = E^3$

In this case E is projection, trijection, tetrajection as well as (4,3)-jection.

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