# PSEUDO $\widetilde{M}$ -PROJECTIVE CURVATURE TENSOR ON A RIEMANNIAN MANIFOLD

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**Abstract:** Yano and Sawaki(1972) introduced quasi conformal curvature tensor in a Riemannian manifold. Recently one of the author Prasad(2002) investigated pseudo projective curvature tensor in a Riemannian manifold. In this paper we introduced a new curvature tensor named as Pseudo $\widetilde{M}$-projective curvature tensor on a Riemannian manifold. Some properties for Pseudo$\widetilde{M}$-projective curvature tensor are investigated. Finally a particular case has been shown.

**Key words and phrases:** Conformal curvature tensor C, projective curvature tensor P, conharmonic curvature tensor H, conciracular curvature tensor V, quasi conformal curvature tensor $\tilde{C}$, pseudo projective curvature tensor $\tilde{P}$, quasi concircular curvature tensor $\tilde{V}$, M-projective curvature tensor M, pseudo M-projective curvature $\widetilde{M}$ and $W_i$ – curvature tensor ($i$=1, 2,----9).

## 1. Introduction

The Weyl conformal curvature tensor (Eisenhart 1949) is the well known tensor, which is invariant of every conformal transformation. In particular, if a conformal transformation transforms a harmonic function into a harmonic function, then it is called a conharmonic transformation, and the conharmonic curvature tensor (Ishii, 1957) is an invariant under conharmonic transformations. Next it is well known that a semi-Riemannian manifold is locally projectively flat if and only if the projective curvature vanishes. Apart from this the concircular curvature tensor (Yano, 1940) is the next most important curvature tensor from the semi – Riemannian point of view, which is an invariant of concircular transformations.

Let conformal curvature tensor C, projective curvature tensor P, conharmonic curvature tensor H and concircular curvature tensor V are given by (Mishra, 1984) .

$$C(X,Y)Z= R(X,Y)\ Z - \frac{1}{n-1}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] + \frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y],$$

where R is the curvature tensor, S is the Ricci tensor and r is the scalar curvature,

'C(X,Y,Z,W)=g(C(X,Y)Z,W)

'R(X,Y,Z,W)=g(R(X,Y)Z,W)

and S(X,Y) = g (QX, Y).

$$P(X,Y)Z=R(X,Y)Z-\frac{1}{n-1}[S(Y,Z)X - S(X,Z)Y]$$

$$H(X,Y)Z=R(X,Y)Z-\frac{1}{n-2}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] \text{ and}$$

$$V(X,Y)Z = R(X,Y)Z-\frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y]$$

These curvatures satisfies the symmetric and skew symmetric property as well as cyclic property possessed by the curvature tensor R(X,Y)Z.

In 1970-72 and 1982 Mishra and Pokhariyal defined the following curvature tensors on Riemannian manifolds

$$W_1(X,Y)Z=R(X,Y)Z + \frac{1}{n-1}[S(Y,Z)X - S(X,Z)Y]$$

$$W_2(X,Y)Z=R(X,Y)Z + \frac{1}{n-1}[g(X,Z)QY - g(Y,Z)QX]$$

$$W_3(X,Y)Z=R(X,Y)Z + \frac{1}{n-1}[g(Y,Z)QX - g(X,Z)QY]$$

$$W_4(X,Y)Z=R(X,Y)Z + \frac{1}{n-1}[g(X,Z)QY - g(X,Y)QZ]$$

$W_5(X,Y)Z=R(X,Y)Z+\frac{1}{n-1}[g(X,Z)QY-S(X,Z)Y]$

$W_6(X,Y)Z=R(X,Y,Z)+\frac{1}{n-1}[g(X,Y)QZ-S(Y,Z)X]$

$W_7(X,Y)Z=R(X,Y)Z+\frac{1}{n-1}[g(Y,Z)QX-S(Y,Z)X]$

$W_8(X,Y)Z=R(X,Y)Z+\frac{1}{n-1}[S(X,Y)Z-S(Y,Z)X]$

$W_9(X,Y)Z=R(X,Y)Z+\frac{1}{n-1}[S(X,Y)Z-g(Y,Z)QY]$

Some properties of these curvature tensor have been studied by many authors such as (De and Ghosh,1994,1995and 1996), (De and Yieldiz, 2010), (Pokhariyal, 1982), (Prasad, 1997, 2002 and 2003), (Prakash, 2010), (Pokharyal etal,2017) and many others.

Yano and Sawaki in 1972 introduced quasi conformal curvature tensor in Riemannian manifold as follows.

$$C(X,Y)Z= aR(X,Y)Z+b[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY]-\frac{r}{n}\left(\frac{a}{n-1}+2b\right).[g(Y,Z)X-g(X,Z)Y],$$

where a and b are constants such that $a\neq 0, b\neq 0$

Some properties quasi conformal curvature tensor have been studied by (Amun and Maralabhavi, 1977), (Chaki and Ghosh, 1997), (De and Shaikh, 1997), (Kumar Prasad and Verma, 2009), (Prakash and Singh, 2009), (Mallick, zhau al De, 2016),(Shukla and Singh, 2011), (Venkatesh and Kumar, 2016) and many other workers.

Recently one of the author (Prasad, 2002) introduced pseudo projective curvature tensor in a Riemannian manifold (n>2) by

$$\tilde{P}(X,Y)Z = a\,R(X,Y)Z+b[S(Y,Z)X-S(X,Z)Y]-\frac{r}{n}\left(\frac{a}{n-1}+b\right).[g(Y,Z)X-g(X,Z)Y]$$

where a and b are constants such that $a,b\neq 0$.

In recent papers (Bagewadi, Prakash and Venkatesh, 2007), (Narain, Prakash and Prasad, 2009), (Jaisaval and Ojha, 2010), (Guler, Demirabag, 2016), (Kumar, 2019) etc explored various geometrical properties by using this curvature tensor on LP – Sasakian manifold, K-contact manifold and Trans-Sasakian manifold, weakly symmetric manifold and Riemannian manifold.

Further in 2004, 2007 and 2008 (Prasad & Maurya) defined pseudo $W_2$ – curvature tensor, quasi – concircular curvature tensor and pseudo $W_4$ – curvature tensor on Riemannian manifold as follows respectively

$$\tilde{W}_2(X,Y)Z=a\,R(X,Y,Z)+b\,[g(Y,Z)QX-g(X,Z)QY]-\frac{r}{n}\left(\frac{a}{n-1}+b\right).[g(Y,Z)X-g(X,Z)Y],$$

$$\tilde{V}(X,Y)Z=aR(X,Y)Z+\frac{r}{n}\left(\frac{a}{n-1}+2b\right).[g(Y,Z)X-g(X,Z)Y],$$

and

$$\tilde{W}_4(X,Y)Z=a\,R(X,Y)Z+b\,[g(X,Y)QZ-g(X,Z)QY]-\frac{r}{n}\left(\frac{a}{n-1}+b\right).[g(X,Y)Z-g(X,Z)Y],$$

(Narain, Prakash and Prasad, 2009), (Kumar, Prasad and Verma 2009), (Prakash, 2011), (Kumar , 2012) and (Prasad etal,2010) extended this notation to LP- Sasakian manifold, P-Sasakian manifold and LP- Sasakian manifold with coefficient $\alpha$.

In continuation of above study, the object of this paper is to define another a new curvature tensor on a Riemannian manifold which we call it Pseudo $\tilde{M}$-projective curvature tensor. The paper is organized as follows. In section 2, the definition of Pseudo $\tilde{M}$-projective curvature tensor is given. We give properties and some identities of this curvature tensor.

In section 3, it is proved that pseudo $\tilde{M}$-projectivelly flat manifold is of zero scalar curvature provided $a(n+1)+bn(3n+2)+c(n-1)\neq 0$

In the last section, a necessary and sufficient condition for $(M^n,g)$ to be Pseudo $\tilde{M}$-projectivelly conservative is obtained.

## 2. Pseudo $\tilde{M}$-projective curvature tensor:

Let $(M^n, g)$ be an n-dimensional Riemannian manifold and $\chi(M^n)$ the Lie algebra of vector fields in $M^n$. Throughout the paper we assume that $X,Y,Z,W \in \chi(M^n)$, unless specifically stated otherwise.

In an n-dimensional Riemannian manifold $(M^n,g)$, pseudo $\tilde{M}$-projective curvature tensor of type (1,3) is defined by

$$\tilde{M}(X,Y)Z = a\,R(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y] + c[g(Y,Z)QX - g(X,Z)QY] + \frac{r}{n}\left(\frac{a}{n-1} + 3b - c\right).[g(Y,Z)X - g(X,Z)Y], \quad \text{---(2.1)}$$

where a,b and c are constants such that a, b, c≠0 and R,S,Q and r are the curvature tensor , the Ricci tensor, the Ricci operator of the type(1,1) and the scalar curvature respectively.

If a=1, $b=c=-\frac{1}{2(n-1)}$, then (2.1) takes the form

$$\tilde{M}(X,Y)Z = R(X,Y,Z) - \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] = M(X,Y)Z,$$

where M(X,Y)Z is the M-Projective curvature tensor (Pokhariyal, 1982). Thus the M-projective curvature tensor is a particular case of the tensor $\tilde{M}(X,Y)Z$. For this reason $\tilde{M}$ is called the $\tilde{M}$-Projective curvature tensor.

Note that Pseudo $\tilde{M}$– projective curvature tensor satisfies the following algebraic properties:

(i) $'\tilde{M}(X,Y,Z,W) + '\tilde{M}(Y,X,Z,W) = 0$

(ii) $'\tilde{M}(X,Y,Z,W) + '\tilde{M}(X,Y,W,Z) = (b-c)[S(Y,Z)g(X,W) - S(X,Z)g(Y,W) + S(Y,W)g(X,Z) - S(X,W)g(Y,Z)]$

(iii) $'\tilde{M}(X,Y,Z,W) + '\tilde{M}(Z,W,X,Y) = (b+c)\,[S(Y,Z)g(X,W) + g(Y,Z)S(X,W)] - 2[b\,S(X,Z)g(Y,W) + c\,S(W,Y)g(Z,X)]$.

(iv) $'\tilde{M}(X,Y,Z,W) + '\tilde{M}(Y,Z,X,W) + '\tilde{M}(Z,X,Y,W) = 0$, where $'\tilde{M}(X,Y,Z,W) = g(M(X,Y)Z,W)$ and $'R(X,Y,Z,W) = g(R(X,Y)Z,W)$.

Let $\{e_i\}$ be an orthonormal basis of the tangent space at each point of the manifold where $1 \le i \le n$. Now from (2.1), we have

$$\sum_{i=1}^{n} {'\tilde{M}}(X,Y,e_i,e_i) = \sum_{i=1}^{n} {'\tilde{M}}(e_i,e_i,Z,W) = 0$$

Particular case:

If the manifold becomes an Eienstein manifold then from (2.1), we get

$$M(X,Y)Z = a\,R(X,Y)Z + \frac{r}{n}\left[\frac{a}{n-1} + 4b\right][g(Y,Z)X - g(X,Z)Y] = \overset{\star}{\tilde{V}}(X,Y)Z \quad \text{-----(2.2)}$$

(a new curvature tensor)

If a=1, $b=\frac{-1}{2(n-1)}$ then (2.2) taken the form $\overset{\star}{\tilde{V}}(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y] = V(X,Y)Z$, where V(X,Y)Z is the concircular curvature tensor. Thus we observe that the

concircular curvature V is a special case of the tensor $\overset{\star}{\tilde{V}}(X,Y)Z$, which seems to be the generalization of concircular curvature tensor.

Hence we call it as generalized concircular curvature tensor $\overset{\star}{\tilde{V}}$.

## 3. Pseudo $\tilde{M}$-projectivelly flat manifold.

For pseudo $\tilde{M}$-projectivelly flat manifold, we have

$$\tilde{M}(X,Y)Z = 0 \quad \text{-----(3.1)}$$

Then from (2.1) and (3.1) we obtain

$$aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y] + c[g(Y,Z)QX - g(X,Z)QY] + \frac{r}{n}\left[\frac{a}{n-1} + 3b - c\right].[g(Y,Z)X - g(X,Z)Y] = 0 \quad \text{-------(3.2)}$$

From (3.2) , we get

$a\,'R(X,Y,Z,W) + b[S(Y,Z)g(X,W) - S(X,Z)g(Y,W)] + c[g(Y,Z)\,S(X,W) -$

$g(X,Z)S(Y,W)]+\frac{r}{n}\left[\frac{a}{n-1}+3b-c\right][g(Y,Z)g(X,W)\text{-}g(X,Z)g(Y,W)]=0$ ------(3.3)

Putting $X=W=e_i$ in (3.3), $\{e_i\}$ is an orthonormal basis of the tangent space at a point of the manifold and taking summation over $e_i$, $1\leq i\leq n$, we get

$$S(Y,Z)[c-a+b(n-1)]=\frac{r}{n}[a+3b(n-1)+c]g(Y,Z),$$

which gives on further contraction

$r[a(n+1)+b(3n+2)n+c(n-1)]=0$ -------(3.4)

If $a(n+1)+b\,n(3n+2)+c(n-1)\neq 0$, then from (3.4) we get r=0.

Hence, we have the following theorem:

**Theorem (3.1):** Pseudo $\tilde{M}$ -projectivelly flat manifold is of zero scalar curvature, provided $a(n+1)+b\,n(3n+2)+c(n-1)\neq 0$.

4. **Pseudo $\tilde{M}$-projectivelly conservative $(M^n, g)$.**

A manifold $(M^n, g)$ shall be called pseudo $\tilde{M}$ -projectivelly conservative if

div $\tilde{M}=0$ (Hicks, 1969) ------(4.1)

Now differentiating (2.1) covariantly , we get

$(D_UR)(X,Y)Z+b[(D_U\tilde{M})(X,Y)Z]=a(D_UR)(X,Y)Z+b[(D_US)(Y,Z)X\text{-}(D_US)(X,Z)Y]+$
$c[g(Y,Z)(D_UQ)(X)\text{-}g(X,Z)(D_UQ)(Y)]\text{-}$
$\frac{D_U r}{n}\left[\frac{a}{n-1}+3b-c\right].[g(Y,Z)X-g(X,Z)Y]$, which gives on contraction

$(\text{div }\tilde{M})(X,Y)Z=a(\text{div }R)(X,Y)Z+b[(D_XS)(Y,Z)\text{-}(D_YS)(X,Z)]+$
$c[g(Y,Z)dr(X)\text{-}g(X,Z)dr(Y)]\text{-}\frac{1}{n}\left[\frac{a}{n-1}+3b-c\right][g(Y,Z)dr(X)-g(X,Z)dr(Y)]$ --(4.2)

From (Eisenhart , 1926), we have $(\text{div }R)(X,Y)Z=(D_XS)(Y,Z)\text{-}(D_YS)(X,Z)$ ---(4.3)

In view of (4.2) and (4.3), we get $(\text{div }\tilde{M})(X,Y)Z=(a+b)[(D_XS)(Y,Z)\text{-}$
$(D_YS)(X,Z)]+[\frac{c(n^2-1)-(a+3b(n-1))}{n(n-1)}][g(Y,Z)dr(X)-g(X,Z)dr(Y)]$ ----(4.4)

Suppose the Ricci tensor S(X,Y) is of Codazzi type i.e

$(D_XS)(Y,Z)=(D_YS)(X,Z)$ ---(4.5)

Then from (4.4) and (4.5), we get

$(\text{div }\tilde{M})(X,Y)Z=[\frac{c(n^2-1)-(a+3b(n-1))}{n(n-1)}][g(Y,Z)dr(X)-g(X,Z)dr(Y)]$ ----(4.6)

Hence if $(\text{div }\tilde{M})(X,Y)Z=0$, then from (4.6), $c(n^2-1)-(a+3b(n-1))[g(Y,Z)dr(X)\text{-}g(X,Z)dr(Y)]=0$

Since $c(n^2-1)-(a+3b(n-1))\neq 0$, here

$g(Y,Z)dr(X)\text{-}g(X,Z)dr(Y)=0$, which shows that r is constant. Again if r is constant than from(4.6), we get $(\text{div }\tilde{M})(X,Y)Z=0$.

Hence we can state the following theorem:

**Theorem (4.1).** If in a Riemannian manifold $(M^n, g)$, the Ricci tensor is Codazzi type then the manifold Pseudo $\tilde{M}$ - projectivelly conservative if and only if the scalar curvature is constant, provided $c(n^2-1)-(a+3b(n-1))\neq 0$,

According to our assumption $a+b\neq 0$, then from (4.4), we have

$\frac{(\text{div }\tilde{M})(X,Y)Z}{a+b}=[(D_XS)(Y,Z)-(D_YS)(X,Y)]+\left[\frac{c(n^2-1)-(a+3b(n-1))}{n(n-1)}\right][g(Y,Z)dr(X)-g(X,Z)dr(Y)]$ −(4.7)

Hence from (4.7), we can state theorem as follows

**Theorem (4.2).** If in a Riemannian manifold $(M^n, g)$, Pseudo $\tilde{M}$-projective curvature tensor is such that $a+b\neq 0$, the manifold is pseudo $\tilde{M}$-projectivelly conservative if and only if

$[(D_XS)(Y,Z)\text{-}(D_YS)(X,Z)]=\left[\frac{(a+3b(n-1))-((n^2-1))}{n(n-1)}\right].[g(Y,Z)dr(X)\text{-}g(X,Z)dr(Y)].$

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