SOME MULTIPLICATIVE TOPOLOGICAL INDICES OF SILICATE NETWORKS

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Abstract : In this paper, we compute some multiplicative topological indices of networks such as rhombus silicate network, chain silicate network, silicate network, honeycomb network and dominating silicate network.

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I. INTRODUCTION

The graphs considered here are finite, undirected, without loops and multiple edges. Let $G = (V, E)$ be a connected graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting vertices u and v will be denoted by uv. The degree $d_G(e)$ denote the degree of an edge $e = uv$ in G, which is defined as $d_G(e) = d_G(u) + d_G(v) - 2$. For further notation and terminology, we refer to [4]. Chemical graph theory is a branch of mathematical chemistry and has an important effect on the development of the chemical sciences, see[2]. A molecular graph of a chemical graph is a simple graph related to the structure of a chemical compound where its vertices correspond to an atom of the molecule and its edges correspond to the bonds between the atoms. A topological index is a numerical parameter mathematically derived from the graph structures. Topological indices are useful for establishing correlation between the structure of a molecular compound and its physico--chemical properties. Numerous topological indices have been considered in Theoretical Chemistry and have found applications, especially in $QSPR/QSAR$ research. Vuki$\check{c}$evic' et. al. observed that many topological indices are simply defined as the sum of individual bond contributions, see[21]. Here they have introduced a class of discrete adriatic indices to study other possible significant topological indices of this form. There exists bond additive descriptors, i.e, descriptors that can be presented as the sum of edge contributions, some important descriptors are defined in this way: Randic index[17], Zagreb index [20], Wiener index [3], ... These concepts give a new class of descriptors that will be called as *Adriatic descriptors*. More precisely, three classes of adriatic descriptors are defined:

(i)extended adriatic descriptors
(ii)variable adriatic descriptors
(iii)discrete adriatic descriptors.

The most restrictive class of these descriptors is the discrete adriatic descriptors. Inverse sum indeg index is one among all these adriatic indices. The inverse sum indeg index [21] of a graph G is defined as

$$ISI(G) = \sum_{uv \in E(G)} \left[\frac{d_G(u) \cdot d_G(v)}{d_G(u) + d_G(v)}\right]$$

This index was also studied in[7,8,18,21]. Motivated by the definition of inverse sum indeg index and its applications, Kulli introduced the first multiplicative inverse sum indeg index [6] of a graph G. The multiplicative inverse sum indeg index of a graph G is defined as

$$ISIII(G) = \prod_{uv \in E(G)} \left[\frac{d_G(u) \cdot d_G(v)}{d_G(u) + d_G(v)}\right]. \qquad (1.1)$$

The first and second multiplicative Zagreb indices of a graph G are defined as

$$II_1(G) = \prod_{u \in V(G)} d_G(u)^2, II_2(G) = \prod_{u,v \in E(G)} d_G(u) \cdot d_G(v).$$

These indices were introduced by Todeshine[20] et al. and were studied. Eliasi[1] et al. proposed a new multiplicative version of the first Zagreb index as

$$II_1^*(G) = \prod_{u,v \in E(G)} [d_G(u) + d_G(v)].$$

The first and second multiplicative hyper--Zagreb indices of a graph G are defined as

$$HII_1(G) = \prod_{u,v \in E(G)} [d_G(u) + d_G(v)]^2,$$

$$HII_2(G) = \prod_{u,v \in E(G)} [d_G(u) \cdot d_G(v)]^2.$$

These indices were introduced by Kulli in [11]. The best known and widely used topological index is the product connectivity index or Randic' index, introduced by Randic' in [17] and is defined as

$$\mathcal{X}(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)\cdot d_G(v)}}.$$

Motivated by the definition of the product connectivity index and its wide applications, Kulli in [9] introduced the multiplicative sum connectivity index, multiplicative product connectivity index, multiplicative atom bond connectivity index and multiplicative geomertic arithmetic index of a graph as follows: The multiplicative sum connectivity index of a graph G is defined as

$$XII(G) = \prod_{u,v\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}}. \quad (1.2)$$

The multiplicative product connectivity index of a graph G is defined as

$$\mathcal{X}^{II(G)} = \prod_{u,v\in E(G)} \frac{1}{\sqrt{d_G(u)\cdot d_G(v)}}. \quad (1.3)$$

The multiplicative atom bond connectivity index of a graph G is defined as

$$ABCII(G) = \prod_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)\cdot d_G(v)}}. \quad (1.4)$$

The multiplicative geomertic--arithmetic index of a graph G is defined as

$$GAII(G) = \prod_{u,v\in E(G)} \left[\frac{2\sqrt{d_G(u)\cdot d_G(v)}}{d_G(u)+d_G(v)}\right]. \quad (1.5)$$

Motivated by the definition of multiplicative geomertic--arithmetic index in [9] Kulli introduced new multiplicative arithmetic--geometric index in [10].The multiplicative arithmetic--geometric index of a graph G is defined as

$$AGII(G) = \prod_{u,v\in E(G)} \left[\frac{d_G(u)+d_G(v)}{2\sqrt{d_G(u)\cdot d_G(v)}}\right]. \quad (1.6)$$

Many other multiplicative topological indices are studied in[6,9,10,11,12]. A fixed correlation architecture is characterized by a graph, with vertices corresponding to processing nodes and edges representing communication links. A network is a fixed interconnection architecture. Interrelationship between networks are ignominiously hard to compare in abstract terms. Researchers in parallel processing are thus persuaded to offer new or enhance interconnection networks, claiming the advantage and for better performance [5]. Some interconnection network topologies are designed and some taken from nature. For example hyper cubes, complete binary tree, butterfly are some of the designed architecture[5]. See Figure(1).

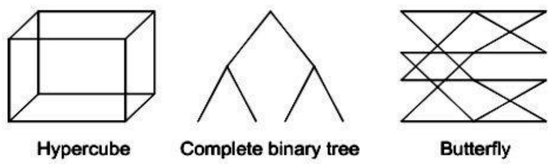

Figure1: Designed Architecture

Figure2: Natural Architecture

Honeycomb, hexagonal network and 4×4 grid networks resemble to atomic or molecular lattice structures and we call them natural architectures. See Figure(2). The silicates are the largest, the most interesting and the most complicated class of minerals so far. Silicates are obtained by fusing metal oxides or metal carbonates with sand. Necessarily all the silicates contain SiO_4 tetrahedra. In chemistry, the corner vertices of SiO_4 tetrahedra mean oxygen ions and the central vertex mean the silicon ions. In graph theory, we refer to corner vertices as oxygen nodes and the central vertex as silicon node. See Figure (3).

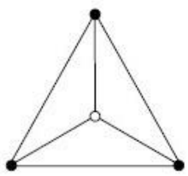

Figure3: SiO_4 tetrahedra where the corner vertices represent oxygen ions and the central vertex the silicon ion.

Minerals are obtained by successively fusing oxygen nodes of two tetrahedra of different silicates, different types of silicate structure arise from the ways in which these tetrahedra are arranged. A few networks such as hexagonal, honeycomb, rhombus, oxide, dominating and many more networks resemble to atomic or molecular lattice structure. These networks are widely used in computer graphics, cellular phone base station, image processing, and in chemistry as the representation of benzenoid hydrocarbons and carbon hexagons of Carbon Nanotubes. These networks have very interesting topological properties which have been studied in different aspects in [5,7,13,14,15,16,22]. In this paper, we compute some multiplicative topological indices such as multiplicative inverse sum indeg index $[ISIII(G)]$, multiplicative sum connectivity index $[XII(G)]$, multiplicative product connectivity index $[\mathcal{X}^{II(G)}]$, multiplicative atom bond connectivity index $[ABCII(G)]$, multiplicative geomertic-- arithmetic index $[GAII(G)]$ and multiplicative arithmetic--geomertic index $[AGII(G)]$ of networks such as rhombus silicate network[14], chain silicate network[15], silicate network[15], honeycomb network[19] and dominating silicate network[22].

II. RHOMBUS SILICATE NETWORKS $[RHSL_n]$

Silicates are obtained by fusing metal oxides or metal carbonates with sand. In this section, we consider a family of rhombus silicate networks. This network is symbolized by $RHSL_n$. A 3--dimensional rhombus silicate network is depicted in Figure (4).

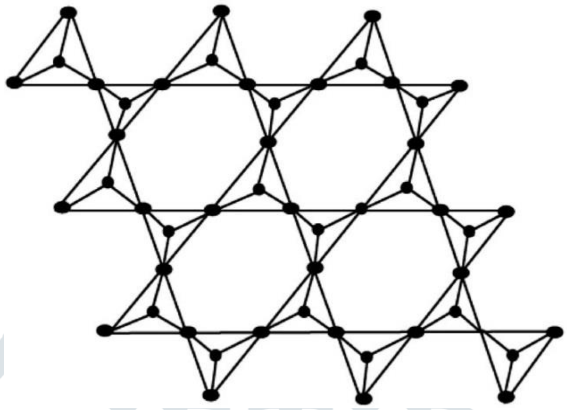

Figure 4: A 3--dimensional rhombus silicate network.

The order and size of rhombus silicate network $RHSL_n$ is $5n^2 + 2n$ and $12n^2$. In $RHSL_n$, by algebraic method, there are three types of edges based on the degree of end vertices. The partition of the edge set of $RHSL_n$ is as follows:

$$E_{33} = \{uv \in E(RHSL_n): d_{RHSL_n}(u) = 3 = d_{RHSL_n}(v)\}, |E_{33}| = 4n + 2$$

$$E_{36} = \{uv \in E(RHSL_n): d_{RHSL_n}(u) = 3, d_{RHSL_n}(v) = 6\}, |E_{36}| = 6n^2 + 4n - 4$$

$$E_{66} = \{uv \in E(RHSL_n): d_{RHSL_n}(u) = 6 = d_{RHSL_n}(v)\}, |E_{66}| = 6n^2 - 8n + 2.$$

Now in the following theorem, we compute multiplicative inverse sum indeg index $[ISIII(G)]$, multiplicative sum connectivity index $[XII(G)]$, multiplicative product connectivity index $[\mathcal{X}^{II(G)}]$, multiplicative atom bond connectivity index $[ABCII(G)]$, multiplicative geomertic-- arithmetic index $[GAII(G)]$ and multiplicative arithmetic--geomertic index $[AGII(G)]$ of $RHSL_n$.

Theorem 2.1. *Considering the rhombus silicate network $RHSL_n$. Then*

$$ISIII(RHSL_n) = 2^{6n^2-2} \times 3^{6n^2-4n+2}$$

$$XII(RHSL_n) = (\sqrt{6})^{-4n-2} \times 3^{4-6n^2-4n} \times (\sqrt{12})^{8n-6n^2-2}$$

$$\mathcal{X}^{II(RHSL_n)} = 3^{-6n^2-8n+2} \times \left(\sqrt{2}\right)^{-6n^2-4n+4} \times 6^{8n-6n^2-2}$$

$$ABCII(RHSL_n) = 2^{12n-6n^2} \times 3^{-12n^2} \times \left(\sqrt{\frac{7}{2}}\right)^{6n^2+4n-4} \times \left(\sqrt{10}\right)^{6n^2-8n+2}$$

$$GAII(RHSL_n) = \left(\frac{2\sqrt{2}}{3}\right)^{6n^2+4n-4}$$

$$AGII(RHSL_n) = \left(\frac{3}{2\sqrt{2}}\right)^{6n^2+4n-4}.$$

Proof. Here let G_1 be the rhombus silicate network. By using definitions, we have

$$\begin{aligned} ISIII(G_1) &= \prod_{uv\in E(G_1)} \left[\frac{d_{G_1}(u)\cdot d_{G_1}(v)}{d_{G_1}(u)+d_{G_1}(v)}\right] \\ &= \left(\frac{3\cdot3}{3+3}\right)^{|E_{33}|} \times \left(\frac{3\cdot6}{3+6}\right)^{|E_{36}|} \times \left(\frac{6\cdot6}{6+6}\right)^{|E_{66}|} \\ &= 2^{6n^2-2} \times 3^{6n^2-4n+2}. \end{aligned}$$

$$XII(G_1) = \prod_{u,v\in E(G_1)} \frac{1}{\sqrt{d_{G_1}(u)+d_{G_1}(v)}}$$
$$= \left(\frac{1}{\sqrt{3+3}}\right)^{|E_{33}|} \times \left(\frac{1}{\sqrt{3+6}}\right)^{|E_{36}|} \times \left(\frac{1}{\sqrt{6+6}}\right)^{|E_{66}|}$$
$$= (\sqrt{6})^{-4n-2} \times 3^{-6n^2-4n+4} \times (\sqrt{12})^{8n-6n^2-2}.$$

$$\chi^{II(G_1)} = \prod_{u,v\in E(G_1)} \frac{1}{\sqrt{d_{G_1}(u)\cdot d_{G_1}(v)}}$$
$$= \left(\frac{1}{\sqrt{3\cdot 3}}\right)^{|E_{33}|} \times \left(\frac{1}{\sqrt{3\cdot 6}}\right)^{|E_{36}|} \times \left(\frac{1}{\sqrt{6\cdot 6}}\right)^{|E_{66}|}$$
$$= 3^{-6n^2-8n+2} \times \left(\sqrt{2}\right)^{-6n^2-4n+4} \times 6^{8n-6n^2-2}.$$

$$ABCII(G_1) = \prod_{u,v\in E(G_1)} \sqrt{\frac{d_{G_1}(u)+d_{G_1}(v)-2}{d_{G_1}(u)\cdot d_{G_1}(v)}}$$
$$= \left(\sqrt{\frac{3+3-2}{3\cdot 3}}\right)^{|E_{33}|} \times \left(\sqrt{\frac{3+6-2}{3\cdot 6}}\right)^{|E_{36}|} \times \left(\sqrt{\frac{6+6-2}{6\cdot 6}}\right)^{|E_{66}|}$$
$$= 2^{12n-6n^2} \times 3^{-12n^2} \times \left(\sqrt{\frac{7}{2}}\right)^{6n^2+4n-4} \times \left(\sqrt{10}\right)^{6n^2-8n+2}$$

$$GAII(G_1) = \prod_{u,v\in E(G_1)} \left[\frac{2\sqrt{d_{G_1}(u)\cdot d_{G_1}(v)}}{d_{G_1}(u)+d_{G_1}(v)}\right]$$
$$= \left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)^{|E_{33}|} \times \left(\frac{2\sqrt{3\cdot 6}}{3+6}\right)^{|E_{36}|} \times \left(\frac{2\sqrt{6\cdot 6}}{6+6}\right)^{|E_{66}|}$$
$$= \left(\frac{2\sqrt{2}}{3}\right)^{6n^2+4n-4}.$$

$$AGII(G_1) = \prod_{u,v\in E(G_1)} \left[\frac{d_{G_1}(u)+d_{G_1}(v)}{2\sqrt{d_{G_1}(u)\cdot d_{G_1}(v)}}\right]$$
$$= \left(\frac{3+3}{2\sqrt{3\cdot 3}}\right)^{|E_{33}|} \times \left(\frac{3+6}{2\sqrt{3\cdot 6}}\right)^{|E_{36}|} \times \left(\frac{6+6}{2\sqrt{6\cdot 6}}\right)^{|E_{66}|}$$
$$= \left(\frac{3}{2\sqrt{2}}\right)^{6n^2+4n-4}.$$

III. Chain Silicate Networks [CS_n]

We now consider, a family of chain silicate networks. The chain silicate network is denoted by CS_n and is obtained by arranging n--tetrahedral linearly. A chain silicate network CS_n is depicted in Figure (5).

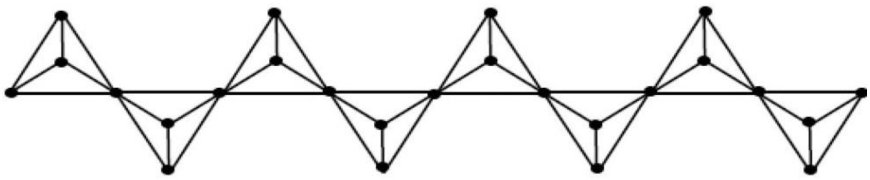

Figure 5: Chain silicate network.

Chain silicate network CS_n has $|V(CS_n)| = 3n + 1$ and $|E(CS_n)| = 6n$. By algebraic method, In CS_n there are three types of edges based on the degree of end vertices. The partition of the edge set of CS_n is as follows:

$$E_{33} = \{uv \in E(CS_n): d_{CS_n}(u) = 3 = d_{CS_n}(v)\}, |E_{33}| = n + 4$$

$$E_{36} = \{uv \in E(CS_n): d_{CS_n}(u) = 3, d_{CS_n}(v) = 6\}, |E_{36}| = 4n - 2$$

$$E_{66} = \{uv \in E(CS_n): d_{CS_n}(u) = 6 = d_{CS_n}(v)\}, |E_{66}| = n - 2.$$

We now compute multiplicative inverse sum indeg index [$ISIII(G)$], multiplicative sum connectivity index [$XII(G)$], multiplicative product connectivity index [$\chi^{II(G)}$], multiplicative atom bond connectivity index [$ABCII(G)$], multiplicative geomertic-- arithmetic index $[GAII(G)]$ and multiplicative arithmetic--geomertic index $[AGII(G)]$ of chain silicate network CS_n.

Theorem 3.1. *For the chain silicate network CS_n, we have*

$$ISIII(CS_n) = 2^{3n-6} \times 3^{2n+2}$$

$$XII(CS_n) = (\sqrt{6})^{-n-4} \times 3^{2-4n} \times (\sqrt{12})^{2-n}$$

$$\chi^{II(CS_n)} = 3^{-5n-2} \times (\sqrt{2})^{2-4n} \times 6^{2-n}$$

$$ABCII(CS_n) = 2^6 \times 3^{-6n} \times \left(\sqrt{\frac{7}{2}}\right)^{4n-2} \times (\sqrt{10})^{n-2}$$

$$GAII(CS_n) = \left(\frac{\sqrt{8}}{3}\right)^{4n-2}$$

$$AGIICS_n) = \left(\frac{3}{\sqrt{8}}\right)^{4n-2}.$$

Proof. Here let G_2 be the chain silicate network CS_n. From Equations (1.1) to (1.6) , we compute

$$\begin{aligned} ISIII(G_2) &= \prod_{uv\in E(G_2)} \left[\frac{d_{G_2}(u)\cdot d_{G_2}(v)}{d_{G_2}(u)+d_{G_2}(v)}\right] \\ &= \left(\frac{3\cdot 3}{3+3}\right)^{|E_{33}|} \times \left(\frac{3\cdot 6}{3+6}\right)^{|E_{36}|} \times \left(\frac{6\cdot 6}{6+6}\right)^{|E_{66}|} \\ &= 2^{3n-6} \times 3^{2n+2}. \end{aligned}$$

$$\begin{aligned} XII(G_2) &= \prod_{u,v\in E(G_2)} \frac{1}{\sqrt{d_{G_2}(u)+d_{G_2}(v)}} \\ &= \left(\frac{1}{\sqrt{3+3}}\right)^{|E_{33}|} \times \left(\frac{1}{\sqrt{3+6}}\right)^{|E_{36}|} \times \left(\frac{1}{\sqrt{6+6}}\right)^{|E_{66}|} \\ &= (\sqrt{6})^{-n-4} \times 3^{2-4n} \times (\sqrt{12})^{2-n}. \end{aligned}$$

$$\begin{aligned} \chi^{II(G_2)} &= \prod_{u,v\in E(G_2)} \frac{1}{\sqrt{d_{G_2}(u)\cdot d_{G_2}(v)}} \\ &= \left(\frac{1}{\sqrt{3\cdot 3}}\right)^{|E_{33}|} \times \left(\frac{1}{\sqrt{3\cdot 6}}\right)^{|E_{36}|} \times \left(\frac{1}{\sqrt{6\cdot 6}}\right)^{|E_{66}|} \\ &= 3^{-5n-2} \times (\sqrt{2})^{2-4n} \times 6^{n-2}. \end{aligned}$$

$$\begin{aligned} ABCII(G_2) &= \prod_{u,v\in E(G_2)} \sqrt{\frac{d_{G_2}(u)+d_{G_2}(v)-2}{d_{G_2}(u)\cdot d_{G_2}(v)}} \\ &= \left(\sqrt{\frac{3+3-2}{3\cdot 3}}\right)^{|E_{33}|} \times \left(\sqrt{\frac{3+6-2}{3\cdot 6}}\right)^{|E_{36}|} \times \left(\sqrt{\frac{6+6-2}{6\cdot 6}}\right)^{|E_{66}|} \\ &= 2^6 \times 3^{-6n} \times \left(\sqrt{\frac{7}{2}}\right)^{4n-2} \times (\sqrt{10})^{n-2} \end{aligned}$$

$$\begin{aligned} GAII(G_2) &= \prod_{u,v\in E(G_2)} \left[\frac{2\sqrt{d_{G_2}(u)\cdot d_{G_2}(v)}}{d_{G_2}(u)+d_{G_2}(v)}\right] \\ &= \left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)^{|E_{33}|} \times \left(\frac{2\sqrt{3\cdot 6}}{3+6}\right)^{|E_{36}|} \times \left(\frac{2\sqrt{6\cdot 6}}{6+6}\right)^{|E_{66}|} \\ &= \left(\frac{\sqrt{8}}{3}\right)^{4n-2}. \end{aligned}$$

$$\begin{aligned} AGII(G_2) &= \prod_{u,v\in E(G_2)} \left[\frac{d_{G_2}(u)+d_{G_2}(v)}{2\sqrt{d_{G_2}(u)\cdot d_{G_2}(v)}}\right] \\ &= \left(\frac{3+3}{2\sqrt{3\cdot 3}}\right)^{|E_{33}|} \times \left(\frac{3+6}{2\sqrt{3\cdot 6}}\right)^{|E_{36}|} \times \left(\frac{6+6}{2\sqrt{6\cdot 6}}\right)^{|E_{66}|} \\ &= \left(\frac{3}{\sqrt{8}}\right)^{4n-2}. \end{aligned}$$

IV. SILICATE NETWORKS $[SL_n]$

Silicate networks are obtained by fusing metal oxide or metal carbonates with sand. A silicate network is denoted by SL_n, where n is the number of hexagons between the centre and boundary of SL_n. A silicate network SL_n is depicted in Figure (6).

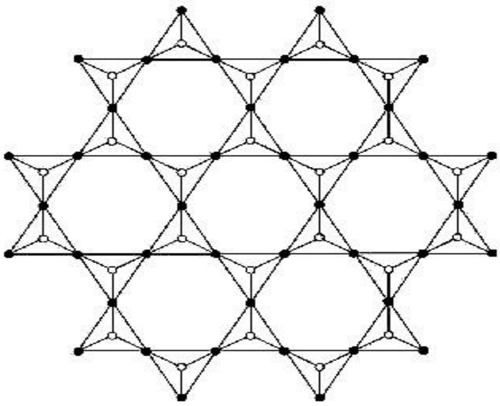

Figure 6: Silicate network

Silicate network SL_n has $|V(SL_n)| = 15n^2 + 3n$ and $|E(SL_n)| = 36n^2$. By algebraic method, there are three types of edges based on the degree of end vertices. The partition of the edge set of SL_n is as follows:

$$E_{33} = \{uv \in E(SL_n): d_{SL_n}(u) = 3 = d_{SL_n}(v)\}, |E_{33}| = 6n$$

$$E_{36} = \{uv \in E(SL_n): d_{SL_n}(u) = 3, d_{SL_n}(v) = 6\}, |E_{36}| = 18n^2 + 6n$$

$$E_{66} = \{uv \in E(SL_n): d_{SL_n}(u) = 6 = d_{SL_n}(v)\}, |E_{66}| = 18n^2 - 12n.$$

We now compute multiplicative inverse sum indeg index $[ISIII(G)]$, multiplicative sum connectivity index $[XII(G)]$, multiplicative product connectivity index $[\mathcal{X}^{II(G)}]$, multiplicative atom bond connectivity index $[ABCII(G)]$, multiplicative geomertic-- arithmetic index $[GAII(G)]$ and multiplicative arithmetic--geomertic index $[AGII(G)]$ of chain silicate network SL_n.

Theorem 4.1. *For the silicate network SL_n, we have*

$$ISIII(SL_n) = 3^{18n^2-6n} \times 2^{18n^2}$$

$$XII(SL_n) = (\sqrt{6})^{-6n} \times 3^{-18n^2-6n} \times (\sqrt{12})^{12n-18n^2}$$

$$\mathcal{X}^{II(SL_n)} = 3^{-36n^2} \times (\sqrt{2})^{-18n^2-6n} \times 2^{12n-18n^2}$$

$$ABCII(SL_n) = 2^{-18n^2+18n} \times 3^{-36n^2} \times \left(\sqrt{\tfrac{7}{2}}\right)^{18n^2+6n} \times (\sqrt{10})^{18n^2-12n}$$

$$GAII(SL_n) = (\sqrt{8})^{18n^2+6n} \times 3^{-18n^2-6n}$$

$$AGII(SL_n) = (\sqrt{8})^{-18n^2-6n} \times 3^{18n^2+6n}.$$

Proof. Here let G_3 be the silicate network SL_n. By making use of definitions and edge partitiion of silicate network SL_n, we compute

$$\begin{aligned} ISIII(G_3) &= \prod_{uv\in E(G_3)} \left[\frac{d_{G_3}(u)\cdot d_{G_3}(v)}{d_{G_3}(u)+d_{G_3}(v)}\right] \\ &= \left(\frac{3\cdot 3}{3+3}\right)^{|E_{33}|} \times \left(\frac{3\cdot 6}{3+6}\right)^{|E_{36}|} \times \left(\frac{6\cdot 6}{6+6}\right)^{|E_{66}|} \\ &= 3^{18n^2-6n} \times 2^{18n^2}. \end{aligned}$$

$$XII(G_3) = \prod_{u,v\in E(G_3)} \frac{1}{\sqrt{d_{G_3}(u)+d_{G_3}(v)}}$$
$$= \left(\frac{1}{\sqrt{3+3}}\right)^{|E_{33}|} \times \left(\frac{1}{\sqrt{3+6}}\right)^{|E_{36}|} \times \left(\frac{1}{\sqrt{6+6}}\right)^{|E_{66}|}$$
$$= (\sqrt{6})^{-6n} \times 3^{-18n^2-6n} \times (\sqrt{12})^{12n-18n^2}.$$

$$\chi^{II(G_3)} = \prod_{u,v\in E(G_3)} \frac{1}{\sqrt{d_{G_3}(u)\cdot d_{G_3}(v)}}$$
$$= \left(\frac{1}{\sqrt{3\cdot 3}}\right)^{|E_{33}|} \times \left(\frac{1}{\sqrt{3\cdot 6}}\right)^{|E_{36}|} \times \left(\frac{1}{\sqrt{6\cdot 6}}\right)^{|E_{66}|}$$
$$= 3^{-36n^2} \times (\sqrt{2})^{-18n^2-6n} \times 2^{12n-18n^2}.$$

$$ABCII(G_3) = \prod_{u,v\in E(G_3)} \sqrt{\frac{d_{G_3}(u)+d_{G_3}(v)-2}{d_{G_3}(u)\cdot d_{G_3}(v)}}$$
$$= \left(\sqrt{\frac{3+3-2}{3\cdot 3}}\right)^{|E_{33}|} \times \left(\sqrt{\frac{3+6-2}{3\cdot 6}}\right)^{|E_{36}|} \times \left(\sqrt{\frac{6+6-2}{6\cdot 6}}\right)^{|E_{66}|}$$
$$= 2^{-18n^2+18n} \times 3^{-36n^2} \times \left(\sqrt{\frac{7}{2}}\right)^{18n^2+6n} \times (\sqrt{10})^{18n^2-12n}.$$

$$GAII(G_3) = \prod_{u,v\in E(G_3)} \left[\frac{2\sqrt{d_{G_3}(u)\cdot d_{G_3}(v)}}{d_{G_3}(u)+d_{G_3}(v)}\right]$$
$$= \left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)^{|E_{33}|} \times \left(\frac{2\sqrt{3\cdot 6}}{3+6}\right)^{|E_{36}|} \times \left(\frac{2\sqrt{6\cdot 6}}{6+6}\right)^{|E_{66}|}$$
$$= (\sqrt{8})^{18n^2+6n} \times 3^{-18n^2-6n}.$$

$$AGII(G_3) = \prod_{u,v\in E(G_3)} \left[\frac{d_{G_3}(u)+d_{G_3}(v)}{2\sqrt{d_{G_3}(u)\cdot d_{G_3}(v)}}\right]$$
$$= \left(\frac{3+3}{2\sqrt{3\cdot 3}}\right)^{|E_{33}|} \times \left(\frac{3+6}{2\sqrt{3\cdot 6}}\right)^{|E_{36}|} \times \left(\frac{6+6}{2\sqrt{6\cdot 6}}\right)^{|E_{66}|}$$
$$= (\sqrt{8})^{-18n^2-6n} \times 3^{18n^2+6n}.$$

V. HONEYCOMB NETWORK [HC_n]

Recursively using hexagonal tiling in a particular pattern, honeycomb networks are formed. These networks are useful in chemistry and also in computer graphics. A honeycomb network of dimensional n is denoted by HC_n where n is the number of hexagons between central and boundary hexagon. A honeycomb network of dimension four is shown in Figure (7).

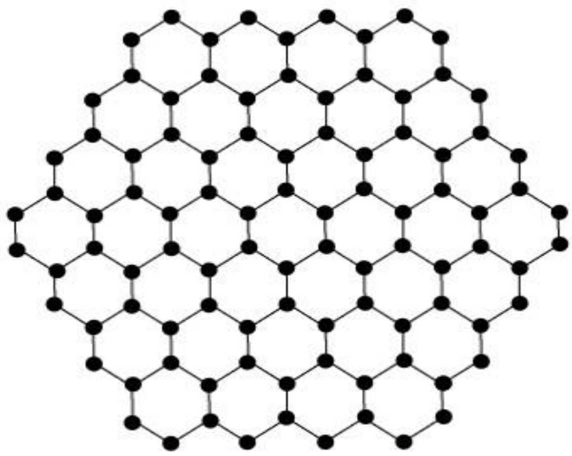

Figure7: Honeycomb network of dimension 4.

Honeycomb network has $|V(HC_n)| = 6n^2$ and $E(HC_n) = 9n^2 - 3n$. The partition of the edge set of HC_n is as follows:

$$E_{22} = \{uv \in E(HC_n): d_{HC_n}(u) = 2 = d_{HC_n}(v)\}, |E_{22}| = 6$$

$$E_{23} = \{uv \in E(HC_n): d_{HC_n}(u) = 2, d_{HC_n}(v) = 4\}, |E_{23}| = 12n - 12$$

$$E_{33} = \{uv \in E(HC_n): d_{HC_n}(u) = 3 = d_{HC_n}(v)\}, |E_{33}| = 9n^2 - 15n + 6.$$

We obtain now multiplicative inverse sum indeg index $[ISIII(G)]$, multiplicative sum connectivity index $[XII(G)]$, multiplicative product connectivity index $[\mathcal{X}^{II(G)}]$, multiplicative atom bond connectivity index $[ABCII(G)]$, multiplicative geomertic--arithmetic index $[GAII(G)]$ and multiplicative arithmetic--geomertic index $[AGII(G)]$ of honeycomb network HC_n.

Theorem 5.1. *Consider the honeycomb network HC_n. Then*

$$ISIII(HC_n) = 2^{-9n^2+27n-18} \times 3^{9n^2-3n-6} \times 5^{12-12n}$$

$$XII(HC_n) = 2^{-6} \times (\sqrt{5})^{12-12n} \times (\sqrt{6})^{15n-9n^2-6}$$

$$\mathcal{X}^{II(HC_n)} = 2^{-6} \times 3^{15n-9n^2-6} \times (\sqrt{6})^{12-12n}$$

$$ABCII(HC_n) = (\sqrt{2})^{6-12n} \times \left(\frac{2}{3}\right)^{9n^2-15n+6}$$

$$GAII(HC_n) = \left(\frac{2\sqrt{6}}{5}\right)^{12n-12}$$

$$AGII(HC_n) = \left(\frac{5}{2\sqrt{6}}\right)^{12n-12}.$$

Proof. Here let G_4 be the honeycomb network HC_n. From Equations (1.1) to (1.2) and the edge partition of HC_n, we compute

$$\begin{aligned} ISIII(G_4) &= \prod_{uv\in E(G_4)} \left[\frac{d_{G_4}(u)\cdot d_{G_4}(v)}{d_{G_4}(u)+d_{G_4}(v)}\right] \\ &= \left(\frac{2\cdot 2}{2+2}\right)^{|E_{22}|} \times \left(\frac{2\cdot 3}{2+3}\right)^{|E_{23}|} \times \left(\frac{3\cdot 3}{3+3}\right)^{|E_{33}|} \\ &= 2^{-9n^2+27n-18} \times 3^{9n^2-3n-6} \times 5^{12-12n}. \end{aligned}$$

$$\begin{aligned} XII(G_4) &= \prod_{u,v\in E(G_4)} \frac{1}{\sqrt{d_{G_4}(u)+d_{G_4}(v)}} \\ &= \left(\frac{1}{\sqrt{2+2}}\right)^{|E_{22}|} \times \left(\frac{1}{\sqrt{2+3}}\right)^{|E_{23}|} \times \left(\frac{1}{\sqrt{3+3}}\right)^{|E_{33}|} \\ &= 2^{-6} \times (\sqrt{5})^{12-12n} \times (\sqrt{6})^{15n-9n^2-6} \end{aligned}$$

$$\begin{aligned} \mathcal{X}^{II(G_4)} &= \prod_{u,v\in E(G_4)} \frac{1}{\sqrt{d_{G_4}(u)\cdot d_{G_4}(v)}} \\ &= \left(\frac{1}{\sqrt{2\cdot 2}}\right)^{|E_{22}|} \times \left(\frac{1}{\sqrt{2\cdot 3}}\right)^{|E_{23}|} \times \left(\frac{1}{\sqrt{3\cdot 3}}\right)^{|E_{33}|} \\ &= 2^{-6} \times 3^{15n-9n^2-6} \times (\sqrt{6})^{12-12n}. \end{aligned}$$

$$\begin{aligned} ABCII(G_4) &= \prod_{u,v\in E(G_4)} \sqrt{\frac{d_{G_4}(u)+d_{G_4}(v)-2}{d_{G_4}(u)\cdot d_{G_4}(v)}} \\ &= \left(\sqrt{\frac{2+2-2}{2\cdot 2}}\right)^{|E_{22}|} \times \left(\sqrt{\frac{2+3-2}{2\cdot 3}}\right)^{|E_{23}|} \times \left(\sqrt{\frac{3+3-2}{3\cdot 3}}\right)^{|E_{33}|} \\ &= (\sqrt{2})^{6-12n} \times \left(\frac{2}{3}\right)^{9n^2-15n+6} \end{aligned}$$

$$\begin{aligned} GAII(G_4) &= \prod_{u,v\in E(G_4)} \left[\frac{2\sqrt{d_{G_4}(u)\cdot d_{G_4}(v)}}{d_{G_4}(u)+d_{G_4}(v)}\right] \\ &= \left(\frac{2\sqrt{2\cdot 2}}{2+2}\right)^{|E_{22}|} \times \left(\frac{2\sqrt{2\cdot 3}}{2+3}\right)^{|E_{23}|} \times \left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)^{|E_{33}|} \\ &= \left(\frac{2\sqrt{6}}{5}\right)^{12n-12}. \end{aligned}$$

$$AGII(G_4) = \prod_{u,v\in E(G_4)} \left[\frac{d_{G_4}(u)+d_{G_4}(v)}{2\sqrt{d_{G_4}(u)\cdot d_{G_4}(v)}}\right]$$
$$= \left(\frac{2+2}{2\sqrt{2\cdot 2}}\right)^{|E_{22}|} \times \left(\frac{2+3}{2\sqrt{2\cdot 3}}\right)^{|E_{23}|} \times \left(\frac{3+3}{2\sqrt{3\cdot 3}}\right)^{|E_{33}|}$$
$$= \left(\frac{5}{2\sqrt{6}}\right)^{12n-12}.$$

VI. Dominating Silicate Network $[DSL(n)]$

Dominating silicate network of dimension n is denoted by $DSL(n)$. A dominating silicate network of dimension two is shown in Figure (8).

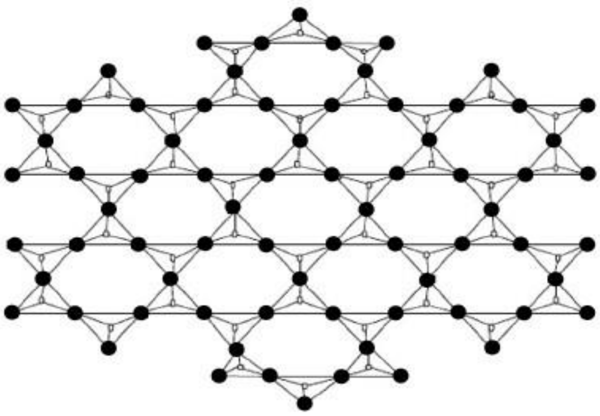

Figure 8: Dominating Silicate network of dimension 2 $[DSL(2)]$.

The number of vertices and the number of edges of the dominating silicate network is $45n^2 - 39n + 12$ and $108n^2 + 36 - 108n$, respectively. By algebraic method, in $DSL(n)$, there are four types of edge partitions based on degrees of end vertices of each edge. The partition of the edge set of $DSL(n)$ is as follows:

$$E_{23} = \{uv \in E(DSL(n)): d_{DSL(n)}(u) = 2, d_{DSL(n)}(v)\} = 3, |E_{23}| = 12n - 6$$

$$E_{26} = \{uv \in E(DSL(n)): d_{DSL(n)}(u) = 2, d_{DSL(n)}(v) = 6\}, |E_{26}| = 24n - 12$$

$$E_{36} = \{uv \in E(DSL(n)): d_{DSL(n)}(u) = 3, d_{DSL(n)}(v) = 6\}, |E_{36}| = 54n^2 - 66n + 24$$

$$E_{66} = \{uv \in E(DSL(n)): d_{DSL(n)}(u) = 6 = d_{DSL(n)}(v)\}, |E_{66}| = 54n^2 - 78n + 30.$$

We obtain now multiplicative inverse sum indeg index $[ISIII(G)]$, multiplicative sum connectivity index $[XII(G)]$, multiplicative product connectivity index $[\chi^{II(G)}]$, multiplicative atom bond connectivity index $[ABCII(G)]$, multiplicative geomertic-- arithmetic index $[GAII(G)]$ and multiplicative arithmetic--geomertic index $[AGII(G)]$ of dominating silicate network $DSL(n)$.

Theorem 6.1. *Let $DSL(n)$ the dominating silicate network be G_5 here, then*

$$ISIII(DSL(n)) = 2^{54n^2-78n+30} \times 3^{54n^2-42n+12} \times 5^{6-12n}$$

$$XII(DSL(n)) = 2^{-54n^2+54n-18} \times (\sqrt{2})^{12-24n} \times (\sqrt{3})^{78n-54n^2-30} \times 3^{66n-54n^2-24} \times (\sqrt{5})^{12n-6}$$

$$\chi^{DSL(n)} = 2^{-54n^2+54n-18} \times (\sqrt{2})^{-54n^2+66n-24} \times (\sqrt{3})^{12-24n} \times 3^{144n-108n^2-54} \times (\sqrt{3})^{12-24n} \times (\sqrt{6})^{-12n+6}$$

$$ABCII(DSL(n)) = 3^{-108n^2+144n-54} \times (\sqrt{2})^{-108n^2+108n-36} \times (\sqrt{5})^{-78n+54n^2+30} \times (\sqrt{7})^{54n^2-66n+24}$$

$$GAII(DSL(n)) = \left(\frac{\sqrt{2}}{3}\right)^{54n^2-66n+24} \times \left(\frac{\sqrt{6}}{5}\right)^{12n-6} \times (\sqrt{3})^{24n-12} \times 2^{54n^2-78n+30}$$

$$AGII(DSL(n)) = \left(\frac{\sqrt{2}}{3}\right)^{-54n^2+66n-24} \times \left(\frac{\sqrt{6}}{5}\right)^{-12n+6} \times (\sqrt{3})^{-24n+12} \times 2^{-54n^2+78n-30}.$$

Proof. By using definitions and the edge partition of $DSL(n)$, we compute

$$\begin{aligned} ISIII(G_5) &= \prod_{uv\in E(G_5)} \left[\frac{d_{G_5}(u)\cdot d_{G_5}(v)}{d_{G_5}(u)+d_{G_5}(v)}\right] \\ &= \left(\frac{2\cdot 3}{2+3}\right)^{|E_{23}|} \times \left(\frac{2\cdot 6}{2+6}\right)^{|E_{26}|} \left(\frac{3\cdot 6}{3+6}\right)^{|E_{36}|} \times \left(\frac{6\cdot 6}{6+6}\right)^{|E_{66}|} \\ &= 2^{54n^2-78n+30} \times 3^{54n^2-42n+12} \times 5^{6-12n}. \end{aligned}$$

$$\begin{aligned} XII(G_5) &= \prod_{u,v\in E(G_5)} \frac{1}{\sqrt{d_{G_5}(u)+d_{G_5}(v)}} \\ &= \left(\frac{1}{\sqrt{2+3}}\right)^{|E_{23}|} \times \left(\frac{1}{\sqrt{2+6}}\right)^{|E_{26}|} \times \left(\frac{1}{\sqrt{3+6}}\right)^{|E_{36}|} \times \left(\frac{1}{\sqrt{6+6}}\right)^{|E_{66}|} \\ &= 2^{-54n^2+54n-18} \times (\sqrt{2})^{12-24n} \times (\sqrt{3})^{78n-54n^2-30} \times 3^{66n-54n^2-24} \times (\sqrt{5})^{12n-6}. \end{aligned}$$

$$\begin{aligned} \chi^{II(G_5)} &= \prod_{u,v\in E(G_5)} \frac{1}{\sqrt{d_{G_5}(u)\cdot d_{G_5}(v)}} \\ &= \left(\frac{1}{\sqrt{2\cdot 3}}\right)^{|E_{23}|} \times \left(\frac{1}{\sqrt{2\cdot 6}}\right)^{|E_{26}|} \times \left(\frac{1}{\sqrt{3\cdot 6}}\right)^{|E_{36}|} \times \left(\frac{1}{\sqrt{6\cdot 6}}\right)^{|E_{66}|} \\ &= 2^{-54n^2-78n-18} \times (\sqrt{2})^{-54n^2-66n-24} \times (\sqrt{3})^{12-24n} \times 3^{144n-108n^2-54} \times (\sqrt{3})^{12-24n} \times \sqrt{6}^{-12n+6}. \end{aligned}$$

$$\begin{aligned} ABCII(G_5) &= \prod_{uv\in E(G_5)} \sqrt{\frac{d_{G_5}(u)+d_{G_5}(v)-2}{d_{G_5}(u)\cdot d_{G_5}(v)}} \\ &= \left(\sqrt{\frac{2+3-2}{2\cdot 3}}\right)^{|E_{23}|} \times \left(\sqrt{\frac{2+6-2}{2\cdot 6}}\right)^{|E_{26}|} \times \left(\sqrt{\frac{3+6-2}{3\cdot 6}}\right)^{|E_{36}|} \times \left(\sqrt{\frac{6+6-2}{6\cdot 6}}\right)^{|E_{66}|} \\ &= 3^{-108n^2+144n-54} \times (\sqrt{2})^{-108n^2+108n-36} \times (\sqrt{5})^{-78n+54n^2+30} \times (\sqrt{7})^{54n^2-66n+24} \end{aligned}$$

$$\begin{aligned} GAII(G_5) &= \prod_{u,v\in E(G_5)} \left[\frac{2\sqrt{d_{G_5}(u)\cdot d_{G_5}(v)}}{d_{G_5}(u)+d_{G_5}(v)}\right] \\ &= \left(\frac{2\sqrt{2\cdot 3}}{2+3}\right)^{|E_{23}|} \times \left(\frac{2\sqrt{2\cdot 6}}{2+6}\right)^{|E_{26}|} \times \left(\frac{2\sqrt{3\cdot 6}}{3+6}\right)^{|E_{36}|} \times \left(\frac{2\sqrt{6\cdot 6}}{6+6}\right)^{|E_{66}|} \\ &= \left(\frac{\sqrt{2}}{3}\right)^{54n^2-66n+24} \times \left(\frac{\sqrt{6}}{5}\right)^{12n-6} \times (\sqrt{3})^{24n-12} \times 2^{54n^2-78n+30}. \end{aligned}$$

$$\begin{aligned} AGII(G_5) &= \prod_{u,v\in E(G_5)} \left[\frac{d_{G_5}(u)+d_{G_5}(v)}{2\sqrt{d_{G_5}(u)\cdot d_{G_5}(v)}}\right] \\ &= \left(\frac{2+3}{2\sqrt{2\cdot 3}}\right)^{|E_{23}|} \times \left(\frac{2+6}{2\sqrt{2\cdot 6}}\right)^{|E_{26}|} \times \left(\frac{3+6}{2\sqrt{3\cdot 6}}\right)^{|E_{36}|} \times \left(\frac{6+6}{2\sqrt{6\cdot 6}}\right)^{|E_{66}|} \\ &= \left(\frac{\sqrt{2}}{3}\right)^{-54n^2+66n-24} \times \left(\frac{\sqrt{6}}{5}\right)^{-12n+6} \times (\sqrt{3})^{-24n+12} \times 2^{-54n^2+78n-30}. \end{aligned}$$

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