A Comparative Study of Mohand and Sumudu Transforms

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ABSTRACT: Mohand and Sumudu transforms are very useful integral transforms for solving many advanced problems of engineering and sciences like heat conduction problems, vibrating beams problems, population growth and decay problems, electric circuit problems etc. In this article, we present a comparative study of two integral transforms namely Mohand and Sumudu transforms. In application section, we solve some systems of differential equations using both the transforms. Results show that Mohand and Sumudu transforms are closely connected.

KEYWORDS: Mohand transform, Sumudu transform, System of differential equations.

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1.INTRODUCTION: In modern time, integral transforms (Laplace transform [1], Fourier transform [1], Hankel transform [1], Mellin transform [1], Z-transform [1], Wavelet transform [1], Mahgoub transform [2], Kamal transform [3], Elzaki transform [4], Aboodh transform [5], Mohand transform [6], Sumudu transform [7], Hermite transform [1] etc.) have very useful role in mathematics, physics, chemistry, social science, biology, radio physics, astronomy, nuclear science, electrical and mechanical engineering for solving the advanced problems of these fields.

Belgacem and Karaballi [8] gave a paper on Sumudu transform fundamental properties investigations and applications. A note on the Sumudu transforms and differential equations was given by Eltayeb and Kilicman [9]. Aggarwal and Chaudhary [10] discussed a comparative study of Mohand and Laplace transforms. A comparative study of Mohand and Kamal transforms was given by Aggarwal et al. [11].

Many scholars [12-38] used these transforms and solve the problems of differential equations, partial differential equations, integral equations, integro-differential equations, partial integro-differential equations, delay differential equations and population growth and decay problems. Aggarwal et al. [39] used Mohand transform and solved population growth and decay problems. Aggarwal et al. [40] defined Mohand transform of Bessel's functions. Kumar et al. [41] solved linear Volterra integral equations of first kind using Mohand transform.

Kumar et al. [42] used Mohand transform and solved the mechanics and electrical circuit problems. Solution of linear Volterra integral equations of second kind using Mohand transform was given by Aggarwal et al. [43]. Aggarwal and Chauhan [44] gave a comparative study of Mohand and Aboodh transforms. A comparative study of Mohand and Elzaki transforms was given by Aggarwal et al. [45].

In this paper, we concentrate mainly on the comparative study of Mohand and Sumudu transforms and we solve some systems of differential equations using these transforms.

2. DEFINITION OF MOHAND AND SUMUDU TRANSFORMS:

2.1 Definition of Mohand transforms:

In year 2017, Mohand and Mahgoub [6] defined "Mohand transform'' of the function $F(t)$ for $t \geq 0$ as

$$M\{F(t)\} = v^2 \int_0^\infty F(t)e^{-vt}dt = R(v), k_1 \leq v \leq k_2$$

where the operator M is called the Mohand transform operator.

2.2 Definition of Sumudu transforms:

The Sumudu transform of the function $F(t)$ for all $t \geq 0$ is defined as [7]:

$S\{F(t)\} = \frac{1}{v}\int_0^\infty F(t)e^{-t/v}dt = T(v), 0 < k_1 \leq v \leq k_2$,where the operator S is called the Sumudu transform operator.

The Mohand and Sumudu transforms of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Mohand and Sumudu transforms of the function $F(t)$.

3. PROPERTIES OF MOHAND AND SUMUDU TRANSFORMS: In this section, we present the linearity property, change of scale property, first shifting theorem, convolution theorem of Mohand and Aboodh transforms.

3.1 Linearity property of Mohand and Sumudu transforms:

a. ***Linearity property of Mohand transforms [10-11, 39-40, 43]:*** If Mohand transform of functions $F_1(t)$ and $F_2(t)$are$R_1(v)$and $R_2(v)$ respectively then Mohand transform of $[aF_1(t) + bF_2(t)]$ is given by

$[aR_1(v) + bR_2(v)]$, where a, b are arbitrary constants.

b. ***Linearity property of Sumudu transforms [8]:*** If Sumudu transform of functions $F_1(t)$ and $F_2(t)$are $T_1(v)$and $T_2(v)$ respectively then Sumudu transform of $[aF_1(t) + bF_2(t)]$ is given by
$[aT_1(v) + bT_2(v)]$, where a, b are arbitrary constants.

3.2 Change of scale property of Mohand and Aboodh transforms:

a. ***Change of scale property of Mohand transforms [10-11, 40, 43]:*** If Mohand transform of function $F(t)$ is $R(v)$ then Mohand transform of function $F(at)$is given by $aR\left(\frac{v}{a}\right)$.

b. ***Change of scale property of Sumudu transforms [8]:*** If Sumudu transform of function $F(t)$ is $T(v)$ then Sumudu transform of function $F(at)$is given by $T(av)$.

3.3 Shifting property of Mohand and Aboodh transforms:

a. ***Shifting property of Mohand transforms [10-11, 43]:*** If Mohand transform of function $F(t)$ is $R(v)$ then Mohand transform of function $e^{at}F(t)$is given by $\left[\frac{v^2}{(v-a)^2}\right]R(v-a)$.

b. ***Shifting property of Sumudu transforms[8]:*** If Sumudu transform of function $F(t)$ is $T(v)$ then Sumudu transform of function $e^{at}F(t)$is given by $\left[\frac{1}{1-av}\right]T\left(\frac{v}{1-av}\right)$.

3.4 Convolution theorem for Mohand and Aboodh transforms:

a. ***Convolution theorem for Mohand transforms [10-11, 41, 43]:*** If Mohand transform of functions $F_1(t)$ and $F_2(t)$are $R_1(v)$and $R_2(v)$ respectively then Mohand transform of their convolution $F_1(t) * F_2(t)$ is given by

$$M\{F_1(t) * F_2(t)\} = \left(\frac{1}{v^2}\right)M\{F_1(t)\}M\{F_2(t)\}$$

$\Rightarrow M\{F_1(t) * F_2(t)\} = \left(\frac{1}{v^2}\right)R_1(v)R_2(v)$, where $F_1(t) * F_2(t)$ is defined by

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x)\,F_2(x)dx = \int_0^t F_1(x)\,F_2(t-x)dx$$

b. ***Convolution theorem for Sumudu transforms[8]:*** If Sumudu transform of functions $F_1(t)$ and $F_2(t)$are $T_1(v)$and $T_2(v)$ respectively then Sumudu transform of their convolution $F_1(t) * F_2(t)$ is given by

$$S\{F_1(t) * F_2(t)\} = vS\{F_1(t)\}S\{F_2(t)\}$$

$\Rightarrow S\{F_1(t) * F_2(t)\} = v\,T_1(v)T_2(v)$, where $F_1(t) * F_2(t)$ is defined by

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x)\,F_2(x)dx = \int_0^t F_1(x)\,F_2(t-x)dx$$

4. MOHAND AND SUMUDU TRANSFORMS OF THE DERIVATIVES OF THE FUNCTION $F(t)$:

4.1 Mohand transforms of the derivatives of the function $F(t)$ [10-11, 41-43]:

If $M\{F(t)\} = R(v)$ then

a) $M\{F'(t)\} = vR(v) - v^2F(0)$
b) $M\{F''(t)\} = v^2R(v) - v^3F(0) - v^2F'(0)$
c) $M\{F^{(n)}(t)\} = v^nR(v) - v^{n+1}F(0) - v^nF'(0) - \cdots \ldots - v^2F^{(n-1)}(0)$

4.2 Sumudu transforms of the derivatives of the function $F(t)$[8]:

If $S\{F(t)\} = T(v)$then

a) $S\{F'(t)\} = \frac{T(v)}{v} - \frac{F(0)}{v}$
b) $S\{F''(t)\} = \frac{T(v)}{v^2} - \frac{F(0)}{v^2} - \frac{F'(0)}{v}$
c) $S\{F^{(n)}(t)\} = \frac{T(v)}{v^n} - \frac{F(0)}{v^n} - \frac{F'(0)}{v^{n-1}} - \cdots \ldots - \frac{F^{(n-1)}(0)}{v}$

5. MOHAND AND SUMUDU TRANSFORMS OF INTEGRAL OF A FUNCTION $F(t)$:

5.1 Mohand transforms of integral of a function $F(t)$ [10]:

If $M\{F(t)\} = R(v)$ then $M\left\{\int_0^t F(t)dt\right\} = \frac{1}{v}R(v)$.

5.2 Sumudu transforms of integral of a function $F(t)$ [8]:
If $S\{F(t)\} = T(v)$ then $S\left\{\int_0^t F(t)dt\right\} = vT(v)$.

6. MOHAND AND SUMUDU TRANSFORMS OF FREQUENTLY USED FUNCTIONS [7-11, 39-43]:

Table: 1

S.N.	$F(t)$	$M\{F(t)\} = R(v)$	$S\{F(t)\} = T(v)$
1.	1	v	1
2.	t	1	v
3.	t^2	$\frac{2!}{v}$	$2!\, v^2$
4.	$t^n, n \in N$	$\frac{n!}{v^{n-1}}$	$n!\, v^n$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^{n-1}}$	$\Gamma(n+1)v^n$
6.	e^{at}	$\frac{v^2}{v-a}$	$\frac{1}{1-av}$
7.	$sinat$	$\frac{av^2}{(v^2+a^2)}$	$\frac{av}{1+a^2v^2}$
8.	$cosat$	$\frac{v^3}{(v^2+a^2)}$	$\frac{1}{1+a^2v^2}$
9.	$sinhat$	$\frac{av^2}{(v^2-a^2)}$	$\frac{av}{1-a^2v^2}$
10.	$coshat$	$\frac{v^3}{(v^2-a^2)}$	$\frac{1}{1-a^2v^2}$
11.	$J_0(t)$	$\frac{v^2}{\sqrt{(1+v^2)}}$	$\frac{1}{\sqrt{(1+v^2)}}$
12.	$J_1(t)$	$v^2 - \frac{v^3}{\sqrt{(1+v^2)}}$	$\frac{1}{v} - \frac{1}{v\sqrt{(1+v^2)}}$

7. INVERSE MOHAND AND SUMUDU TRANSFORMS:

7.1 Inverse Mohand transforms [10-11, 39, 43]: If $R(v)$is the Mohand transform of $F(t)$then $F(t)$is called the inverse Mohand transform of $R(v)$and in mathematical terms, it can be expressed as
$F(t) = M^{-1}\{R(v)\}$, where M^{-1} is an operator and it is called as inverse Mohand transform operator.

7.2 Inverse Sumudu transforms: If $T(v)$ is the Sumudu transforms of $F(t)$ then $F(t)$ is called the inverse Sumudu transform of $T(v)$and in mathematical terms, it can be expressed as
$F(t) = S^{-1}\{K(v)\}$, where S^{-1} is an operator and it is called as inverse Sumudu transform operator.

8. INVERSE MOHAND AND SUMUDU TRANSFORMS OF FREQUENTLY USED FUNCTIONS [10-11, 39-40]:

Table: 2

S.N.	$R(v)$	$F(t)=M^{-1}\{R(v)\}=S^{-1}\{T(v)\}$	$T(v)$
1.	v	1	1
2.	1	t	v
3.	$\frac{1}{v}$	$\frac{t^2}{2}$	v^2
4.	$\frac{1}{v^{n-1}}, n\epsilon N$	$\frac{t^n}{n!}$	$v^n, n\epsilon N$
5.	$\frac{1}{v^{n-1}}, n>-1$	$\frac{t^n}{\Gamma(n+1)}$	$v^n, n>-1$
6.	$\frac{v^2}{v-a}$	e^{at}	$\frac{1}{1-av}$
7.	$\frac{v^2}{(v^2+a^2)}$	$\frac{sinat}{a}$	$\frac{v}{1+a^2v^2}$
8.	$\frac{v^3}{(v^2+a^2)}$	$cosat$	$\frac{1}{1+a^2v^2}$
9.	$\frac{v^2}{(v^2-a^2)}$	$\frac{sinhat}{a}$	$\frac{v}{1-a^2v^2}$
10.	$\frac{v^3}{(v^2-a^2)}$	$coshat$	$\frac{1}{1-a^2v^2}$
11.	$\frac{v^2}{\sqrt{(1+v^2)}}$	$J_0(t)$	$\frac{1}{\sqrt{(1+v^2)}}$
12.	$v^2-\frac{v^3}{\sqrt{(1+v^2)}}$	$J_1(t)$	$\frac{1}{v}-\frac{1}{v\sqrt{(1+v^2)}}$

9. APPLICATIONS OF MOHAND AND SUMUDU TRANSFORMS FOR SOLVING SYSTEM OF DIFFERENTIAL EQUATIONS:

In this section some numerical applications are give to solve the systems of differential equations using Mohand and Sumudu transforms.

9.1 Consider a system of linear ordinary differential equations

$$\left.\begin{aligned}\frac{d^2x}{dt^2}+3x-2y=0\\ \frac{d^2x}{dt^2}+\frac{d^2y}{dt^2}-3x+5y=0\end{aligned}\right\} \quad (1)$$

with $x(0)=0, y(0)=0, x'(0)=3, y'(0)=2$ (2)

Solution using Mohand transforms:

Taking Mohand transform of "Eq. (1)", we have

$$\left.\begin{aligned}M\left\{\frac{d^2x}{dt^2}\right\}+3M\{x\}-2M\{y\}=0\\ M\left\{\frac{d^2x}{dt^2}\right\}+M\left\{\frac{d^2y}{dt^2}\right\}-3M\{x\}+5M\{y\}=0\end{aligned}\right\} \quad (3)$$

Now using the property, Mohand transform of the derivatives of the function, in "Eq. (3)", we have

$$\left.\begin{aligned}v^2M\{x\}-v^3x(0)-v^2x'(0)+3M\{x\}-2M\{y\}=0\\ v^2M\{x\}-v^3x(0)-v^2x'(0)+v^2M\{y\}-v^3y(0)-v^2y'(0)-3M\{x\}+5M\{y\}=0\end{aligned}\right\} \quad (4)$$

Using "Eq. (2)" in "Eq. (4)", we have

$$\left.\begin{aligned}(v^2+3)M\{x\}-2M\{y\}=3v^2\\ (v^2-3)M\{x\}+(v^2+5)M\{y\}=5v^2\end{aligned}\right\} \quad (5)$$

Solving the "Eq. (5)" for $M\{x\}$ and $M\{y\}$, we have

$$\left.\begin{aligned} M\{x\} &= \frac{11}{4}\left[\frac{v^2}{(v^2+1)}\right] + \frac{1}{4}\left[\frac{v^2}{(v^2+9)}\right] \\ M\{y\} &= \frac{11}{4}\left[\frac{v^2}{(v^2+1)}\right] - \frac{3}{4}\left[\frac{v^2}{(v^2+9)}\right] \end{aligned}\right\} \quad (6)$$

Now taking inverse Mohand transform of "Eq. (6)", we have

$$\left.\begin{aligned} x &= \frac{11}{4} sint + \frac{1}{12} sin3t \\ y &= \frac{11}{4} sint - \frac{1}{4} sin3t \end{aligned}\right\} \quad (7)$$

which is the required solution of "Eq. (1)" with "Eq. (2)".

Solution using Sumudu transforms:

Taking Sumudu transform of "Eq. (1)", we have

$$\left.\begin{aligned} S\left\{\frac{d^2x}{dt^2}\right\} + 3S\{x\} - 2S\{y\} = 0 \\ S\left\{\frac{d^2x}{dt^2}\right\} + S\left\{\frac{d^2y}{dt^2}\right\} - 3S\{x\} + 5S\{y\} = 0 \end{aligned}\right\} \quad (8)$$

Now using the property, Sumudu transform of the derivatives of the function, in "Eq. (8)", we have

$$\left.\begin{aligned} \frac{1}{v^2}S\{x\} - \frac{x(0)}{v^2} - \frac{x'(0)}{v} + 3S\{x\} - 2S\{y\} = 0 \\ \frac{1}{v^2}S\{x\} - \frac{x(0)}{v^2} - \frac{x'(0)}{v} + \frac{1}{v^2}S\{y\} - \frac{y(0)}{v^2} - \frac{y'(0)}{v} - 3S\{x\} + 5S\{y\} = 0 \end{aligned}\right\} \quad (9)$$

Using "Eq. (2)" in "Eq. (9)", we have

$$\left.\begin{aligned} \left(\frac{1}{v^2}+3\right)S\{x\} - 2S\{y\} = \frac{3}{v} \\ \left(\frac{1}{v^2}-3\right)S\{x\} + \left(\frac{1}{v^2}+5\right)S\{y\} = \frac{5}{v} \end{aligned}\right\} \quad (10)$$

Solving the "Eq. (10)" for $S\{x\}$ and $S\{y\}$, we have

$$\left.\begin{aligned} S\{x\} &= \frac{11}{4}\left[\frac{v}{1+v^2}\right] + \frac{1}{4}\left[\frac{v}{1+9v^2}\right] \\ S\{y\} &= \frac{11}{4}\left[\frac{v}{1+v^2}\right] - \frac{3}{4}\left[\frac{v}{1+9v^2}\right] \end{aligned}\right\} \quad (11)$$

Now taking inverse Sumudu transform of "Eq. (11)", we have

$$\left.\begin{aligned} x &= \frac{11}{4} sint + \frac{1}{12} sin3t \\ y &= \frac{11}{4} sint - \frac{1}{4} sin3t \end{aligned}\right\} \quad (12)$$

which is the required solution of "Eq. (1)" with "Eq. (2)".

9.2 Consider a system of linear ordinary differential equations

$$\left.\begin{aligned} \frac{dx}{dt} + y = 2cost \\ x + \frac{dy}{dt} = 0 \end{aligned}\right\} \quad (13)$$

with $x(0) = 0, y(0) = 1$ (14)

Solution using Mohand transforms:

Taking Mohand transform of "Eq. (13)", we have

$$\left.\begin{aligned} M\left\{\frac{dx}{dt}\right\} + M\{y\} = 2M\{cost\} \\ M\{x\} + M\left\{\frac{dy}{dt}\right\} = 0 \end{aligned}\right\} \quad (15)$$

Now using the property, Mohand transform of the derivatives of the function, in "Eq. (15)", we have

$$\left.\begin{aligned} vM\{x\} - v^2x(0) + M\{y\} = \frac{2v^3}{(v^2+1)} \\ M\{x\} + vM\{y\} - v^2y(0) = 0 \end{aligned}\right\} \quad (16)$$

Using "Eq. (14)" in "Eq. (16)", we have

$$\left.\begin{aligned} vM\{x\} + M\{y\} &= \frac{2v^3}{(v^2+1)} \\ M\{x\} + vM\{y\} &= v^2 \end{aligned}\right\} \quad (17)$$

Solving the "Eq. (17)" for $M\{x\}$and $M\{y\}$, we have

$$\left.\begin{aligned} M\{x\} &= \left[\frac{v^2}{(v^2+1)}\right] \\ M\{y\} &= \left[\frac{v^3}{(v^2+1)}\right] \end{aligned}\right\} \quad (18)$$

Now taking inverse Mohand transform of "Eq. (18)", we have

$$\left.\begin{aligned} x &= sint \\ y &= cost \end{aligned}\right\} \quad (19)$$

which is the required solution of "Eq. (13)" with "Eq. (14)".

Solution using Sumudu transforms:

Taking Sumudu transform of "Eq. (13)", we have

$$\left.\begin{aligned} S\left\{\frac{dx}{dt}\right\} + S\{y\} &= 2S\{cost\} \\ S\{x\} + S\left\{\frac{dy}{dt}\right\} &= 0 \end{aligned}\right\} \quad (20)$$

Now using the property, Sumudu transform of the derivatives of the function, in "Eq. (20)", we have

$$\left.\begin{aligned} \frac{1}{v}S\{x\} - \frac{x(0)}{v} + S\{y\} &= \frac{2}{1+v^2} \\ S\{x\} + \frac{1}{v}S\{y\} - \frac{y(0)}{v} &= 0 \end{aligned}\right\} \quad (21)$$

Using "Eq. (14)" in "Eq. (21)", we have

$$\left.\begin{aligned} \frac{1}{v}S\{x\} + S\{y\} &= \frac{2}{1+v^2} \\ S\{x\} + \frac{1}{v}S\{y\} &= \frac{1}{v} \end{aligned}\right\} \quad (22)$$

Solving the "Eq. (22)" for $S\{x\}$and $S\{y\}$, we have

$$\left.\begin{aligned} S\{x\} &= \left[\frac{v}{1+v^2}\right] \\ S\{y\} &= \left[\frac{1}{1+v^2}\right] \end{aligned}\right\} \quad (23)$$

Now taking inverse Sumudu transform of "Eq. (23)", we have

$$\left.\begin{aligned} x &= sint \\ y &= cost \end{aligned}\right\} \quad (24)$$

which is the required solution of "Eq. (13)" with "Eq. (14)".

9.3 Consider a system of linear ordinary differential equations

$$\left.\begin{aligned} \frac{dz}{dt} + x &= sint \\ \frac{dx}{dt} - y &= e^t \\ \frac{dy}{dt} + z + x &= 1 \end{aligned}\right\} \quad (25)$$

with $x(0) = 1, y(0) = 1, z(0) = 0$ (26)

Solution using Mohand transforms:

Taking Mohand transform of "Eq. (25)", we have

$$\left.\begin{aligned} M\left\{\frac{dz}{dt}\right\} + M\{x\} &= M\{sint\} \\ M\left\{\frac{dx}{dt}\right\} - M\{y\} &= M\{e^t\} \\ M\left\{\frac{dy}{dt}\right\} + M\{z\} + M\{x\} &= M\{1\} \end{aligned}\right\} \quad (27)$$

Now using the property, Mohand transform of the derivatives of the function, in "Eq. (27)", we have

$$\left.\begin{aligned} vM\{z\} - v^2 z(0) + M\{x\} &= \left[\frac{v^2}{(v^2+1)}\right] \\ vM\{x\} - v^2 x(0) - M\{y\} &= \left[\frac{v^2}{v-1}\right] \\ vM\{y\} - v^2 y(0) + M\{z\} + M\{x\} &= v \end{aligned}\right\} \quad (28)$$

Using “Eq. (26)” in “Eq. (28)”, we have

$$\left.\begin{aligned} vM\{z\} + M\{x\} &= \left[\frac{v^2}{(v^2+1)}\right] \\ vM\{x\} - M\{y\} &= \left[\frac{v^3}{v-1}\right] \\ vM\{y\} + M\{z\} + M\{x\} &= v + v^2 \end{aligned}\right\} \quad (29)$$

Solving the “Eq. (29)” for $M\{x\}$, $M\{y\}$and $M\{z\}$, we have

$$\left.\begin{aligned} M\{x\} &= \left[\frac{v^2}{v-1}\right] + \left[\frac{v^2}{(v^2+1)}\right] \\ M\{y\} &= \left[\frac{v^3}{(v^2+1)}\right] \\ M\{z\} &= v - \left[\frac{v^2}{v-1}\right] \end{aligned}\right\} \quad (30)$$

Now taking inverse Mohand transform of “Eq. (30)”, we have

$$\left.\begin{aligned} x &= e^t + sint \\ y &= cost \\ z &= 1 - e^t \end{aligned}\right\} \quad (31)$$

which is the required solution of “Eq. (25)” with “Eq. (26)”.

Solution using Sumudu transforms:

Taking Sumudu transform of “Eq. (25)”, we have

$$\left.\begin{aligned} S\left\{\frac{dz}{dt}\right\} + S\{x\} &= S\{sint\} \\ S\left\{\frac{dx}{dt}\right\} - S\{y\} &= S\{e^t\} \\ S\left\{\frac{dy}{dt}\right\} + S\{z\} + S\{x\} &= S\{1\} \end{aligned}\right\} \quad (32)$$

Now using the property, Sumudu transform of the derivatives of the function, in “Eq. (32)”, we have

$$\left.\begin{aligned} \frac{1}{v}S\{z\} - \frac{z(0)}{v} + S\{x\} &= \left[\frac{v}{1+v^2}\right] \\ \frac{1}{v}S\{x\} - \frac{x(0)}{v} - S\{y\} &= \left[\frac{1}{1-v}\right] \\ \frac{1}{v}S\{y\} - \frac{y(0)}{v} + S\{z\} + S\{x\} &= 1 \end{aligned}\right\} \quad (33)$$

Using “Eq. (26)” in “Eq. (33)”, we have

$$\left.\begin{aligned} \frac{1}{v}S\{z\} + S\{x\} &= \left[\frac{v}{1+v^2}\right] \\ \frac{1}{v}S\{x\} - S\{y\} &= \left[\frac{1}{v(1-v)}\right] \\ \frac{1}{v}S\{y\} + S\{z\} + S\{x\} &= 1 + \frac{1}{v} \end{aligned}\right\} \quad (34)$$

Solving the “Eq. (34)” for $S\{x\}$, $S\{y\}$ and $S\{z\}$, we have

$$\left.\begin{aligned} S\{x\} &= \left[\frac{1}{1-v}\right] + \left[\frac{v}{1+v^2}\right] \\ S\{y\} &= \left[\frac{1}{1+v^2}\right] \\ S\{z\} &= 1 - \left[\frac{1}{1-v}\right] \end{aligned}\right\} \quad (35)$$

Now taking inverse Sumudu transform of “Eq. (35)”, we have

$$\left.\begin{aligned} x &= e^t + sint \\ y &= cost \\ z &= 1 - e^t \end{aligned}\right\} \quad (36)$$

which is the required solution of “Eq. (25)” with “Eq. (26)”.

10. CONCLUSIONS:

In this paper, we have successfully discussed the comparative study of Mohand and Sumudu transforms. In application section, we solve systems of differential equations comparatively using both Mohand and Sumudu transforms. The given numerical applications in application section show that both the transforms (Mohand and Sumudu transforms) are closely connected to each other.

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