GRAPH RUBBLING PROBLEMS

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Abstract: Graph pebbling is a mathematical game played over a fixed graph. A pebbling distribution D is a function that assigns a number of pebbles to each vertex in a graph. A pebbling move removes two pebbles from one vertex and places a single pebble on an adjacent vertex. Thus, one pebble is lost with every move. A pebbling sequence is a list of pebbling moves performed in order under a distribution. A vertex is reachable under a distribution if it is possible to place a pebble on that vertex after some pebbling sequence.A vertex is reachable under a distribution if it is possible to place a pebble on that vertex after some pebbling sequence.A pebbling distribution is solvable if any chosen target vertex is reachable. In 2007, Christain Belford introduced an extension of graph pebbling called rubbling, Rubbling is similar to graph pebbling, but it allows for an additional type of move. A rubbling move is either a pebbling move or a strict rubbling move. A strict rubbling move onto a vertex v removes one pebble each from two of v's neighbors and places a single pebble on v. The *rubbling number of a graph* G, denoted ρ(G), as the smallest k ∈ N such that every initial distribution of k pebbles to G results in any vertex of G being reachable using rubbling moves. The rubbling number of tree,star,diameter two graphs are proved in this paper.

Index Terms:Pebbling,rubbling,diameter two graphs

INTRODUCTION: A pebbling Graph pebbling is a mathematical game played over a fixed graph[1]. A pebbling distribution D is a function that assigns a number of pebbles to each vertex in a graph. For instance, if a vertex v in a graph G contains three pebbles under a distribution D, we say D(v) = 3. A pebbling move removes two pebbles from one vertex and places a single pebble on an adjacent vertex. Thus, one pebble is lost with every move. A pebbling sequence is a list of pebbling moves performed in order under a distribution. A vertex is reachable under a distribution if it is possible to place a pebble on that vertex after some pebbling sequence. A pebbling distribution is solvable if any chosen target vertex is reachable. In 2007, Christain Belford introduced an extension of graph pebbling called rubbling , Rubbling is similar to graph pebbling, but it allows for an additional type of move. A rubbling move is either a pebbling move or a strict rubbling move. A strict rubbling move onto a vertex v removes one pebble each from two of v's neighbors and places a single pebble on v.The *rubbling number of a graph* G, denoted ρ(G), as the smallest k ∈ N such that every initial distribution of k pebbles to G results in any vertex of G being reachable using rubbling moves.
Belford[1] established many results and bounds for rubbling. But there are still many concepts known in pebbling that have not been explored in terms of rubbling, and relationships between pebbling and rubbling that have not been investigated. Further in [1] there are some open problems. In this paper determine the rubbling number of trees and diameter two graphs. find solutions to a few of them.

Pebbling number of Trees

We use notation and theme from Matt Mohorn[2]. A tree is a connected graph on n - 1 vertices. A root of a tree is a chosen vertex of degree one. A path partition of a tree T is a partition of the edges of G into sets of paths, where each path is directed towards a root r. The length list of a path partition is the list of all path lengths in the path partition in non-increasing order. We say that a path partition L majorizes path partition $L^{/}$if the length list of L is larger than that of $L^{/}$ in the first position where they are not equal. A path partition with root r is r-optimal if its length list is not majorized by that of any other path partition. The following theorem states the value of the rubbling and pebbling number of a tree are the same.

Fact [2]: $\rho(Pn) = 2^{n-1}$ where P_n is the path on n vertices

Theorem : Let L= (l_1; l_2; : : : ; l_m) be the length list of an r-optimal path partition L of a tree T with root r. Then

$$\left(\sum_{i=1}^{m} 2^{l_i}\right) - m + 1. = \rho\ (T)$$

Proof: Every path in L begins at a pendant (end) vertex. every path of length li in L, place 2^{li} -1 pebbles on its starting leaf. We know that $\rho(P_n) = 2^{n-1}$ where P_n is the path on n vertices each path of length n. As l_i is a path on $l_i + 1$ vertices, each rubbling sequence will place a pebble on the last vertex of any path in L. Also addition of one pebble to any path leaves one pebble to the target vertex.

Theorem: If G is a diameter two graph, ρ (G)=4.

Proof:

Case 1. G has a cut vertex v.Let V(G), the vertex set of G be such that

$V(G)=U \cup V,\ U \cap V=v$

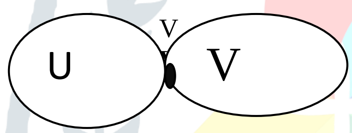

Figure (1):The partitioning of the vertex set V(G)of G having a cut vertex v

Let w be the target vertex. If w is in V, D (V)=2,then w can be reached.

If w is in U, D (V)=4 ,two pebbles can be moved to v ,from there a pebble can be moved two w.

Case 2:G has no cut vertex.

Let u be the target vertex. If D (u)=1, then over.

If two pebbles are there in the vertices adjacent to u, then done.

Let W=Set of vertices 'w'at a distance 2 from u.

If D (w)=4 then a pebble can be moved to u .Let w_1,w_2 be at a distance two from u. There must be a vertex k adjacent to w_1 and w_2 and u

If D (w_1)=3,D (w_2)=1, then a pebble can be moved to u through k.

If not, there are at least three vertices where there are pebbles. Now a pebble can be moved to u.

If w_1,w_2 are not connected ,then w_1 adjacent to k_1 which is adjacent to u,and w_2 adjacent to k_2 which is adjacent to u and k_1k_2 is an edge. Now u can be pebbled.

Theorem :$\rho(S_n) = 4$ where S_n is the star on n+1vertices

Proof: Let s_1,s_2. …,sn be the end vertices of Sn and c , the internal vertex.Then it is easy to see that from any distribution of 4 pebbles on the vertices of Sn ,a pebble can be moved to any desired vertex.

2 –rubbling property: The analogue of 2 –pebbling property is not applicable to rubbling. Since the rubbling number of many graphs (example Sn)are less than the number of vertices , with 2 $\rho(G)$-q+1 pebbles,q is the number of vertices with at least one pebble each to place 2 pebbles to any desired vertex.

An unknown situation: Both pebbling and rubbling moves excludes the situation in which in which adjacent vertices in a path have one pebble

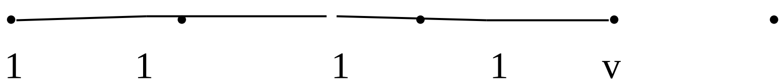

Figure (2): A situation where a pebble cannot be moved to v,either in pebbling or rubbling.

References : [1] Graph rubbling: An extension of graph pebbling, Christopher Andrew Belford,[December 2006],Northern Arizona University

[2] Matt Mohorn :An Introduction to Graph Pebbling ,[May 13, 2014],Honors Thesis in Mathematics, Davidson College