

# A STUDY AN EFFICIENT APPROACH FOR ONLINE MULTI-ROBOT TERRAIN COVERAGE

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## ABSTRACT:

Terrain coverage offers a broad range of applications in our daily lives, from small-scale activities such as floor cleaning, grass mowing, and harvesting to large-scale missions such as hazardous terrain inspection and combat surveillance. These jobs are tedious, time-consuming, and occasionally dangerous. This thesis investigates the issue of online terrain coverage (when the map of the environment is unknown in advance) utilizing numerous mobile robots. Cluster, Allocate, Cover (CAC), a centralized method based on frontier propagation, is presented. Using an optimum allocation method, the CAC algorithm generates clusters of known border cells and assigns them to specific robots. Despite the benefits provided by CAC, it has two drawbacks: it is not resistant to failures and it does not cover the landscape in a continuous manner. Another method suggested in this thesis, known as FAST, is fully distributed and resistant to numerous robot failures. FAST robots follow a predefined path while covering the unknown territory in a mutually exclusive way, resulting in vast, unbroken, and continuous regions to be covered.

**KEYWORDS:** Terrain coverage, range small-scale activities, hazardous, Multi-Robot,

## INTRODUCTION

During the past two decades, multi-robot terrain coverage has gotten a lot of attention. For the activity to be completed, the robots must cross the whole landscape. Various activities that require terrain covering may be boring (floor cleaning), time consuming (harvesting), and/or dangerous (inspection of hazardous terrains). As a result, automating these processes is extremely desired. The terrain coverage methods used online do not depend on topographical knowledge. As a result, the robots build their trajectories and maps gradually in real time. Faster coverage is the consequence of an effective coordinating approach. However, since the robots must do several tasks in simultaneously, such as obstacle avoidance, mapping, localization, route planning, and communication with peers, it poses a number of difficulties. Furthermore, the sheer nature of the deployment of mobile multi-robot systems necessitates the robots' computing, sensor, storage, and communication capabilities being basic. Knowing that one robot's plan has an inherent effect on the plans of other robots, it's critical for the robot team to coordinate correctly.

This paper provides a practical method for assigning a set of borders to each robot. It is suggested that the robots use a route planning technique that enables them to cover the set of border cells assigned to them in a less redundant way. When compared to existing state-of-the-art methods, empirical findings in simulation and on a multi-robot test-bed show that the suggested methodology delivers quicker coverage completion and greater robot utilization.

## LITERATURE REVIEW

Alyssa Pierson et al 2015 — In this article, a novel method is proposed for a group of robots for whom tasks need to be completed cooperatively in order to dynamically adjust online to the differences in actuation performance among the robots. When several robots must roam about to cover the area, we call this a multi-robot coverage issue. We think that certain robots are characterised by low actuation performance (such as weak motors, gear-train friction losses, or wheel slide) whereas others are characterised by high actuation performance (powerful motors, little friction, favourable terrain, etc.). While robots do not know the relative powers of their actuation ahead of time, they do not make assumptions about their teammates' capabilities. This article presents an algorithm that learns how actuation performance varies across robots, with no central coordination, and makes compensatory adjustments by distributing less to the weak robots and more to the strong ones. We demonstrate that the robots converge to locally optimum locations for coverage using a Lyapunov approach. This method is shown via the simulation of Matlab programmes and by conducting Pololu m3pi robot tests.

Pooyan Fazli 2013 We address the problem of repeated coverage of a target area, of any polygonal shape, by a team of robots having a limited visual range. Three distributed Cluster-based algorithms, and a method called Cyclic Coverage are introduced for the problem. The goal is to evaluate the performance of the repeated coverage algorithms under the effects of the variables: Environment Representation, and the Robots' Visual Range. A comprehensive set of performance metrics are considered, including the distance the robots travel, the frequency of visiting points in the target area, and the degree of balance in workload distribution among the robots. The Cyclic Coverage approach, used as a benchmark to compare the algorithms, produces optimal or near-optimal solutions for the single robot case under some criteria. The results can be used as a framework for choosing an appropriate combination of repeated coverage algorithm, environment representation, and the robots' visual range based on the particular scenario and the metric to be optimized.

Han-ye Zhang 2018 Good path planning technology of mobile robot can not only save a lot of time, but also reduce the wear and capital investment of mobile robot. Several methodologies have been proposed and reported in the literature for the path planning of mobile robot. Although these methodologies do not guarantee an optimal solution, they have been successfully applied in their works. The purpose of this

paper is to review the modeling, optimization criteria and solution algorithms for the path planning of mobile robot. The survey shows GA (genetic algorithm), PSO (particle swarm optimization algorithm), APF (artificial potential field), and ACO (ant colony optimization algorithm) are the most used approaches to solve the path planning of mobile robot. Finally, future research is discussed which could provide reference for the path planning of mobile robot.

HONGLIANG GUO 2012 Embryonic development of multicellular organisms, also known as morphogenesis, is regarded as a robust self-organization process for pattern generation. Inspired by the recent findings in biology indicating that morphogen gradients, together with a Gene Regulatory Network (GRN), play a key role in biological patterning, we propose a framework for self-organized multirobot pattern formation and boundary coverage based on an artificial GRN model. The proposed framework does not need a global coordinate system, which makes it more practical to be implemented in a physical robotic system. Moreover, an adaptation mechanism is included in the framework so that the self-organization algorithm is robust to changes in the number of robots. Various case studies of multirobot pattern formation and boundary coverage show the effectiveness of the framework.

Avinash Gautam.et.al 2015 This article presents an algorithm for online multirobot coverage that proceeds with minimal knowledge of the already explored region and the frontier cells. It creates clusters of frontier cells which are designated to robots using an optimal assignment scheme. Coverage is then performed using a novel path planning technique. Many approaches that use clustering for multi-robot coverage do not specify strict time criteria for re-clustering. Moreover, the motion plans they use result in redundant coverage. To overcome these limitations, an appropriate motion plan for the robots is chosen based on the context of already covered frontiers. Dispersion of robots is vital for efficient coverage and is an emergent behavior in our approach. The efficacy of the proposed approach is tested in simulation and on a multi-robot test-bed. The algorithm performs better than some state of the art approaches.

## EXPLORATION VS. COVERAGE

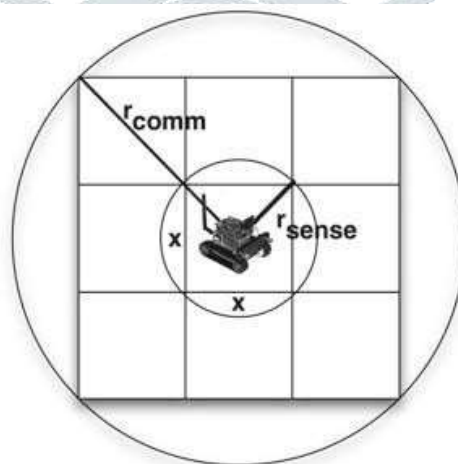
Before we get started on some of the most cutting-edge methods, it's essential to clarify the difference between terrain coverage and terrain exploration. Terrain coverage necessitates the robots traversing the unfamiliar terrain in a shorter amount of time, while exploration necessitates the creation of a high-quality global map of the unknown area. After scanning an area, the robots simply move towards unknown regions; they are not obliged to cross the whole region. Despite the fact that they are two fundamentally distinct applications, they have a lot in common in terms of robot operations, which are as follows:

- The robots have no prior knowledge of the landscape layout.
- The landscape is decomposed into equal-sized grid cells using approximate cell decomposition. This

isn't the only way to decompose terrain, but we've restricted ourselves to frontier exploring methods.

- The robots should be able to self-localize. Simultaneous localization and mapping (SLAM) methods are utilized to construct the map in the absence of a global map.
- The robots plan their next move using map data produced locally by their sensors and/or information obtained from their peers. In this way, the robots combine valuable map information received from other robots for navigation purposes.
- The robots interact with one another to share information about the local instance of the global map that each robot is aware of at the moment, and to coordinate their activities on occasion. They may also communicate information such as their present posture, the current job given to them and its progress, and their current health condition.

One significant distinction between the two applications is that in the exploration phase, the robots have a sensing radius greater than one cell, while in the terrain coverage job, the robots can only detect the eight cells around the robot's location. Before accepting this argument as truth, it's necessary to go through some of the most current state-of-the-art multi-robot exploration methods. Are there three distinct ant-based methods where the environment is split into square cells in a grid? The cell's size is determined by two factors: (a) the distance recorded by the range sensors (sense), and (b) the agents' communication range (comm.). It is believed that the agent sitting in the centre of some cell  $c$  will be able to completely cover that cell, thus the size of the cell  $c$ , i.e.,  $x$ , should be less than or equal to that of the cell  $c$ . to  $\frac{2*r_{sense}}{3\sqrt{2}}$  and the agent is able to communicate within eight cells surrounding  $c$ ,  $\sqrt{2}$  therefore, the communication range of the robot is less than or equal to  $\frac{2*r_{comm}}{3\sqrt{2}}$ , as shown in Figure 1.



**Figure 1 Sensing ( $r_{sense}$ ) and Communication Range ( $r_{comm}$ ) of Robots**

RAPID is an extension and BMI is an improvement over Brick&Mortar. These three are suggested as exploration approaches by the authors. Yet the robot's coverage is limited to only one cell in which it is



currently located and it is able to sense the status of eight surrounding cells by probing the smart tags lying in those cells. Therefore, if the sensing range of the robots is restricted to one cell all exploration algorithms will transform into coverage algorithms. We argue that terrain coverage is also a form of exploration with one additional requirement that the robots have to physically traverse whole of the free space and which is a single connected component. We have compared some of the state of the art approaches of exploration (restricting the sensing range of robots to one cell) with the approach.

## DISCUSSION ON REPRESENTATIVE APPROACHES

In this section, a representative set of important multi-robot exploration approaches which are compared with the proposed approach are discussed. Three highly praised techniques have been considered:

- (a) Pure frontier exploration - It is the most basic form of frontier exploration wherein the robots do not coordinate with each other. They just exchange their map data with their peers and greedily select the target frontier cells.
- (b) Coordinated exploration - This is an extension of Pure Frontier Exploration. The robots coordinate with each other for the selection of the frontier cells. However, the coordination mechanism is not very sophisticated.
- (c) Dispersion based exploration - This algorithm exhibits stronger form of coordination than the other two. The unknown region is partitioned and optimally assigned to individual robots for further exploration.

These techniques are different from each other in the level of multi-robot coordination. It is worth comparing these methods within a common framework and evaluates how fruitful they will be for the purpose of online terrain coverage task. This helps us in identifying the advantages and disadvantages of each method so that the most suitable method can be selected for the said task.

A popular technique referred to as frontier-based exploration for a single mobile robot is proposed. The border between the known and the unknown region is referred to as a frontier. As the robot moves closer to the nearest frontier, the map unfolds. Formally the process is described as follows: Suppose  $F = \{f_1, f_2, f_3, \dots, f_k\}$  is the collection of frontier cells currently being evaluated. The current position of a robot  $r_i$  is  $pos(r_i)$  ( $r_i$ ). The nearest frontier, say  $nf$ , is the one that can be reached in least cost from  $pos(r_i)$  and is obtained as follows:

Path () is a function that returns the length of the shortest path from some cell  $c_i$  to  $c_j$ . In this case both the nearest frontier and the shortest path are determined using Dijkstra's Algorithm. The robot then reaches the nearest frontier and continues exploration. The procedure is repeated until no more frontiers are visible. This technique has been extended to multiple robots in. Each robot has its own local instance of the global

grid map. Every robot after reaching its nearest frontier observes the environment using its sensors and generates a local evidence grid. The robot integrates its local impression with its own local instance of the global grid map. Also, the robot broadcasts this information to all the other robots. The other robots receiving these broadcasts stores them and integrate this information in their own global grid map after reaching their destinations i.e., the frontiers cells chosen by them. Based on the knowledge of the global grid map the robots avoid visiting already explored frontiers.

From the experimental perspective this method sequences the frontier cells by clustering those frontier cells which are in the immediate neighborhood. Small frontier sequences are ignored. The mid-points of the frontier sequences are selected as potential target for exploration. Let us call them target frontier cells. Navigation cost starting from a robot's position to all the target frontier cells is propagated in the multi-robot system.

Euclidean distance cost is assumed between frontier cells. Infinite cost is associated to those cells which correspond to obstacles and unknown cells and some penalty is associated to those cells which are too close to obstacles for the robot to derive and follow a safe path. Each robot selects a target frontier cell with the minimum cost path (mcp) found after backtracking.

The low-level planner of the robot makes the robot follow the mcp. The planner executes again and reassesses the path in three situations (a) after a specified time (b) when the robot reaches its destination and (c) when the path is blocked by an obstacle. This revolutionary work has a limitation that it allocates the frontiers to the robots, which may lead to improper utilization of resources by assigning the same frontier to multiple robots. This is due to the fact that while selecting a particular target frontier cell  $x$  only the mcp to  $x$  is considered and not the information gain that can be obtained by visiting  $x$ . The robots stay close to each other confined within the communication range and disperse locally. Despite the aforementioned limitation, the frontier-based exploration technique has become a de-facto standard for multi-robot exploration.

### **Coordinated exploration**

It is a decision theoretic approach which extends so that the robots coordinate with each other for faster completion of the exploration task. A function is defined those trades of the utility with the cost of reaching a particular target frontier cell. The cost is nothing but the length of the shortest optimal path from the robot's position to the target frontier cell. The utility of a target frontier is the area that can be discovered when the robot arrives at it. The utility of target frontier cell  $x$  is lesser when the target frontier cells in the neighbourhood of  $x$  are assigned to other robots. The target frontiers cells with the maximum utility are chosen by the robots. This ensures that the robots select the target frontier cells that are far from each other. Following is the formal summarization of the whole idea:

Let  $F = \{f_1, f_2, f_3, \dots, f_n\}$  be a set of target frontier cells. Let  $f_r$  be a target frontier cell for some robot  $r$  with a position  $pos(r)$ . Let  $cost(f_r, pos(r))$  be a function that returns the cost of robot  $r$  reaching the target frontier cell  $f_r$ . The cost is calculated as a ratio of the shortest optimal path from the position of the robot  $r$  to the target frontier  $f_r$  and the maximum of the shortest optimal path length of robot  $r$  reaching all the other target frontiers  $F' = \{F - f_r\}$ . The path function is defined above in the Yamauchi's algorithm. The utility of a robot  $r$  Reaching the target frontier  $f_r$  is calculated as follows:  $util(f_r, r) = \sum_{Z \in \{R-r\}} p(f_r, Z)$

Here  $R$  is the set of all robots including robot  $r$  and  $Z$  is the set of all robot excluding robot  $r$ . Function  $p$  returns the distance between the target frontier cells chosen by other robots and the target frontier cell  $f_r$  selected by robot. Where  $f^i$  is the target frontier cell assigned to robot  $i \in Z$ ,  $\square$  is a parameter that models the robots sphere of influence. Now the benefit  $b$  obtained by robot  $r$  when it selects the target frontier cell  $f_r$  is given below:

$$b(f_r, r) = util(f_r, r) - cost(f_r, pos(r))$$

Each robot sends a bid (the benefit) for each target frontier cell to all the other robots. Each robot selects a target frontier cell which has a maximum bid for itself. Finally, the robot plans a path to the chosen target frontier cell. Although the overlap between the robots reduces and the approach tends to minimize the completion time but it confines the robots locally within the sensing range i.e., the robots disperse locally until there is no overlap in the newly discovered region and therefore is similar to Yamauchi's algorithm. Moreover, the algorithm does not scale well even in a graph of moderate size. Another shortcoming of this work is that it does not try to optimize the allocation of the target frontier cells to the robots.

### Dispersion based exploration

For the purpose of exploration dispersion of robots is particularly important. In fact, dispersion remains at the heart of most of the exploration algorithms. The more the robots are dispersed better will they be able to discover new areas and thus complete the exploration faster. This is the main theme of the above algorithm. Instead of allocating the frontiers cells (which are many in large maps) using auctions the cells in the unknown region of the occupancy grid map are clustered into disjoint regions using the K-means algorithm. The number of clusters is equal to the number of robots. The initial seeds for the K-means algorithm are chosen randomly. Euclidean distance to the centric of a region from the current position of the robot is considered for computing an optimal assignment of robots to distinct regions using linear programming. The next task is that of leading the robot to their assigned regions. For each robot-region pair if there is a direct path from the robot's position to the regions centric via the free space, the region is accessible to the robot it is assigned to. Otherwise the region is inaccessible. In that case the robot selects a

frontier cell in the accessible region which is nearest to the region assigned to it. Separate distances are defined for reaching to both accessible and inaccessible regions as follows:

- Let us denote the accessible region by  $A$  and the robot by  $r$ . The distance from  $r$  to  $A$  is the distance from  $r$  to the closest cell  $c \in A$  and is a shortest real path distance (minimum cost path in the previously discussed approaches) from  $r$  to all frontier cells in the neighborhood of  $c$ . If  $F_c$  is the neighborhood of  $c$  then the distance is calculated as follows:

$$(r, c) = \min \{ (r, f) \mid f \in F_c \}, c \in A \text{ where function } p \text{ returns the shortest path.}$$

Let us denote the inaccessible region with  $I$ . For region  $I$  geometric distance is defined from  $r$  to  $c$ . If there is an obstacle on a direct line connecting  $r$  and  $c$  a penalty  $\rho$  is added to the distance calculation as follows.

$$d(r, c) = g(r, c) + \rho, c \in I$$

he final objective is to find a robot-region pair which is the minimum of the distances to all regions (say  $AR$ ) as shown below:  $d(r, AR) = \min d(r, c), c \in A \text{ or } c \in I$

After the region assignment goals are assigned to the robots. The robots which are assigned to inaccessible regions have higher precedence. Some other robot another penalty  $\rho_2$  i.e.,  $\mathbb{E}_2(f_i)$  is added. When the robots arrive at their destinations Burgard's algorithm is used for the purpose of exploration within the region. The main difference with the that this approach targets global dispersion. Global dispersion is successfully achieved by penalizing the robots when they choose frontiers that do not belong to their assigned regions or are chosen by other robots. Although, this approach is successful in achieving global dispersion and claims to complete the exploration faster by reducing the regional waiting time, it has few significant limitations. The limitations are:

The K-means algorithm uses Euclidian distance for clustering. The quality of the clusters obtained in a free space (the region without any obstacles and walls) and the region with obstacles of arbitrary size, shapes and walls will be completely different. Suppose the map is a known map, such that, the planner has complete knowledge of the geometry of the obstacles and the walls in the environment. In this situation instead of using traditional K-means algorithm, Geodesic K-means clustering can be used. This only allows the grid cells in neighborhood i.e., those that are not separated by obstacles to be clustered together as shown in Figure 4.4. Since the map is unknown and the size, shape and location of the obstacles and the walls is also not known a prior, geodesic distance cannot be used. This surely results in producing clusters of grid cells that are separated by obstacles as shown in Figure 3. As a result, the robots may have to travel much longer distances. This issue is not addressed by the authors and therefore all the three claims of minimum completion time, minimum total path length travelled by the robots, and minimum variance of regional waiting time are questionable.



The primary reason for the above situation is that the robots are directly sent to explore unknown territories therefore they were not able to exploit the global knowledge which is getting generated as they explore and discover walls and obstacles. Since no re-planning or re-clustering is done the robots have to travel longer distances.

In order to guarantee the completion of exploration task either the individual robots will have to communicate their task completion status and the map information to all the other robots, like or it should be a centralized algorithm. The algorithm sends the robots in far off regions thus it requires long range inter-robot communication. In its present state the algorithm is centralized as it states nothing about who is running the clustering algorithm.

The algorithm is not robust to failure of robots. Let us assume that robots fail with some random probability and one robot survives till the end to finish the exploration task. This approach will have to repeatedly cluster the unknown region every time a robot fails (let us call it iteration). In large maps the volume of frontier cells to be clustered remains high and therefore the algorithm progresses slowly.

At this point it is very important to state that when these algorithms are employed for the purpose of online terrain coverage, they are required to visit each and every frontier cell discovered in the process of execution until no more frontier cells are visible. Therefore, the algorithms are improvised in the manner that they do not ignore any frontier cell. Moreover, each frontier cell is a potential target for coverage and continues to remain a frontier until it is not traversed by a robot. Each frontier cell gets equal opportunity for it to be selected by some robot for coverage. The path planning algorithm executes only when the frontiers are more than two grid cells away from the robot's position.

## RESEARCH MOTIVATION AND CONTRIBUTION

The main motivation behind this work comes from the fact that the potential of reuse of algorithms designed for terrain exploration has not been considered for the purpose of terrain coverage. But it is possible to improvise exploration algorithms and achieve online terrain coverage. The main limitations of previous approaches are:

Many previous approaches which are based on frontier exploration allow the robots to disperse locally within the sensing range of the range sensors. This increases the overlapping sensing impression of the robots which in turn increases the chance of robots selecting nearby target frontiers for exploration. This produces difficulty in managing the exploration process due to the following reasons:

## Interference of sensors.

The robots spend a lot of time in collision avoidance with other robots thus affecting the speed of the exploration algorithm.

In a larger terrain some portion of the terrain will be explored much later than the portion from where the robots have actually started exploring.

These approaches are explicatory in nature. The robots tend to exploit their knowledge to its maximum and explore newer regions in a conservative manner. As a result, exploration becomes slower.

Approaches which are more exploratory in nature target global dispersion. The environment is partitioned into bigger segments which are then assigned to the robots for mutually exclusive exploration or coverage. Various techniques are used for the purpose of partitioning including data clustering algorithms like K-means graph coloring graph partitioning and Voronoi partitioning. These methods have obvious advantage over the pure frontier-based approaches in terms of faster task completion time but also have many challenges:

Fixed partitions: the robots plan longer paths as they do not exploit the knowledge of the structure of the environment that gets generated when they explore or cover the environment. Moreover, the motion plans they use result in redundant coverage.

Fault tolerance: handling robot(s) failure has not been addressed by the above approaches. To detect the same either long range communication is required or the robots should develop consensus and go to a rendezvous point periodically.

Re-planning: is an essential component of a good algorithm. None of these approaches specify strict time criteria for re-planning.

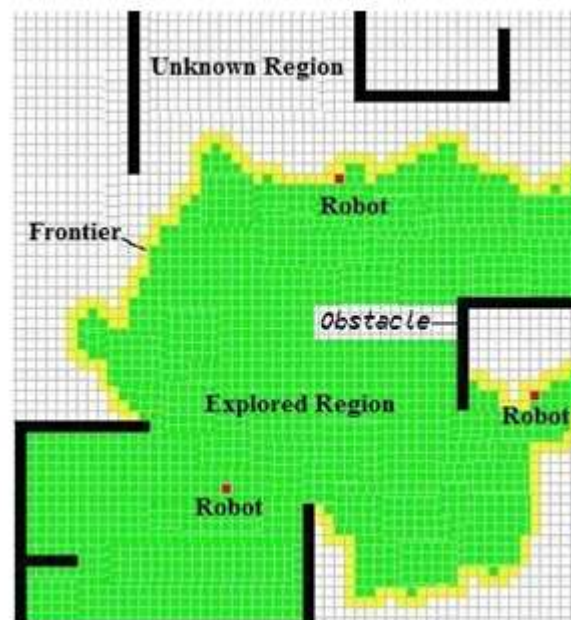
The essence of an effective algorithm should be both exploratory and explicatory. The strategy should be updated in the event of robot malfunctions or when significant new information becomes available. The suggested method develops a suitable mobility plan for the robots based on the context of previously covered borders, which overcomes some of the constraints of the above-mentioned algorithms. Robot dispersion is an emergent characteristic that is necessary for efficient coverage. As a result, the robots are not immediately sent to cover the unknown area. In reality, they are the ones who take use of the information of known border cells first. As the map expands and new borders are found, the robots become more aggressive in their exploration. The suggested method is put to the test in a simulation and on a multi-robot tested. The proposed method outperforms certain state-of-the-art methods and is ideally suited for use in an indoor setting for tasks such as floor cleaning.

## TARGETS FOR TERRAIN COVERAGE

Most of the methods proposed in the literature for multi-robot terrain coverage advise creating a collection of coverage targets, intelligently clubbing these targets together to create task subsets, and assigning these task subsets to the robots. A coverage goal is just some specified location(s) in the environment that a robot must visit. To cover a target, the robot must physically visit the target's area and execute some job, such as cleaning it.

The way the environment is represented has a big effect on how coverage objectives are generated. An occupancy grid is a common depiction that divides the environment into grid cells. The binary Bayes filter is used to assess the status of each cell, whether it is occupied or not. Each cell is given an unknown value at the start. The occupancy of each cell is determined only after the POSTERIOR measurements have been combined, and is classified as either occupied (if the detected region includes the barrier) or free (if it is a free space). One of the first studies uses occupancy grid based maps for robot exploration and considers sensor data to define the area of the map that is sensed and what is still unknown. Frontier cells are cells that lie on the border between known and unknown areas, and they may be excellent targets for the robot to explore. Furthermore, in the exploration job, each cell may be in one of four states: unknown, explored, obstacle, or frontier. Figure 4.2 depicts a frontier-based exploration scenario involving several mobile robots. It is well known that when exploring, the robots scan their surroundings and proclaim any unknown cells within their sensing range to be explored, removing them from consideration as an exploration goal for the robots.

Terrain coverage, on the other hand, necessitates the robots physically traversing the whole free space, whether in the known or unknown area, and therefore an extra flag variable, referred to as visited, is associated with each cell in the free space. When the robot physically crosses a cell, the visited flag of that cell is set to true. As the robots continue their coverage mission, they become more aware of their environment and explore new horizons. The status of all previously identified border cells that have yet to be explored by the robot switches to investigate, but their visited flag stays false. These cells will continue to exist as possible coverage targets.



**Figure 2 Frontiers based exploration**

## CONCLUSION

The idea behind utilizing multi-robot systems to solve complicated issues is to allocate smaller sub-problems to individual robots while enabling them to communicate with one another for knowledge exchange. Geometric pattern creation by numerous mobile robots is a topic that has gotten a lot of interest recently because of its broad range of applications, such as sensor arrays for monitoring the condition of the environment, area exploration, and so on. Individual robots may be given particular responsibilities if they can reach a decentralized agreement on their own and their peers' positions. Researchers in theoretical computer science have looked at the uniform circle formation issue in this regard. For developing distributed systems that have been shown to be sound and comprehensive, simplified assumptions have been examined because these methods have not been tested in the lab, they cannot be compared to other empirical techniques.

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