A CONTRA HARMONIC MEAN LABELING OF DERIVED GRAPHS

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Abstract : A graph $G = (V, E)$ is called a Contra Harmonic mean graph with p vertices and q edges, if it is possible to label the vertices $x \in V$ with distinct element $f(x)$ from $0,1,\dots,q$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ or $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ with distinct edge labels. Then f is a called Contra Harmonic mean labeling of G.

IndexTerms - contra harmonic mean graph, a contra harmonic mean labeling of derived graphs.

I. INTRODUCTION

The field of graph theory plays an important role in various areas of pure and applied sciences. By a graph $G = (V(G), E(G))$ with p vertices and q edges we mean a simple, connected and undirected graph. In this paper, a brief summary of definitions and other information is given in order to maintain compactness. The term is not defined here are used in the sense of Harary [3]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by J.A. Gallian (2016) can be found in [1]. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling). For all other standard terminology and notation we follow Harary [3]. S.Somasundaram and R.Ponraj [4] have introduced the notion of mean labeling of graphs. R. Ponraj and D. Ramya introduced super mean labeling of graphs in [5]. S.S. Sandhya and S. Somasundaram introduced the concept of Harmonic mean labeling in [6] and studied their behavior in [7,8,9]. S.S.Sandhya, S.Somasundaram and J. Rajeshni Golda introduced Contra Harmonic mean labeling of graphs in [2].In this paper, we introduce the concept of Contra Harmonic mean labeling in derived graph and we investigate Contra Harmonic mean labeling in derived graph of some graphs. The following definitions are useful for our present study. The **derived of a simple graph** G denoted by G^+, is the graph having the same vertex set as G, in which two vertices are adjacent if and only if their distance in G is two. A graph $G\ (V, E)$ is called a **Contra Harmonic mean graph** with p vertices and q edges, if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $0,1,\dots,q$ in such a way that when each edge $e = uv$ is labelled with $f(e = uv) = \left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ or $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ with distinct edge labels. Then f is a called Contra Harmonic mean labeling of G. The **n- pan graph** is a graph obtained from the cycle C_n and singleton graph K_1 with the bridge. The $(\boldsymbol{n}, \boldsymbol{m})$ **– tad pole graph** is the graph obtained by joining a cycle C_n to a path P_m with a bridge.

II. MAIN RESULTS FOR SOME STANDARD GRAPHS

Theorem 2.1. The derived graph of cycle $G^\dagger(C_n), \forall n \geq 5$ admits a contra harmonic mean labeling.
Proof: Let G be a cycle C_n of length n, $\forall n \geq 5$.
Let $V(C_n) = \{w_i | 1 \leq i \leq n\}$ and $E(C_n) = \{w_i w_{i+1} | 1 \leq i \leq n-1\} \cup \{w_n w_1\}$
Then the derived graph $G^\dagger(C_n)$ has the following two distinct cases of labeling
Case 1: When n is even
The derived graph $G^\dagger(C_n)$ has 2 copies of cycle each of which has $\frac{n}{2}$ vertices and $\frac{q}{2}$ edges and the vertices of $G^\dagger(C_n)$ are denoted by u_i & v_i respectively, for each $1 \leq i \leq \frac{n}{2}$. First we label the vertices as follows,
Define a function $f: V(G^\dagger) \to \{0,1,\dots,q\}$ by

$$f(u_i) = i, \qquad for\ 1 \leq i \leq \frac{n}{2}$$
$$f(v_1) = 0$$
$$f(v_i) = \frac{q}{2} + i - 1, \qquad for\ 1 < i \leq \frac{n}{2} - 1$$
$$f(v_i) = q, \qquad for\ i = \frac{n}{2}$$

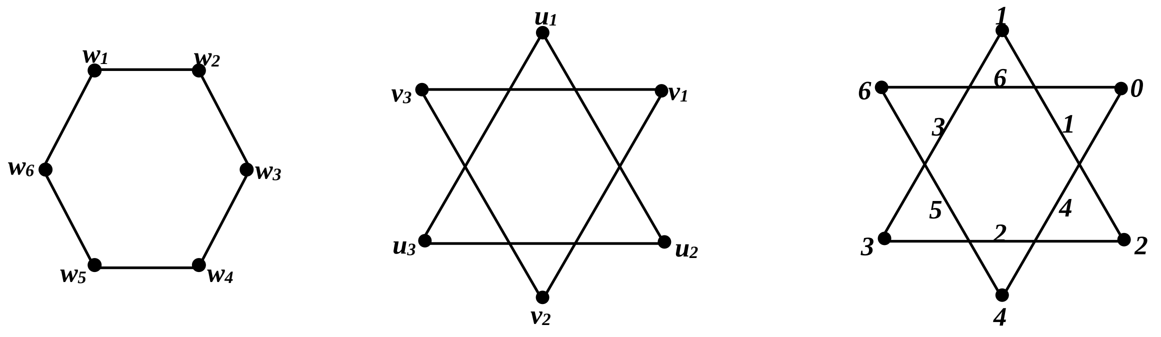
Then the distinct edge labels are,

$$f(u_i u_{i+1}) = i, \qquad for\ 1 \leq i \leq \frac{n}{2} - 2$$
$$f(u_1 u_{n/2}) = \frac{n}{2} - 1,$$
$$f(u_i u_{i+1}) = \frac{n}{2}, \qquad for\ i = \frac{n}{2} - 1$$

$f(v_i v_{i+1}) = \frac{q}{2} + i, \quad for\ 1 \leq i \leq \frac{n}{2}$

Thus f is a contra harmonic mean labeling of $G^{\dagger}$.

Illustration 2.2. When $n = 6$



G Derived graph of G Contra harmonic mean labeling of G

Case 2: When n is odd

The derived graph $G^{\dagger}(C_n)$ is a cycle with n vertices and q edges, the vertices of $G^{\dagger}(C_n)$ is denoted by $v_1, v_2, \ldots, v_n$.

Define a function $f: V(G^{\dagger}) \rightarrow \{0,1, \ldots, q\}$ by

$f(v_i) = i - 1, \quad for\ 1 \leq i \leq n - 1$

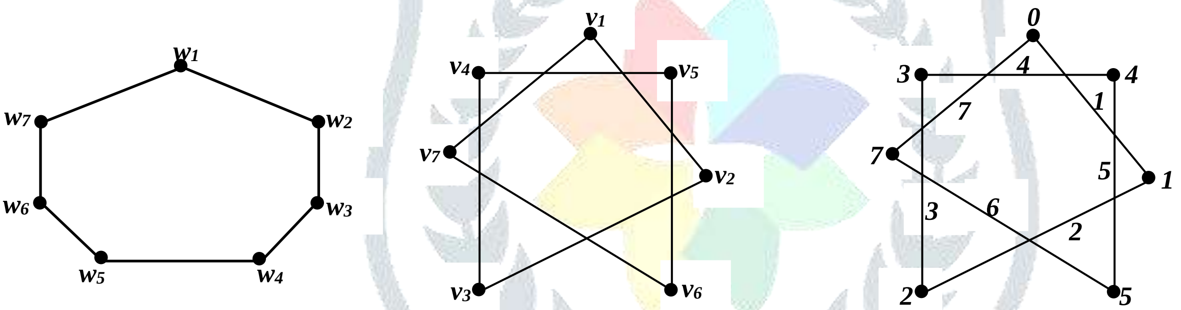
$f(v_i) = n, \quad for\ i = n$

Then the distinct edge labels are,

$f(v_i v_{i+1}) = i, \quad for\ 1 \leq i \leq n$

Thus f is a contra harmonic mean labeling of $G^{\dagger}$.

Illustration 2.3. When $n = 7$



G Derived graph of G Contra harmonic mean labeling of G

The above defined function f provides contra harmonic mean labeling of the derived graph of C_n.

Hence, $G^{\dagger}(C_n)$ is a contra harmonic mean graph.

Theorem 2.4. The derived graph G^{+} of a path P_n $(n \geq 3)$ is not a contra harmonic mean graph.

Proof: Let G be a path $P_n (n \geq 3)$.

Let $V(G) = \{w_i \mid 1 \leq i \leq n\}$ and $E(G) = \{w_i w_{i+1} \mid 1 \leq i \leq n - 1\}$.

If possible, let there be a contra harmonic mean labeling f for a derived graph G^{+} of G.

Case 1: When n is even

Derived graph of G has 2 components each of which has $\frac{n}{2}$ vertices with $\frac{n-1}{2}$ edges.

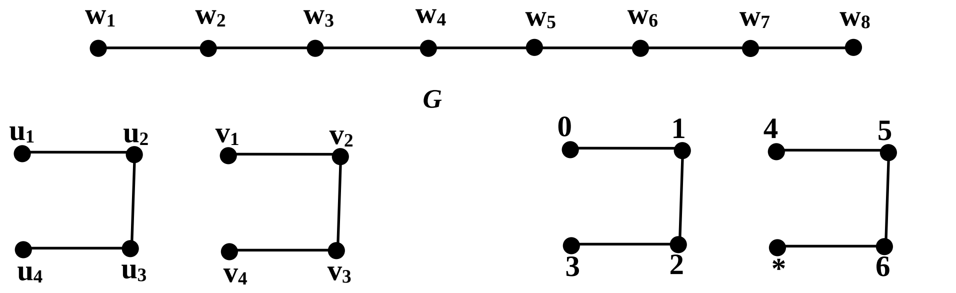
Therefore $V(G^{+}) = \left\{u_i, v_j \mid 1 \leq i \leq \frac{n}{2} \ \&\ 1 \leq j \leq \frac{n}{2}\right\}$ and $E(G^{+}) = \left\{u_i u_{i+1}, v_j v_{j+1} \mid 1 \leq i \leq \frac{n-1}{2} \ \&\ 1 \leq j \leq \frac{n-1}{2}\right\}$.

Define a function $f: V(G^{+}) \rightarrow \{0,1, \ldots, q\}$

Here $q = n - 2$, so $q < n$.

Thus the function f is not a contra harmonic mean labeling of G^{+}.

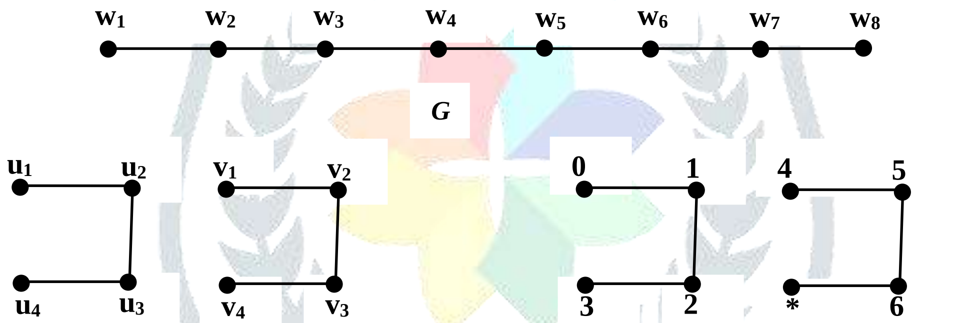
Illustration 2.5. When $n = 8$



Derived graph G+ of G *Contra harmonic mean labeling of G+*

Case 2: When n *is odd*
In this case also derived graph of G has 2 components.
One component has $\frac{n+1}{2}$ vertices with $\frac{n-1}{2}$ edges and
another component has $\frac{n-1}{2}$ vertices with $\frac{n-3}{2}$ edges.
Therefore, $V(G^+) = \left\{u_i, v_j \mid 1 \le i \le \frac{n+1}{2} \;\&\; 1 \le j \le \frac{n-1}{2}\right\}$ and $E(G^+) = \left\{u_i u_{i+1}, v_j v_{j+1} \mid 1 \le i \le \frac{n-1}{2} \;\&\; 1 \le j \le \frac{n-3}{2}\right\}$.
Define a function $f: V(G^+) \rightarrow \{0,1,\dots,q\}$.
Here $q = n-2$ so edges of G^+ is less than the vertices of G^+. Therefore, it is impossible to label the vertices of $G^+(P_n)$. Thus the function f is not a contra harmonic mean labeling of G^+.The above defined function f provides contra harmonic mean labeling of the derived graph of G. Hence, the derived graph G^+ of a path P_n $(n \ge 3)$ is not a contra harmonic mean graph.

Illustration 2.6. When $n = 7$



Derived graph G+ of G *Contra harmonic mean labeling of G+*

Theorem 2.7. The derived graph G^+ of complete graph K_n is not a contra harmonic mean graph.
Proof: Let G be a complete graph K_n.
Let $V(G) = \{v_i \mid 1 \le i \le n\}$ .If possible, let there be a contra harmonic mean labeling of derived graph of G.
Thus the derived graph G^+ of G has n vertices with zero degree. That means derived graph G^+ of G has only isolated vertices.
Let $V(G^+) = \{u_i \mid 1 \le i \le n\}$.
Define a function $f: V(G^+) \rightarrow \{0,1,\dots,q\}$.
Since derived graph of G has only isolated vertices, we have $q = 0$. So this is impossible to label the vertices of G^+. Thus the function f is not a contra harmonic mean labeling of G^+. Hence, the derived graph G^+ of complete graph K_n is not a contra harmonic mean graph.

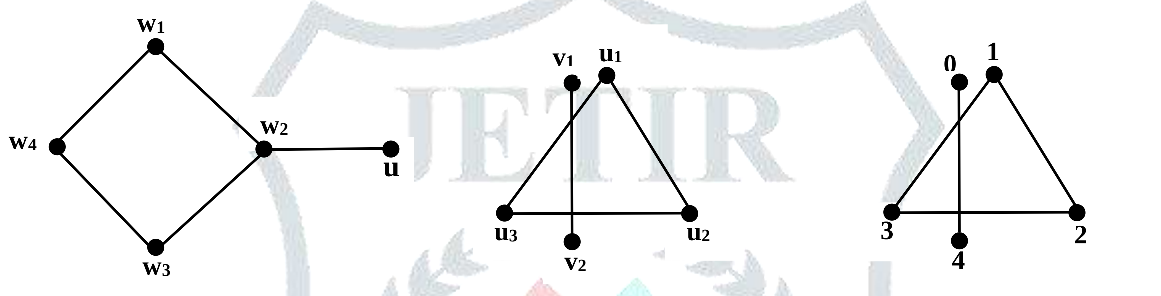
Illustration 2.8. When $n = 4$

G *Derived graph G+ of G*

III. RESULTS FOR SOME SPECIAL GRAPHS

Theorem 3.1. The Derived graph G^+ of n- pan graph $(n \geq 4)$ is a contra harmonic mean graph.
Proof: Let G be an n-pan graph.
Let V (G) = $\{ w_i, u | 1 \leq i \leq n\}$ and E (G) =$\{ w_i w_{i+1} | 1 \leq i \leq n-1\} \cup \{ w_i u \,| for\ some\ i\} \cup \{ w_n w_1\}$.
Then the derived graph G^+ of G has the following cases.
Case 1: When n = 4
If $n = 4$, then the derived graph G^+ of 4- pan graph has a cycle C_3 and a path P_2 and are denoted by u_i $(1 \leq i \leq 3)$ and v_j $(1 \leq j \leq 2)$ respectively.
Define a function f:$V(G^+) \rightarrow \{0,1,\dots,q\}$.
$f(u_i) = i, \quad 1 \leq i \leq 3$
$f(v_1) = 0$
$f(v_2) = q$
Then the distinct edge labels are,
$f(u_i u_{i+1}) = i, \quad 1 \leq i \leq 3$
$f(v_1 v_2) = q$
Thus the function f is a contra harmonic mean labeling ofG^+.

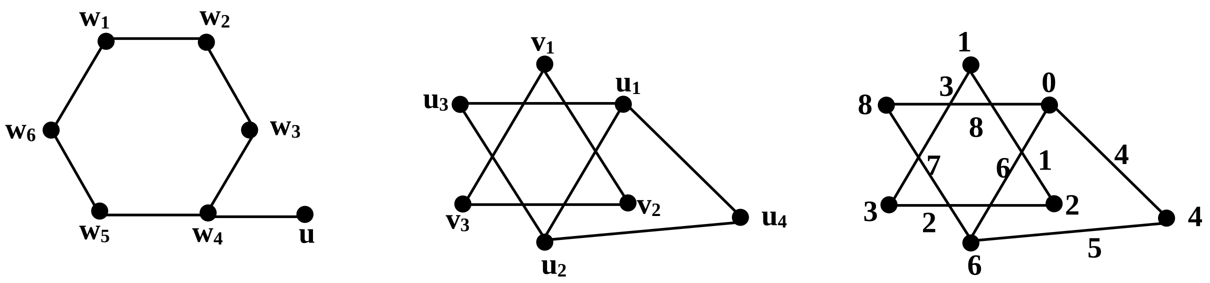
Illustration 3.2. When n = 4



G *Derived graph of G* *Contra harmonic mean labeling of G*

Case 2: When n is even, where $n \geq 5$
If n is even then the derived graph $G^+$of G has 2 copies of cycle each of which has $\frac{n}{2}$ vertices with $\frac{q}{2}$ edges and are denoted by $v_i\ and\ u_i, \forall\ 1 \leq i \leq \frac{n}{2}$ respectively. In that, vertices $u_1 and\ u_2$ of one copy are adjacent to vertex u is in G and this vertex is denoted by $u_{i+1}, i = \frac{n}{2}$.
Define a function f:$V(G^+) \rightarrow \{0,1,\dots,q\}$.
$f(v_i) = 1, \quad 1 \leq i \leq \frac{n}{2}$
$f(u_1) = 0$
$f(u_i) = \frac{q}{2} + i\,, \quad 1 < i \leq \frac{n}{2} - 1$
$f(u_i) = q\,, \quad i = \frac{n}{2}$
$f(u_i) = \frac{q}{2}\,, \quad i = \frac{n}{2} + 1$
Then the distinct edge labels are,
$f(v_i v_{i+1}) = i, \quad 1 \leq i \leq \frac{n}{2}$
$f(u_i u_{i+1}) = \frac{q}{2} + i + 1, \quad 1 \leq i \leq \frac{n}{2}$
$f(u_i u_1) = \frac{q}{2}, \quad i = \frac{n}{2} + 1$
$f(u_i u_2) = \frac{q}{2} + 1, \quad i = \frac{n}{2} + 1$
Thus the function f is a contra harmonic mean labeling of G^+.

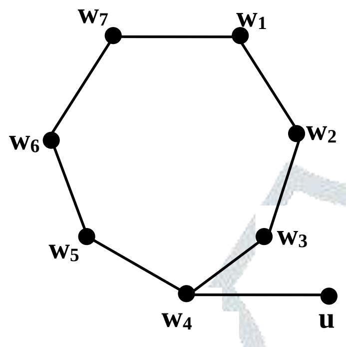
Illustration 3.3. When n = 6 & $m = 1$



Case 3: When n is *G* *Derived graph of G* *Contra harmonic mean labeling of G*

If n is odd, then the derived graph G^+ of G has a cycle which has n vertices with q edges and is denoted by $u_i \ for\ 1 \leq i \leq n$. In that, cycle $u_1 and\ u_2$ vertices are adjacent to the vertex u is in G and this vertex is denoted by v_1.
Define a function f: $V(G^+) \rightarrow \{0,1,\ldots,q\}$
$f(u_i) = i, \quad 1 \leq i \leq n$
$f(u_i) = n, \quad i = n$
$f(v_1) = q$
Then the distinct edge labels are,
$f(u_i u_{i+1}) = i, \quad 1 \leq i \leq n$
$f(u_1 v_1) = q$
$f(u_2 v_1) = q - 1$
Thus the function f is a contra harmonic mean labeling of G^+.

Illustration: **3.4** when n = 7 & $m = 1$



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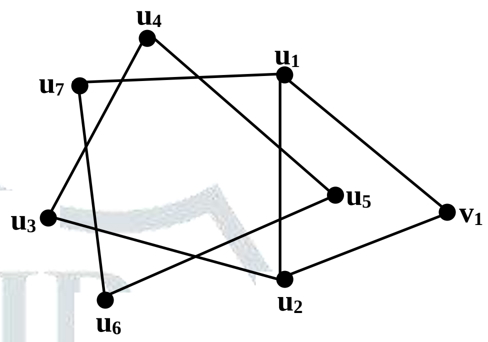
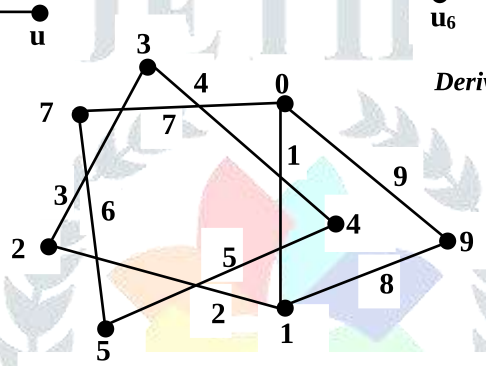


Derived graph G⁺ of G



Contra harmonic mean labeling of G^+

The above defined function f provides contra harmonic mean labeling of the derived graph of G. Hence, the derived graph G^+ of n- pan graph $(n \geq 4)$ is a contra harmonic mean graph.

Theorem 3.5. The derived graph of (n,m)-tadpole graph is a contra harmonic mean graph.
Proof: Let G be the (n,m)-tadpole graph,$\forall n \geq 3, m \geq 1$.
Let $V(G) = \{n_i,, m_j | 1 \leq i \leq n, 1 \leq j \leq m\}$
and $E(G) = \{n_i n_{i+1} | 1 \leq i \leq n-1\} \cup \{m_j m_{j+1} | 1 \leq j \leq m\} \cup \{n_n n_1\} \cup \{n_i m_1 | for\ some\ i\}$.
Then the derived graph of (n,m)-tadpole graph has the following cases;
Case 1: When $n \geq 4 and m = 1$,(n,m)-tadpole graph is a contra harmonic mean graph.
This result is follows from the above theorem.3.1.
Case 2: $When\ n = 4\ and\ m \geq 2$.
Sub case (i): $n = 4\ and\ m = 2$.
We have the derived graph $G^\dagger$ of (4,2)-tadpole graph has a cycle C_3 and the path P_3 whose vertices are denoted by $u_i\ and\ v_i$,$1 \leq i \leq 3$.
Define a function $f: V(G^\dagger) \rightarrow \{0,1,\ldots,q\}$
$f(u_i) = i, \quad for\ 1 \leq i \leq 3,$
$f(v_1) = 0,$
$f(v_i) = 2 + i, \quad for\ 2 \leq i \leq 3.$
Then the distinct edge labels are,
$f(u_i u_{i+1}) = i, \quad for\ 1 \leq i \leq 3,$
$f(v_i v_{i+1}) = 3 + i, \quad for\ 1 \leq i \leq 2.$
Thus f is a contra harmonic mean labeling of $G^\dagger$.

Sub case (ii): When $n = 4\ \&\ m \geq 3$.

Then we have the derived graph $G^{\dagger}$ of $(3,p)$-tadpole graph, where $p = \left\lceil\frac{n+m}{2}\right\rceil - 3$ and a path with $\left\lfloor\frac{n+m}{2}\right\rfloor$ vertices. The vertices of $(3,p)$-tadpole graph is denoted by u_i ,$1 \le i \le 3$ and v_i ,$1 \le i \le \left\lceil\frac{n+m}{2}\right\rceil - 3$. The vertices of path are denoted by w_i, $1 \le i \le \left\lfloor\frac{n+m}{2}\right\rfloor$.

Define a function $f: V(G) \rightarrow \{0,1,2,\dots,q\}$,

$f(u_i) = i, \quad for\ 1 \le i \le 3$

$f(v_i) = i + 3, \quad for\ 1 \le i \le \left\lceil\frac{n+m}{2}\right\rceil - 3$

$f(w_1) = 0$

$f(w_i) = (i-1) + \left\lceil\frac{n+m}{2}\right\rceil, \quad for\ 2 \le i \le \left\lfloor\frac{n+m}{2}\right\rfloor$

Then the distinct edge labels are,

$f(u_i u_{i+1}) = i, \quad for\ 1 \le i \le 3$

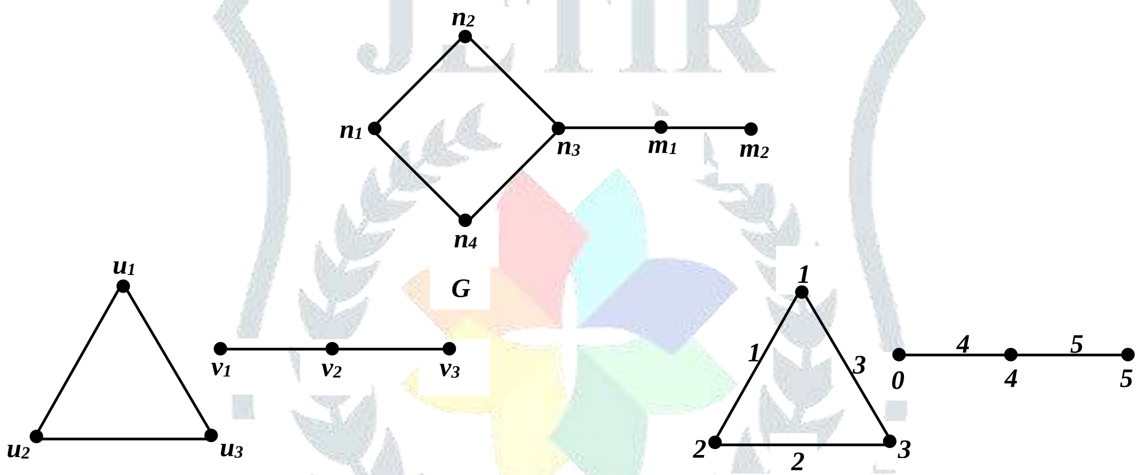
$f(u_3 v_1) = 4$

$f(v_i v_{i+1}) = i + 4, \quad for\ 1 \le i \le \left\lfloor\frac{n+m-1}{2}\right\rfloor - 3$

$f(w_i w_{i+1}) = i + \left\lceil\frac{n+m}{2}\right\rceil, \quad for\ 1 \le i \le \left\lfloor\frac{n+m}{2}\right\rfloor - 1$

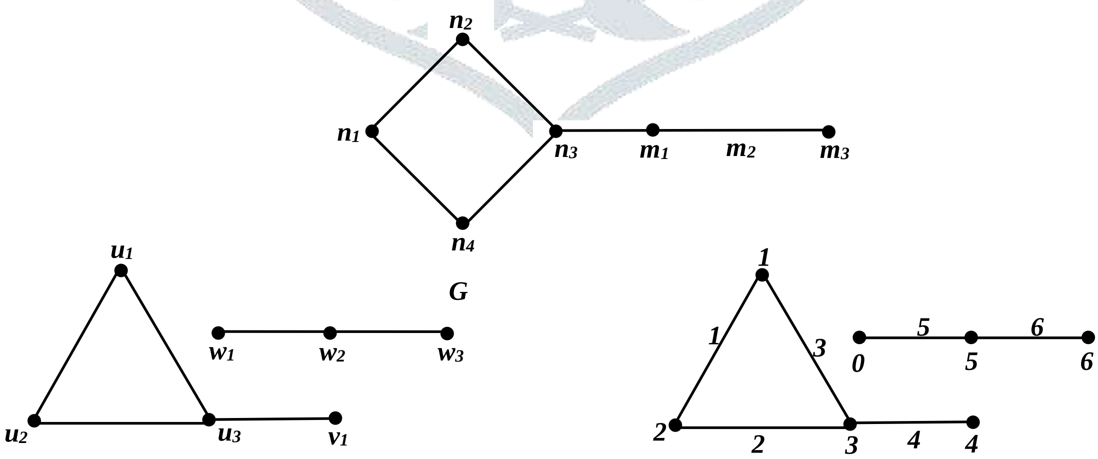
Thus f is a contra harmonic mean labeling of $G^{\dagger}$.

Illustration 3.6. when $n = 4$ & $m = 2$



Derived graph of G *Contra harmonic mean labeling of G⁺*

Illustration 3.7. when n $= 4$ & $m = 3$



Derived graph of G *Contra harmonic mean labeling of G⁺*

Case 3: When n is even

Then we have the derived graph $G^{\dagger}$ of (n,m)-tadpole graph has 2 components. One is $\left(\frac{n}{2},p\right)$-tadpole graph where $p=\left\lfloor\frac{m}{2}\right\rfloor$ and vertices are denoted by $v_i\left(1\le i\le\frac{n}{2}\right)$ and $x_i\left(1\le i\le\left\lfloor\frac{m}{2}\right\rfloor\right)$ another one is cycle of which has $\frac{n}{2}$ vertices and $\frac{q}{2}$ edges denoted by $u_i\left(1\le i\le\frac{n}{2}\right)$ and the vertices $u_1 and\, u_2$ are adjacent to vertex $k_1$is followed by a path if $m\ge 3$ having vertices $\left\lceil\frac{m}{2}\right\rceil$ and is denoted by $w_i\left(1\le i\le\left\lceil\frac{m}{2}\right\rceil\right)$.

Define a function$f: V(G^{\dagger})\to\{0,1,2,\dots,q\}$

$f(v_i)=i,\quad for\ 1\le i\le\frac{n}{2}$

$f(u_1)=0$

$f(u_i)=\left\lfloor\frac{q}{2}\right\rfloor+i,\ \ for\ 2\le i\le\frac{n}{2}-1$

$f(u_i)=q,\quad for\ i=\frac{n}{2}$

$f(w_i)=i+3,\ \ for\ \ 1\le i\le\left\lceil\frac{m}{2}\right\rceil$

$f(x_i)=q-\left\lfloor\frac{m}{2}\right\rfloor+(i-1),\quad for\ 1\le i\le\left\lfloor\frac{m}{2}\right\rfloor$

Then the distinct edge labels are,

$f(v_iv_{i+1})=i,\quad for\ 1\le i\le\frac{n}{2}$

$f(u_{i-1}u_i)=\left\lfloor\frac{q}{2}\right\rfloor+i,\quad for\ 2\le i\le\frac{n}{2}-1$

$f(u_iu_{i+1})=q-2,\quad for\ i=\frac{n}{2}-1$

$f(u_iu_1)=q,\quad for\ i=\frac{n}{2}$

$f(u_1w_1)=4$

$f(u_2w_1)=\left\lfloor\frac{q}{2}\right\rfloor+1$

$f(v_2x_1)=q-\left\lceil\frac{m+1}{2}\right\rceil$

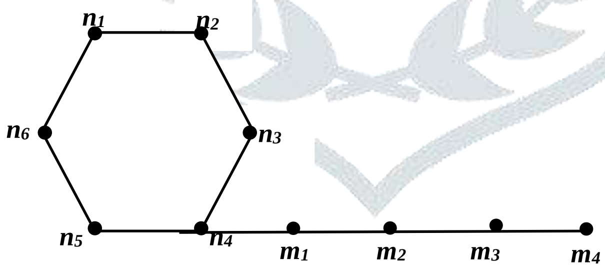
$f(w_iw_{i+1})=i+4,\quad for\ 1\le i\le\left\lceil\frac{m-1}{2}\right\rceil$

$f(x_ix_{i+1})=q-\left\lfloor\frac{m}{2}\right\rfloor+(i-1), for\ 1\le i\le\left\lfloor\frac{m}{2}\right\rfloor-2$

$f(x_ix_{i+1})=q-1,\quad for\ i=\left\lfloor\frac{m}{2}\right\rfloor-1$

Thus f is a contra harmonic mean labeling of $G^{\dagger}$.

Illustration 3.8. when $n=6\ \&\ m=4$



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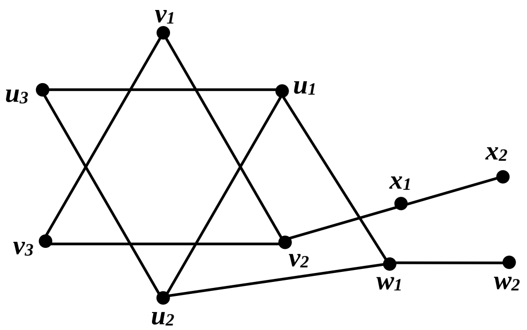
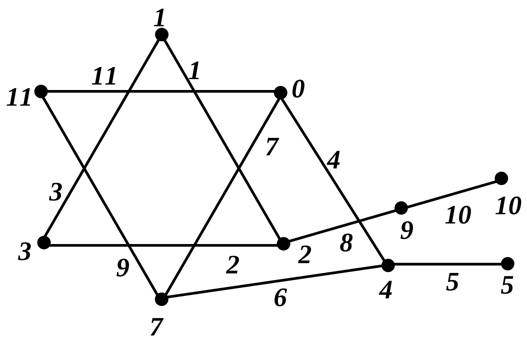


Derived graph of G



Contra harmonic mean labeling of G

Case 4: When n is odd

We have the derived graph $G^{\dagger}$ of (n,m)-tadpole graph has cycle which has n vertices and q edges denoted by $u_i(1 \le i \le n)$. In this cycle u_1 & u_2 vertices are adjacent to the vertex $k_1$and that k_1 followed by a path if $m \ge 3$ having $\left\lceil \frac{m}{2} \right\rceil$ vertices denoted by $w_i\left(1 \le i \le \left\lceil \frac{m}{2} \right\rceil\right)$. Again in this cycle $\left(\left\lceil \frac{n}{2} \right\rceil + 1\right)^{th}$vertex and a path with $\left\lfloor \frac{m}{2} \right\rfloor$ vertices and they are join by a bridge denoted by $v_i\left(1 \le i \le \left\lfloor \frac{m}{2} \right\rfloor\right)$.

Define a function $f: V(G^{\dagger}) \to \{0,1,2, \dots, q\}$

$f(u_i) = i - 1, \qquad for\ 1 \le i \le n-1$

$f(u_i) = n, \qquad for\ i = n$

$f(v_i) = q + i - m, \qquad for\ 1 \le i \le \left\lfloor \frac{m}{2} \right\rfloor$

$f(w_i) = q + i - \left\lceil \frac{m}{2} \right\rceil, \qquad for\ 1 \le i \le \left\lfloor \frac{m+1}{2} \right\rfloor$

Then the distinct edge labels are,

$f(u_i u_{i+1}) = i, \qquad for\ 1 \le i \le n$

$f(u_1 w_1) = q + 1 - \left\lceil \frac{m}{2} \right\rceil$

$f(u_2 w_1) = q - \left\lceil \frac{m}{2} \right\rceil$

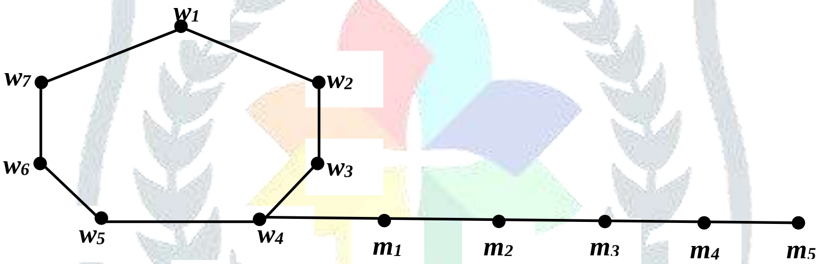
$f(u_i v_1) = q - m, \qquad for\ i = \left\lceil \frac{n}{2} \right\rceil$

$f(w_i w_{i+1}) = q + i - \left\lfloor \frac{m-1}{2} \right\rfloor, for\ 1 \le i \le \left\lfloor \frac{m-1}{2} \right\rfloor$

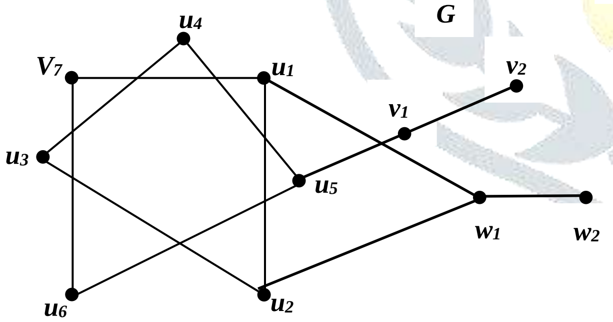
$f(v_i v_{i+1}) = q - m + i, for\ 1 \le i \le \left\lfloor \frac{m}{2} \right\rfloor - 1$

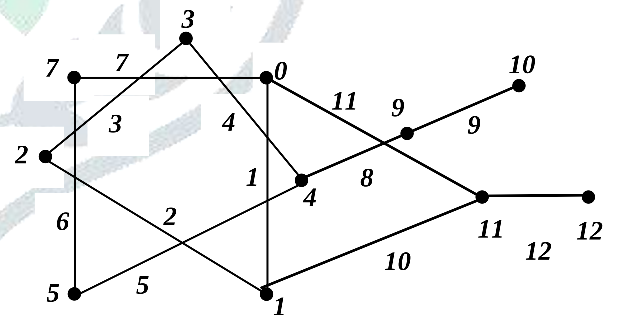
Thus f is a contra harmonic mean labeling of $G^{\dagger}$.

Illustration 3.9. When $n = 7$ & $m = 5$



G





Derived graph of G *Contra harmonic mean labeling of G*

The above defined function f provides contra harmonic mean labeling of the derived graph of G. Hence, the derived graph of (n,m)-tadpole graph is a contra harmonic mean graph.

IV. CONCLUSION

The Study of labeled graph is important due to its diversified applications. It is very interesting to investigate graphs which admit Contra Harmonic Mean Labeling of Derived Graphs. In this paper, we proved that Path, cycle, complete graph, n-pan graph and (n,m)-tad pole graph are Contra Harmonic Mean Labeling of derived Graph or not. The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

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