A new algorithm for solving single valued Trapezoidal Neutrosophic Shortest Path Problems

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Abstract: Shortest path problem is one of the most important and well-known problem that appears in various fields of science, engineering etc. The main objective of the shortest path problem is to find a path with minimum length or cost between any two vertices. The edge length of the network may represent many real life situations which includes uncertainity, incompleteness, indeterminate and inconsistent information in the real world. To deal with all these real life situations we use the concept of neutrosophic sets and logic. But in real life situations we are not certain about the quantities used. There may exist uncertainity always. Here in this paper we are proposing a neutrosophic network method for finding the shortest path between the nodes using single valued trapezoidal neutrosophic numbers.

Key Words: Floyd's algorithm, single valued trapezoidal neutrosophic number, shortest path problem.

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1.INTRODUCTION

The concept of the Neutrosophic set and Neutrosophic logic were introduced by Smarandache[1,2] in order to handle many real life situations such as indeterminacy , incompleteness etc. Neutrology is a new branch of philosophy that deals with the origin, nature and scope of neutralities. The concept of neutrosophic set can generalize many of the existing logic such as Fuzzy sets[3],intuitionistic fuzzy sets[4],interval valued fuzzy sets[5] etc. The main aim of neutrosophic logic is to characterize the logical statement in 3D space, with each dimension representing the truth-membership function, the indeterminacy –membership function and the falsity membership function, which are within the real standard or non-standard unit interval]0^{-},1^{+}[. But the neutrosophic set as itself is very difficult to apply in real science and engineering areas. SoWanetal[7] proposed the concept of SVNS where functions of truth, indeterminacy and falsity lie in [0,1].

A paper based on the neutrosophic set theory Babas[9] presented the concept of triangular and trapezoidal neutrosophic numbers and applied to multiple-attribute decision making problems Deli and Subas[11] presented the single valued trapezoidal neutrosophic numbers as a generalization of the intuitionistic trapezoidal fuzzy numbers and proposed a methodology for solving multiple attribute decision making problems with SVN numbers. In addition ThamaraSelvi and Santhi[12] introduced a mathematical representation of a transportation problems in neutrosophic environment based on single valued trapezoidal neutrosophic numbers and also provided the solution method.

Shortest path problem is a problem of finding shortest path between two vertices, so that the sum of the weight of their corresponding edge is minimized. It is one of the most fundamental and well known combinatorial problems that appear in various field of science and engineering viz, road network application, transportation etc. In a classic shortest path problem, the distance of the edge between different nodes of a network are assumed to be certain. Numerous algorithms have been developed where weight edges on network being fuzzy numbers, intuitionistic fuzzy numbers, vague numbers etc[13-16]. Dijkstra's algorithm, Floyd's algorithm are different methods to find the shortest path or distance between the nodes. Here we are applying Floyd's algorithm to find the shortest path between the nodes, where the distance between the nodes are represented by single valued trapezoidal neutrosophic numbers.

SECTION 2: Definitions based on neutrosophic sets and single valued trapezoidal neutrosophic numbers

SECTION 3: New version of Floyd's algorithm for solving shortest path problem with connected edges using trapezoidal neutrosophic data

SECTION 4: Practical example of Floyd's algorithm using trapezoidal neutrosophic data

SECTION 5: Conclusions

II.PRELIMINARIES

In this section a review of neutrosophic sets, single valued neutrosophic sets and single valued trapezoidal neutrosophic sets have been explained and discussed how to solve Floyd's algorithm when the values are single valued trapezoidal numbers.

Definition2.1: Let A be a set of points with generic element in A denoted by x. Then the neutrosophic set K is an object having the form K=$\{< x: T_k(x), I_k(x), F_k(x); x \in A\}$ where the functions T,I and F are the truth-membership function, indeterminacy function and falsity membership function of the element x∈A to the set K with the condition

$$-0 \leq T_k(x) + I_k(x) + F_k(x) \leq 3^+$$

The functions $T_k(x), I_k(x), F_k(x)$ are real standard or non-standard subsets of $]^-0,1+[$.

Definition 2.2 : Let x be a set of points with generic elements in $\hat{A}$ denoted by x. A single valued neutrosophic set K is characterized by truth membership function $T_k(x)$, an indeterminacy membership function $I_k(x)$ and a falsity-membership function $F_k(x)$.For each x in $\hat{A}$, $T_k(x), I_k(x), F_k(x) \in [0,1]$. A single valued neutrosophic number can be written as

K= $\{< x: T_k(x), I_k(x), F_k(x); x \in A\}$

Definition 2.3: A single valued trapezoidal neutrosophic number $\hat{A}$= < (x_1, x_2 ,x_3,x_4);T_k, I_k, F_k> is a special neutrosophic set on the real number set R, whose truth membership function, indeterminacy function, and falsity membership function are given as follows

$$T_k(x) = f(x) = \begin{cases} \frac{(x-x_1)T_k}{x_2-x_1}, & (x_1 \le x \le x2) \\ T_k \quad , & (x2 \le x \le x3) \\ \frac{(x_4-x)T_k}{(x_4-x_3)}, & (x3 \le x \le x4) \\ 0 & \text{Otherwise} \end{cases}$$

$$I_k(x) = f(x) = \begin{cases} \frac{(x_2-x\ \ )+I_k(x-x_1)}{x_2-x_4}, & (x_1 \le x \le x2) \\ I_k \quad , & (x2 \le x \le x3) \\ \frac{(x-x_3)+I_k(x_4-x)}{(x_4-x\ \ )}, & (x3 \le x \le x4) \\ 1 & \text{Otherwise} \end{cases}$$

$$F_k(x) = f(x) = \begin{cases} \frac{[(x_2-x)+F_k(x-x_1)]}{x_2-x_1}, & (x_1 \le x \le x2) \\ F_k \quad , & (x2 \le x \le x3) \\ \frac{(x-x_3)+F_k(x_4-x)]}{(x_4-x_3)}, & (x3 \le x \le x4) \\ 1 & \text{Otherwise} \end{cases}$$

When $0\le T_k \le 1$

When $0\le I_k \le 1$

When $0\le F_k \le 1$ & $0\le T_k + I_k + F_k \le 3$; x_1 , x_2 , x_3 , x_4 € R

Definition 2.4: Let $\tilde{A}_1$=< (x_1, x_2, x_3, x_4);T_1, I_1, F_1> and $\tilde{A}_2$= < (y_1, y_2, y_3, y_4);T_2, I_2, F_2> be two single valued trapezoidal neutrosophic numbers. Then the operation of SVTN are defined as follows.

(i)$\tilde{A}_1 \oplus \tilde{A}_2$ = {<x_1+y_1 , x_2+y_2 ,x_3+y_3, x_4+y_4>; min(T_1,T_2); max(I_1,I_2); max(F_1,F_2)}

(ii)$\tilde{A}_1 \otimes \tilde{A}_2$ = {<x_1y_1 , x_2y_2 ,x_3y_3, x_4y_4>; min(T_1,T_2); max(I_1,I_2); max(F_1,F_2)}

(iii)$\lambda\tilde{A}_1$ ={<λx_1 , λx_2,λx_3, λx_4>; min(T_1,T_2); max(I_1,I_2); max(F_1,F_2)}

A convenient method for comparing of single valued trapezoidal neutrosophic number is by use of score function

Definition 2.5: Let $\tilde{A}_1$=<(x_1, x_2, x_3, x_4);T_1, I_1, F_1> be a single valued trapezoidal neutrosophic number. Then the score function $\tilde{A}_1$=< (x_1, x_2, x_3, x_4);T_1, I_1, F_1>be a single valued trapezoidal neutrosophic numbers. Then the score function S($\tilde{A}_1$) and accuracy function($\tilde{A}_1$) of a SVTN numbers are as follows;

$$S(\tilde{A}_1)=\frac{1}{12}[(a_1+a_2+a_3+a_4)][2+T_1-I_1-F_1]$$

$$a(\tilde{A}_1)=\frac{1}{12}[(a_1+a_2+a_3+a_4)][2+T_1-I_1+F_1]$$

Arithmetic operations between two single valued trapezoidal neutrosophic numbers. A slight modified form of single valued trapezoidal neutrosophic number proposed by Deli and Subas [11]is given below.

Let $\tilde{A}_1$=< (x_1, x_2, x_3, x_4);T_1, I_1, F_1> and $\tilde{A}_2$=< (y_1, y_2, y_3, y_4); T_2, I_2, F_2> be two single valued trapezoidal neutrosophic numbers. Then the operation of SVTN are defined as follows.

(i)$\tilde{A}_1 \oplus \tilde{A}_2$ =<x_1+y_1 , x_2+y_2 ,x_3+y_3, x_4+y_4>; $T_1+T_2-T_1T_2$, I_1I_2, F_1F_2)

(ii)$\tilde{A}_1 \otimes \tilde{A}_2$ =<x_1y_1 , x_2y_2 ,x_3y_3, x_4y_4>;T_1T_2, $I_1+I_2-I_1I_2$, $F_1+F_2-F_1F_2$)

(iii)$\lambda\tilde{A}_1$ =<λx_1 , λx_2,λx_3, λx_4>;$1-(1-T_1)^{\lambda}$, I_1^{λ}, F_1^{λ})

Network Terminology:

Consider a directed network G= (V, E) consisting of a finite set of nodes V={1,2,3,4,…..n } and a set of m directed edges E $\subseteq$ VxV. Each edge is denoted by an ordered pair (i, j) where i, j$\in$V and i$\neq$j. In this network we specify two nodes, denoted by s and t, which are the source node and destination node respectively. We define a path as a sequence P_{ij}= {i=i_1, (i_1, i_2), i_2 ……(i_{l-1}, i_l) i_l=j } of alternating nodes and edges. The existence of atleast one path P_{si} in G(V,E) is assumed for every i$\in$V-{s}. d_{ij} denote a single valued trapezoidal neutrosophic number associated with the edge (i, j), corresponding to the length necessary to transverse (i, j) from i to j. In real problems the length corresponds to the cost, the time and the distance etc. Then the neutrosophic distance along the path p is denoted as

$$d(p)=\sum_{i,j\in P} d_{ij} .$$

III.SINGLE VALUED TRAPEZOIDAL FLOYD'S ALGORITHM

Floyd's algorithm is more general than Dijkstra's algorithm because it determines the shortest route between any two nodes in the network. Here most of the routes are undirected. In this paper the distance or weights are represented by single valued trapezoidal neutrosophic numbers.

The algorithm represents an 'm' node network as a square matrix with 'm' rows and 'm' columns. The distance from x to y is denoted by C_{xy} that is the entry (x, y) of the matrix

(i) if x is directly linked to y the distance is finite.

(ii) Otherwise it is infinite.

Given three nodes x, y, z in the figure, we want to find the shortest distance from x to z. Here x is directly connected to z by the distance C_{xy} but while we are calculating we found that it is shorter to reach z from x passing through y, if $\mathbf{C_{xy} + C_{yx} < C_{xz}}$.

In this case it is optimal to replace the direct route from X $\rightarrow$ Z with the indirect route

x $\rightarrow$ y $\rightarrow$ z. The triple operation network is applied systematically to the network.

General procedure is:

Step 1: Let us start by defining the starting matrix C_0 and node sequence matrix S_0 as given below. The diagonal elements are blocked since no loops are allowed. Diagonal cells are marked with (-) symbol.

Set k = 1

Define the matrices as:

C_0	1	2	3	….	y	……	m
1	-	C_{12}	C_{13}	…..	C_{1y}	……	C_{1m}
2	C_{21}	-	C_{23}	…..	C_{2y}	……	C_{2m}
3	C_{31}	C_{32}	-	…..	C_{3y}	…..	C_{3m}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
X	C_{x1}	C_{x2}	C_{x3}	…..	-	…...	C_{xm}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
M	C_{m1}	C_{m2}	C_{m3}	…..	C_{my}	…..	-

Table 1: General distance matrix C_0

S_0	1	2	3	…..	y	….	M
1	-	2	3	…..	y	….	M
2	1	-	3	…..	y	….	M
3	1	2	-	…..	y	….	M
:	:	:	:	-	:	….	:
X	1	2	3	…..	-	….	M
:	:	:	:	…..	:	-	:
M	1	2	3	…..	y	….	-

Table 2: Node sequence matrix S_0

General step k:

Define row k and column k as pivot row and pivot column and apply the triple operation to each element C_{xy} in C_{k-1} for all x and y. If the condition

$C_{xk} + C_{ky} < C_{xy}$ $(x \neq k, y \neq k, z \neq y)$ is satisfied then

(a) Create C_{ky} by replacing C_{xy} in C_{k-1} with $C_{xk} \neq C_{ky}$

(b) Create S_k by replacing S_{xy} in S_{k-1} with k.

(c) Set k = k+1 and repeat step K.

Note: If the sum of the elements on the pivot row and pivot column is smaller than the corresponding intersection element, then it is optimal to replace the intersection element, by the sum of the pivot elements. After these m steps, we can determine the shortest route between x and y using the following rules from the matrices C_m and $S_{m.}$

(i) From C_k and C_{xy} we get the shortest distance between nodes x and y.

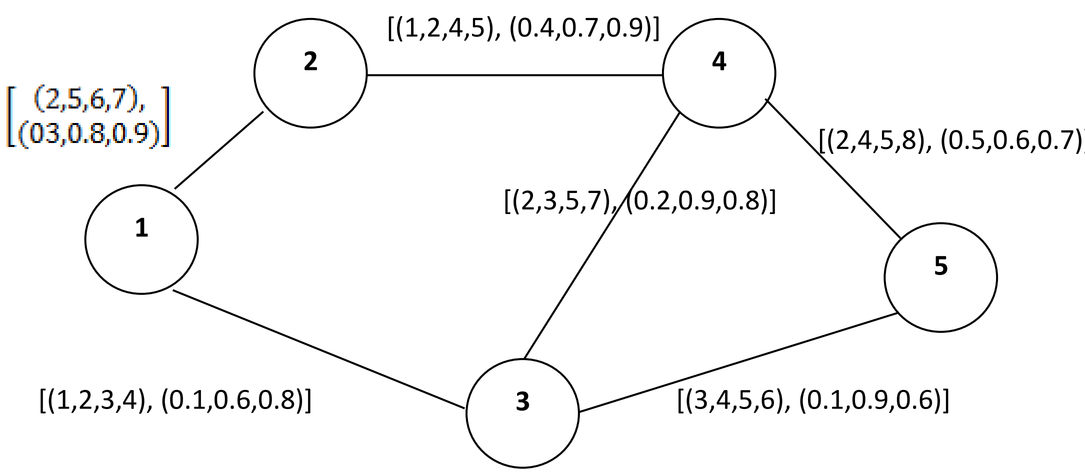
(ii) From S_n determine the intermediate node $k = S_{xy}$ that yields the route $x \rightarrow k \rightarrow y$

If $S_{xk} = k$ and $S_{ky} = y$, then stop the process otherwise repeat the process between nodes x and k and between nodes k and y.

IV.ILLUSTTRATIVE EXAMPLES

Find the shortest route between every two nodes. The Distance are represented by single value trapezoidal neutrosophic numbers.

Let us now apply the proposed algorithm to the network given in figure below:



Step 1: First let us convert the single valued trapezoidal neutrosophic numbers into a single digit for comparison by using the Score Function

$S(B)=\frac{1}{12}[(a_1+a_2+a_3+a_4)][2+T_1-I_1-F_1]$

Construct Matrices C_0&S_0, using the corresponding Score function. In the initial matrix C_{14}, C_{15} , C_{32} , C_{23} , C_{51} , C_{52} , C_{25} have no direct connections, so we can substitute all these cells as infinity.

S[(2,5,6,7)(0.3,0.8,0.9)] = $\frac{1}{12}$(20) (0.6) = 1

S[(1,2,4,5),(0.4,0.7,0.9)] = $\frac{1}{12}$(12) (0.8) = 0.8

S[(2,4,5,8),(0.5,0.6,0.7))] = $\frac{1}{12}$(19) (1.2) = 1.9

S[(1,2,3,4),(0.1,0.6,0.8)] = $\frac{1}{12}$(10) (0.7) = 0.58

S[(3,4,5,6),(0.1,0.9,0.6)]= $\frac{1}{12}$(18) (0.6) = 0.9

S[(2,3,5,7), (0.2,0.9,0.8)]= $\frac{1}{12}$(17)(0.5) = 0.70

C_0	**1**	**2**	**3**	**4**	**5**
1	-	1	0.58	∞	∞
2	1	-	∞	0.8	∞
3	0.58	∞	-	0.7	0.9
4	∞	0.8	0.7	-	1.9
5	∞	∞	0.9	1.9	-

S_0	**1**	**2**	**3**	**4**	**5**
1	-	2	3	4	5
2	1	-	3	4	5
3	1	2	-	4	5
4	1	2	3	-	5
5	1	2	3	4	-

Step 2: Set k=1 in the above matrix the lightly shaded first row and first row represent the Pivot row and pivot column. The dark cells C_{23}, C_{32} are the only cells that can be improved by triple operation. Thus C_1&S_1 are obtained by the operation

Replace C_{23} with C_{21}+C_{13} = 1+0.58 = 1.58 & set s_{23} = 1

Replace C_{32} with C_{31}+C_{12} = 0.58+1 = 1.58 & set s_{32} = 1

These changes are shown in the next matrix.

C1	1	2	3	4	5
1	-	1	0.58	∞	∞
2	1	-	**1.58**	0.8	∞
3	0.58	**1.58**	-	0.7	0.9
4	∞	0.8	0.7	-	1.9
5	∞	∞	0.9	1.9	-

S1	1	2	3	4	5
1	-	2	3	4	5
2	1	-	**1**	4	5
3	1	**1**	-	4	5
4	1	2	3	-	5
5	1	2	3	4	-

Step3: Set k=2; here the Pivot row and Pivot column are the second row and second column, the darker cells C_{14}&C_{41} can be improved by triple operation. The resulting changes are shown bold in C_2&S_2.

Replace $C_{14}$with C_{12}+C_{24} = 1+0.8 = 1.8 & set s_{14} = 2

Replace $C_{41}$with C_{42}+C_{21} = 0.8+1 = 1.8 & set s_{41} = 2

These changes are shown in the next matrix.

C_2	1	2	3	4	5
1	-	1	0.58	**1.8**	∞
2	1	-	1.58	0.8	∞
3	0.58	1.58	-	0.7	0.9
4	**1.8**	0.8	0.7	-	1.9
5	∞	∞	0.9	1.9	-

S_2	1	2	3	4	5
1	-	2	3	**2**	5
2	1	-	1	4	5
3	1	1	-	4	5
4	**2**	2	3	-	5
5	1	2	3	4	-

Step 4: Set k=3, the pivot row & pivot column are 3rd column. The darkened cells C_{14} , C_{15} , C_{25} ,C_{41} , C_{51} , C_{52} can be improved by triple operation. Thus C_3&$S_3$are obtained by the operations

Replace $C_{14}$with C_{13}+C_{34} = 0.58+0.7 = 1.28

Replace $C_{15}$with 3+C_{35} = 0.58+0.9 = 1.48

Replace $C_{25}$with C_{23}+C_{35} = 1.58+0.9 = 2.48

Replace $C_{41}$with C_{43}+C_{31} = 0.7+0.58 = 1.28

Replace $C_{51}$with C_{53}+C_{31} = 0.9+0.58 = 1.48

Replace $C_{52}$with C_{53}+C_{32} = 0.9+1.5 =2.48

Replace $C_{45}$with C_{43}+C_{35} = 0.7+0.9 = 1.6

Replace $C_{54}$with C_{53}+C_{34} = 0.9+0.7 =1.6

C_3	1	2	3	4	5
1	-	1	0.58	**1.28**	**1.48**
2	1	-	1.58	0.8	**2.48**
3	0.58	1.58	-	0.7	0.9
4	**1.28**	0.8	0.7	-	**1.6**
5	**1.48**	**2.48**	0.9	**1.6**	-

S_3	1	2	3	4	5
1	-	2	3	**3**	**3**
2	1	-	1	4	**3**
3	1	1	-	4	5
4	**3**	2	3	-	**3**
5	**3**	**3**	3	**3**	-

Step 5: Set k=4, here 4th row and column are the pivot row and pivot column the new matrix

C_4& S_4 are given by replacing

C_{23}= C_{24} + C_{43}= 0.8+0.7 = 1.5

C_{25} = C_{24} + C_{45} =4.8+0.64 =1.44

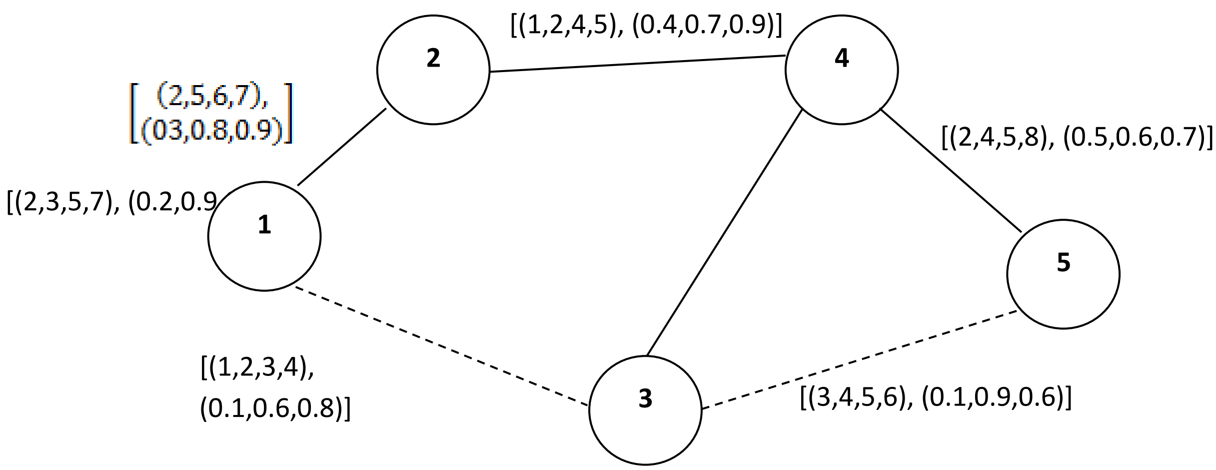
C_{32} = C_{34} + C_{42}= 0.7+0.8 = 1.5

C_{52} = C_{54} + C_{42}=0.64+0.87 = 1.44

C_4	1	2	3	4	5
1	-	1	0.58	1.28	1.48
2	1	-	**1.5**	0.8	**1.44**
3	0.58	**1.5**	-	0.7	0.9
4	1.28	0.8	0.7	-	1.6
5	1.48	**1.44**	0.9	1.6	-

S_4	1	2	3	4	5
1	-	2	3	**3**	**3**
2	1	-	**4**	4	**4**
3	1	**4**	-	4	5
4	**3**	2	3	-	3
5	**3**	**4**	3	3	-

Step6: The final matrix contains all the information needed to determine the shortest route from node 1 from node 5. To determine the shortest route check whether S_{xy} = y or not. Otherwise x and y are linked through atleast one other intermediate node. The shortest route is 1→3→5. That is S_{13} = 3, S_{35}=5 and the value of the route is 1.48. Therefore the process ends.



V.CONCLUSIONS

In this paper, we have developed an algorithm for solving shortest path problem on a network with single valued trapezoidal neutrosophic edge lengths. In the algorithm the edge weights are uncertain. We can compare the weights of two different paths using score function. This is very useful to make decisions in choosing the best of all possible alternative paths. The proposed algorithm can be applied to many real life situations. In future we can do research in the applications of this algorithm.

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