OUTER SUM LABELING OF FRIENDSHIP GRAPH

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Abstract: All sum graphs are disconnected in order for a connected graph to bear a sum labelling the graph is considered in conjuction with a number of isolated vertices , the labels of which complete the sum labeling for the disjoint union. The smallest number of isolated vertices required to make a graph is called sum number of G denoted by $\sigma(G)$.An outer sum labelling is a labelling of graph G is an injective function $f: V(G) \to Z^{+}$ with the property that for each vertex $v \in V(G)$, there exist a vertex $w \in V(G)$ such that $f(w) = \sum_{u \in N(v)} f(u)$ where $N(v) = \{x: vx \in E(G)\}$. A graph G which admits an outer sum labeling is called an outer sum graph. If G is not an outer sum graph then minimum number of isolated verices required to make G a outer sum graph, is called outer sum number of G and is denoted by $on(G)$. In this paper we obtain outer sum number for some generalised friendship graph.

Keywords : Sum labeling, Sum graph, Outer sum labeling, Outer sum graph, Friendship graph.

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1. Introduction

All the graphs considered here are undirected, finite, connected and simple. The length of a shorted path between two vertices u & v and is denoted by $d(u, v)$. We use the standard terminology, the terms not defined here may be found in [1].

A sum labelling λ of a graphs is a mapping of the vertices of G in to distinct positive integers such that for $u, v \in V(G), uv \in (G)$ if and only if the sum of the labels assigned to w of G. In such a case w is called working vertex. A graph which has a sum labelling is called sum graphs. Sum graph were originally proposed by Harary [2] and later extended to include all integers in[3].

Sum graphs can not be connected graphs. Graphs which are not sum graphs can be made to support a sum labelling by considering the graph in conjunction with a number of $N(v) = \{x: vx \in E(G)\}$. A graph G which admits outer sum labelling is called outer sum graph.

If G is not outer sum graph then by adding certain isolated vertices to G we can convert G to outer sum graph. The minimum number of isolated vertices required by the graph to support outer sum labelling is called the outer sum number of the graph and is denoted by $on(G)$.

2 . Definitions

2.1 Sum labeling

A sum labelling is an injective mapping f from the vertices of G into the positive integers such that , for any two vertices u ,v belongs $V(G)$ with labels $f(u)$ and $f(v)$, respectively, uv is an edge if and only if $f(u) + f(v)$ is the label of another vertex in $V(G)$ [2].

2.2 Exclusive sum labelling

The exclusive sum number was introduced by Mirka Miller et.al [6].A sum labelling L is called an exclusive sum labelling with respect to a sub graph H of G if L is a sum labelling of G where H contains no working vertex [6].

A graph G will require some isolated vertices to be labelled exclusively. The least possible number of such isolated vertices is called Exclusively sum number of G.

2.3 Outer sum labeling

The outer sum labelling was introduced by Sooryanarayana et.al [5]. An outer sum labelling is a labelling of a graph G is an injective function $f: V(g) \to Z^+$ with the property that for vertex $v \in V(G)$, there exsits a vertex $w \in V(G)$ such that $f(w) = \sum_{u \in N(v)} f(u)$, where $N(v) = \{x: x \in E(G)\}$ [5].

A graph G which admits an outer sum labelling is called an Outer sum graph. If G is not an outer sum labeling , then by adding certain number of isolated vertices required for a graph G, to make the resultant graph an outer sum graph, is called the Outer sum number of G and is denoted by $on(G)$.

3 Earlier results

Remark 3.1 for any G , $\in (G) \geq \sigma(G)$ [4].

Remark 3.2[4]

Let $\Delta(G)$ be the maximum degree of the vertices of a graph G. Then $\in (G) \geq \Delta(G)$.

Theorem 3.3[4]

The exclusive sum number of paths , $\in (P_n) = 2$ for $n \geq 3$.

Theorem 3.4[4]

$$\in (C_N) = 3 \; for \; n \geq 3$$

Theorem 3.5[5]

For any integers ≥ 3 ,$on(C_n) = \begin{cases} 1 & if \quad n = 4 \\ 2 & otherwise \end{cases}$

Remark 3.6

A graph G is an outer sum graphs if and only if outer sum number is zero.

Theorem 3.7

A connected graph G is an outer sum graph if and only if $G \equiv K_{1,n}$

Theorem 3.8

For any tree T on n vertices $on(T) = \begin{cases} 0 & if \quad T \quad is \quad star \\ 1 & otherwise \end{cases}$

4 Main Results

Theorem 4.1

f_{3P} is optional 2 outer summable.

Proof : By theorem 3.8, outer sum number of G is one if and only if G is a tree or unicyclic graph. Since f_{3P} is not either tree or unicyclic graph tree hence $on(f_{3P}) \geq 2$.Now to prove the inequality we define a labelling procedure as follows .

Label the centre vertex $c = 1$.

$f(c) = 1, f(v_i) = 2P$

$f(v_1^i) = f(v_1^{i-1}) + 1 \quad , \quad 2 \leq i \leq P$

$f(v_2^1) = f(v_1^P) + 1$

$f(v_2^i) = f(v_1^{i-1}) + 1 \quad , \quad 2 \leq i \leq P$

Now the neighbourhood Sum of each vertices assigned as follows

$N(v_1^i) = f(v_2^i) + f(c) = f(v_2^{i+1}) \quad 1 \leq i \leq P\text{-}1$

$N(v_2^i) = f(v_1^i) + f(c) = f(v_1^{i+1}) \quad 1 \leq i \leq P\text{-}1$

$N(v_2^P) = f(v_1^P) + f(c) = f(v_1^2)$

It is clear that neighbourhood sum of two vertices has been not assigned to any of the vertices of f_{3P} , so we need two isolated vertices say u and v.

$N(v_1^P) = f(v_2^P) + f(c) = f(u)$

$$N(c) = \sum_{i=1}^{P} [f(v_1^i) + f(v_2^i)] = f(v)$$

And hence $on(f_{3P}) \leq 2$

From (1) and (2)

$$on(f_{3P}) = 2$$

Theorem 4.2

f_{4P} is optional 2 outer Summable. $on(f_{4P}) = 2$

Proof : By theorem 3.8 , outer sum number of G is one if G is a tree, unicyclic graph. Since f_{4P} is not a either tree or unicyclic graph, hence $on(f_{4P}) \geq 2$.

To prove the reverse inequality we define a labelling procedure as follows,

$$f(c) = 1, f(v_1^1) = 3P, f(v_2^1) = 3P\text{-}1 \text{ and } f(v_2^1) = 3P\text{+}1$$

$$f(v_1^i) = 2f(v_1^{i-1}) + 1 \quad , \quad 2 \leq i \leq P$$

$$f(v_2^i) = 2f(v_2^{i-1}) + 2 \quad , \quad 2 \leq i \leq P$$

$$f(v_3^i) = 2f(v_3^{i-1}) \quad , \quad 2 \leq i \leq P$$

Now the neighbourhood Sum of each vertices assigned as follow

$N(v_1^i) = f(v_2^i) + f(c) = f(v_1^i) \quad , \quad 1 \le i \le P$

$N(v_2^i) = f(v_1^i) + f(v_3^i) = f(v_1^{i+1}) \quad , \quad 1 \le i \le P\text{-}1$

$N(v_3^i) = f(v_2^i) + f(c) = f(v_1^i) \quad , \ 1 \le i \le P$

It is clear that neighbourhood sum of two vertices has been not assigned, so we need two isolated vertices say *u* and *v*.

$N(v_2^P) = f(v_1^P) + f(v_3^P) = f(u)$

$$N(c) = \sum_{i=1}^{P} [f(v_1^i) + f(v_2^i) + f(v_3^i)] = f(v)$$

And hence $on(f_{4P}) \le 2$

From (1) and (2)

$$on(f_{4P}) = 2$$

Theorem 4.4

f_{5P} is optional 2 outer Summable .

Proof : By theorem 3.8 , outer Sum number of *G* is one if *G* is a tree, unicyclic graph. Since f_{5P} is not a either tree or unicyclic graph, hence $on(f_{5P}) \ge 2$.

To prove the reverse inequality we define a labelling procedure as follows,

$$f(v_2^1) = 4P, f(v_3^1) = 4P + 1 , f(v_4^1) = 4P + 3 \text{ and } f(v_1^1) = 4P + 2$$

$$f(v_2^i) = f(v_1^{i-1}) + f(v_3^{i-1})1 \quad , \quad 2 \le i \le P$$

$$f(v_3^i) = f(v_2^i) + 1 \quad , \quad 2 \le i \le P$$

$$f(v_4^i) = f(v_1^i) + 1 \quad , \quad 2 \le i \le P$$

Now the neighbourhood Sum of each vertices assigned as follow

$N(v_1^i) = f(v_2^i) + f(c) = f(v_3^i) \quad , \quad 2 \le i \le P$

$N(v_2^i) = f(v_1^i) + f(v_3^i) = f(v_2^{i+1}) \quad , \quad 2 \le i \le P\text{-}1$

$N(v_3^i) = f(v_2^i) + f(v_4^i) = f(v_2^{i+1}) \quad , \ 2 \le i \le P - 1$

$N(v_4^i) = f(v_3^i) + f(c) = f(v_1^i) \quad , \ 2 \le i \le P$

It is clear that neighbourhood Sum of two vertices has been not assigned, so we need two isolated vertices say *u* and *v*.

$N(v_3^P) = f(v_2^P) + f(v_4^P) = f(u)$

$$N(c) = \sum_{i=1}^{P} [f(v_1^i) + f(v_2^i) + f(v_3^i) + f(v_4^i)] = f(v)$$

And hence $on(f_{5P}) \le 2$

From (1) and (2)

$$on(f_{5P}) = 2$$

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