Instability phenomenon in two phase heterogeneous porous medium and numerical solution of the governing equation

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Abstract: During water flooding process, with the injection of water into the petroleum reservoir, oil is displaced in such a pattern that unstable fingers (water filled) rise, these fingers are unstable and consequently an important phenomenon viz. instability (fingering) arises. There is a two-phase immiscible flow, one phase being water and other being oil. This phenomenon is of a great practical importance and it has been extensively studied by the researchers and scientists working in the petroleum production industry especially during secondary oil recovery process, from different viewpoints in the recent past. In the present paper for a systematic study, this phenomenon is numerically discussed for the heterogeneous porous media. The heterogeneous porous medium is considered with a steady permeability, whereas porosity is assumed varying with the space co-ordinate. It makes possible to derive the non-linear governing equation for the Instability phenomenon in the heterogeneous porous medium. The governing equation is based on the mass conservation principle and important Darcy's law. Under the specific standard relationships and basic assumptions considered, the numerical solution is obtained by using Crank-Nicolson finite difference scheme for the specified initial and boundary conditions. The numerical results are obtained by generating a MATLAB code. The oil recovery factor is shown in accordance with the standard relationship from the experimental results. The sensitivity of the basic parameters is studied to observe the corresponding effect on the obtained numerical solution for the saturation of water. The numerical results behave well with the physical phenomena and the basic properties of the parameters are preserved.

Index Terms–**Darcy's law, mass conservation principle, capillary pressure, heterogeneous porous medium, Crank-Nicolson finite difference method.**

I. INTRODUCTION

In the early stage, the pressure at a petroleum reservoir is so high that the gas or oil is produced by simple natural decompression without any pumping efforts at the wells, it is called primary recovery and it ends when a pressure equilibrium between the oil field and atmosphere occurs, it recovers 12% to 16% of oil. The part of the remaining oil is recovered by secondary oil recovery process, in which usually water is injected into oil formatted homogeneous porous medium to drive oil and consequently, oil is produced through production wells. During secondary oil recovery process, when water is injected in oil formatted homogeneous porous medium to recover the part of remaining oil, the important phenomenon viz. instability occurs due to the injecting force, difference of viscosity and wettability of the injected water and native oil. The moment water is injected into the oil-filled region instead of regular displacement of common interface protuberances occur due to the force of injecting water and difference in viscosities of water and native oil. It gives rise to the shape of fingers (protuberances) at common interface. The injected water, intended to push the native fluid forward, it tends to penetrate the more viscous native oil through spontaneously formed multi-branched fingers. discussed instability in water-oil displacement, numerically formulated a non-linear evolution equation for water oil displacement front (Figure 2(a)) [5, 8, 11].

Figure 4.1(a) Irregular Instabilities (fingers)

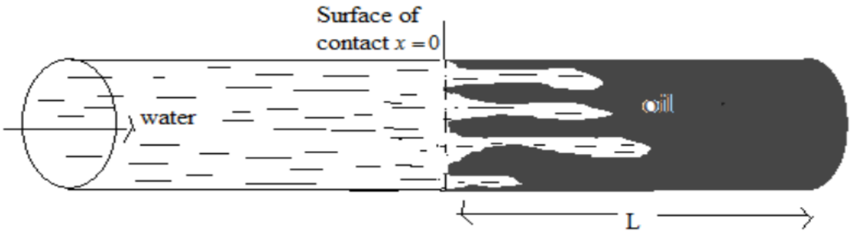

Figure 4.1(b) Instability phenomenon in the cylindrical porous medium

These resembling water filled fingers in the oil field porous medium are continuously unstable, hence, this phenomenon is known as instability phenomenon. The small amplitude fingers either grow (the unstable case) or decay (the stable case) with an exponential rate. For oil reservoir equations, the dimensionless number which governs the division between these cases is called the mobility ratio M defined to be the ratio of transmissibility of the fluid just behind the discontinuity, to the transmissibility of the fluid just ahead of the discontinuity. The value $M = 1$ is the threshold for instability growth; small perturbations grow for $M > 1$ and disappear for $M < 1$. The mobility ratio defined here is called the "frontal mobility ratio" in the petroleum reservoir literature to differentiate it from a similar ratio involving fluid transmissibility at points away from the discontinuity. In the statistical treatment of fingers, only the average behavior of the two fluids involved is taken into consideration. The saturation of displacing fluid in porous medium represents the average cross sectional area occupied by fingers [1, 4, 6, 7].

II. MATHEMATICAL FORMULATION

The instability phenomenon in heterogeneous porous medium has been studied under the necessary basic assumptions: Immiscible flow of two incompressible fluids, water and oil, porous medium is isotropic, incompressible; finite in extent, bottom boundary is horizontal and impermeable, mass is conserved and gravity force is neglected. The natural oil field is very large to study the effect of water injection (water flooding) in oil recovery process. To study the phenomenon of instability in heterogeneous porous medium we choose a cylindrical piece of porous matrix of length L whose three sides are impermeable except one end, from where water is injected (Figure 4.1(b)).

For the mathematical study, we take vertical cross sectional area of the cylindrical piece of porous matrix, which is rectangle, and open will be common interface $x = 0$ from where water is injected. The length x of the fingers is to be measured in the direction of displacement.

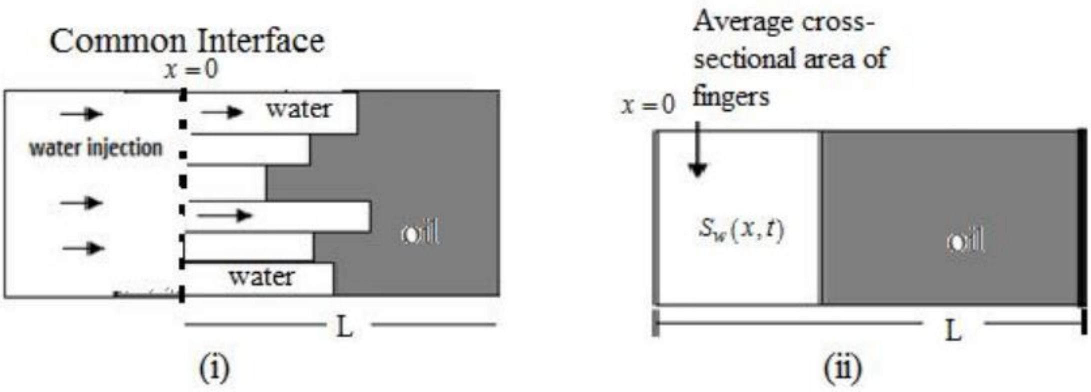

Figure: 4.2(i) Fingers in the Rectangular porous medium [6] 4.2(ii) Average cross-sectional area covered by fingers [6]

The reservoir heterogeneity is defined as a variation in reservoir properties as a function of space. The heterogeneity is represented by the fact that porosity changes with the space variable. A specific relation for the porosity as a function of space variable and permeability varying linearly with the porosity is considered in the mathematical formulation of the problem. Following Verma [16] the variation in the porosity (say, m) is assumed as follows.

$$m = \frac{1}{a - bx} \tag{4.18}$$

With the condition $a - bx > 1$; where, a and b are constants.

Since, $m < 1$, the values of a and b are chosen to preserve this fact. In the present section, the mathematical formulation is obtained by neglecting the small inclination, which makes possible to formulate under this simplified assumption. In order to study the saturation of the water, Scheidegger and Johnson suggested replacing irregular fingers by the rectangular schematic fingers [7, 8] (Figure 4(b)). The seepage velocities of injected water V_w and native oil V_o are expressed by Darcy's law as

$$V_w = -\frac{k_w}{\mu_w} K \left(\frac{\partial P_w}{\partial x} \right) \tag{4.19}$$

$$V_o = -\frac{k_o}{\mu_o} K \left(\frac{\partial P_o}{\partial x} \right) \tag{4.20}$$

where, K is the permeability of the heterogeneous porous medium, k_w and k_o are the respective relative permeability of water and oil, P_w and P_o are the respective pressure of water and oil, μ_w and μ_o are the respective viscosity of water and oil.
The equations of continuity of two phases are given as,

$$\frac{\partial\left(m\rho_w S_w\right)}{\partial t}+\frac{\partial\left(\rho_w V_w\right)}{\partial x}=0 \tag{4.21}$$

$$\frac{\partial\left(m\rho_o S_o\right)}{\partial t}+\frac{\partial\left(\rho_o V_o\right)}{\partial x}=0 \tag{4.22}$$

Since the porous medium is considered heterogeneous, the porosity m is defined as above. The two immiscible fluids water and oil are assumed to be incompressible their density ρ_w and ρ_o are constants. Hence, the above equations become

$$m\frac{\partial S_w}{\partial t}+\frac{\partial V_w}{\partial x}=0 \tag{4.23}$$

$$m\frac{\partial S_o}{\partial t}+\frac{\partial V_o}{\partial x}=0 \tag{4.24}$$

where, S_w and S_o are the saturation of water and oil respectively and m is the porosity of the medium. From the definition of phase saturation,

$$S_w+S_o=1 \tag{4.25}$$

As an empirical fact; we accept that capillary pressure P_c is a unique function of wetting phase saturation [3, 7].

$$P_c\left(S_w\right)=P_o-P_w \tag{4.26}$$

For definiteness, we assume capillary pressure P_c as a linear function of the saturation of water S_w as [2, 5, 12]

$$P_c=-\beta S_w \tag{4.27}$$

where, β is a capillary pressure parameter defined as $\beta=\sigma\sqrt{m}/\sqrt{K}$ where, σ is the surface tension exerted between the two immiscible fluids.

The relative permeability of water and oil are considered from the standard relationship due to Scheidegger and Johnson [14] given by

$$k_w=S_w \tag{4.28}$$

$$k_o=S_o=1-S_w \tag{4.29}$$

The equations of motion for saturation are obtained by substituting the values of equation (4.19) and (4.20) in equation (4.23) and (4.24) respectively as,

$$m\frac{\partial S_w}{\partial t}=\frac{\partial}{\partial x}\left[K\frac{k_w}{\mu_w}\frac{\partial P_w}{\partial x}\right] \tag{4.30}$$

$$m\frac{\partial S_o}{\partial t}=\frac{\partial}{\partial x}\left[K\frac{k_o}{\mu_o}\frac{\partial P_o}{\partial x}\right] \tag{4.31}$$

Eliminating $\frac{\partial P_w}{\partial x}$ from equation (4.26) and (4.30) we get,

$$m\frac{\partial S_w}{\partial t}=\frac{\partial}{\partial x}\left[K\frac{k_w}{\mu_w}\left(\frac{\partial P_o}{\partial x}-\frac{\partial P_c}{\partial x}\right)\right] \tag{4.32}$$

Combining equation (4.31) and (4.32) and using equation (4.25) we get,

$$\frac{\partial}{\partial x}\left[\left(K\frac{k_w}{\mu_w}+K\frac{k_o}{\mu_o}\right)\frac{\partial P_o}{\partial x}-K\frac{k_w}{\mu_w}\frac{\partial P_c}{\partial x}\right]=0 \tag{4.33}$$

Integrating equation (4.33) with respect to x,

$$C_1=\left(K\frac{k_w}{\mu_w}+K\frac{k_o}{\mu_o}\right)\frac{\partial P_o}{\partial x}-K\frac{k_w}{\mu_w}\frac{\partial P_c}{\partial x} \tag{4.34}$$

where, C_1 is a constant of integration. On simplifying, we have

$$\frac{\partial P_o}{\partial x}=\frac{C_1}{\left(K\frac{k_w}{\mu_w}+K\frac{k_o}{\mu_o}\right)}+\frac{K\frac{k_w}{\mu_w}}{\left(K\frac{k_w}{\mu_w}+K\frac{k_o}{\mu_o}\right)}\cdot\frac{\partial P_c}{\partial x} \tag{4.35}$$

Substituting the value of equation (4.35) in (4.32),

$$m\frac{\partial S_w}{\partial t}=\frac{\partial}{\partial x}\left[K\frac{k_w}{\mu_w}\left(\frac{C_1}{\left(K\frac{k_w}{\mu_w}+K\frac{k_o}{\mu_o}\right)}+\frac{K\frac{k_w}{\mu_w}}{\left(K\frac{k_w}{\mu_w}+K\frac{k_o}{\mu_o}\right)}\frac{\partial P_c}{\partial x}-\frac{\partial P_c}{\partial x}\right)\right] \tag{4.36}$$

Expressing P_o as $P_o=\bar{P}+\frac{1}{2}P_c$, where $\bar{P}=\frac{P_w+P_o}{2}$ is a constant mean pressure, we have,

$$\frac{\partial P_o}{\partial x}=\frac{1}{2}\frac{\partial P_c}{\partial x} \tag{4.37}$$

Thus from equation (4.34) and (4.37) we get,

$$C_1=\left(K\frac{k_w}{\mu_w}+K\frac{k_o}{\mu_o}\right)\frac{1}{2}\frac{\partial P_c}{\partial x}-K\frac{k_w}{\mu_w}\frac{\partial P_c}{\partial x} \tag{4.38}$$

Substituting the value of C_1 in equation (4.36) and using equation (4.28), on simplification we have,

$$m\frac{\partial S_w}{\partial t}=\frac{\partial}{\partial x}\left[-\frac{Kk_w}{2\mu_w}\frac{\partial P_c}{\partial x}\right] \tag{4.39}$$

For further simplification, considering permeability $K=K_c\,m$, in equation (4.39). We have,

$$m\frac{\partial S_w}{\partial t}=\frac{\beta K_c}{2\mu_w}\frac{\partial}{\partial x}\left(mS_w\frac{\partial S_w}{\partial x}\right) \tag{4.40}$$

Choosing new dimensionless variables as $X=\frac{x}{L}$ and $T=\frac{K_c\beta}{2\mu_w L^2}t$, the equation (4.40) reduces to

$$\frac{\partial S_w}{\partial T}=\frac{\partial}{\partial X}\left(S_w\frac{\partial S_w}{\partial X}\right)+\frac{1}{m^*}\frac{dm^*}{\partial X}S_w\frac{\partial S_w}{\partial X} \qquad \text{where, } m^*=\frac{1}{a-bXL} \tag{4.41}$$

Equation (4.41) is a non-linear partial differential equation. For the sake of simplicity dropping the asterisk, the term

$\frac{1}{m}\frac{dm}{dX}$ appearing in the equation (4.41), is approximated as follows:

$$\frac{1}{m}\frac{dm}{dX}=\frac{d\left(\log m\right)}{dX}=\frac{d\log\left(a-b^{*}X\right)^{-1}}{dX}=-\frac{d\log\left(1-\frac{b^{*}}{a}X\right)}{dX}\approx-\frac{d}{dX}\left(-\frac{b^{*}}{a}X\right)=\frac{b^{*}}{a} \tag{4.42}$$

where, $b^{*}=bL$ (Neglecting higher order terms of $\frac{b^{*}}{a}X$ in the expansion of $\log\left(1-bX/a\right)$) [11,16].

Using equation (4.42), the equation (4.41) is written as

$$\frac{\partial S_w}{\partial T}=\frac{\partial}{\partial X}\left(S_w\frac{\partial S_w}{\partial X}\right)+R_1 S_w\frac{\partial S_w}{\partial X} \qquad \text{where, } R_1=b^{*}/a \tag{4.43}$$

Equation (4.43) is the required governing non-linear equation, which is solved by the Crank-Nicolson finite difference method under the following initial and boundary conditions.

4.5 Initial and boundary conditions

$$S_w\left(X,0\right)=0 \qquad ; \qquad 0\le X\le 1 \tag{4.44}$$

$$S_w\left(0,T\right)=1=S_{w0} \quad ; \quad T>0 \tag{4.45}$$

$$S_w\left(1,T\right)=0=S_{w1} \quad ; \quad T>0 \tag{4.46}$$

It is assumed that initially the porous medium is fully saturated with oil. Since, $S_w+S_o=1$, we have the initial condition (4.44). It is assumed that the entire oil from the initial boundary of the formation, $x=0$, has been displaced for all time by the impact of the injected water. Since $X=x/L$, the boundary condition (4.45) indicates that the saturation of water is 1 (100%) for $T>0$. This means that the relative permeability of oil at the common interface is zero for all time. At the boundary $x=L$ it is assumed that the saturation of water is zero i.e. water is not reaching to the boundary. Hence, $S_w(1,T)=0$, for all time $T>0$.

IV. CRANK-NICOLSON SCHEMES

Equation (4.43) is numerically solved by applying Crank-Nicolson finite difference scheme for the initial condition (4.44) and boundary conditions (4.45) and (4.46). Applying forward projection for $S_{w_{i,\,n+1/2}}$ and introducing, the ratio $r=\Delta T/(\Delta X)^2$, the resulting central difference Crank-Nicolson scheme for the governing equation (4.43) is given [9,10,13,14,15]: For $2\le i\le\left(R-1\right)$,

$$\begin{aligned}
&\left[\left(4-2R_1\Delta X\right)S_{w_{i,\,n+1/2}}+S_{w_{i-1,\,n+1/2}}-S_{w_{i+1,\,n+1/2}}\right]S_{w_{i-1,\,n+1}}+\left[-8S_{w_{i,\,n+1/2}}-\frac{8}{r}\right]S_{w_{i,\,n+1}}\\
&\qquad+\left[\left(4+2R_1\Delta X\right)S_{w_{i,\,n+1/2}}-S_{w_{i-1,\,n+1/2}}+S_{w_{i+1,\,n+1/2}}\right]S_{w_{i+1,\,n+1}}\\
&=-\left[\left(4-2R_1\Delta X\right)S_{w_{i,\,n+1/2}}+S_{w_{i-1,\,n+1/2}}-S_{w_{i+1,\,n+1/2}}\right]S_{w_{i-1,\,n}}+\left[8S_{w_{i,\,n+1/2}}-\frac{8}{r}\right]S_{w_{i,\,n}}\\
&\qquad-\left[\left(4+2R_1\Delta X\right)S_{w_{i,\,n+1/2}}-S_{w_{i-1,\,n+1/2}}+S_{w_{i+1,\,n+1/2}}\right]S_{w_{i+1,\,n}}
\end{aligned} \tag{4.47}$$

with

$$S_{w_{i,\,n+1/2}} = S_{w_{i,\,n}} + \frac{r}{2}\left[\begin{array}{l} S_{w_{i,\,n}}\cdot\left(S_{w_{i+1,\,n}} - 2S_{w_{i,\,n}} + S_{w_{i-1,\,n}}\right) \\ +\frac{1}{4}\left(S_{w_{i+1,\,n}} - S_{w_{i-1,\,n}}\right)^2 + \left(R_1 S_{w_{i,\,n}}\right)\frac{\Delta X}{2}\left(S_{w_{i+1,\,n}} - S_{w_{i-1,\,n}}\right) \end{array}\right] \tag{4.47.1}$$

For $i=1$:

$$\begin{aligned} &\left[-\left(11-2R_1\Delta X\right)S_{w_{1,\,n+1/2}} + S_{w_{2,\,n+1/2}} - 2S_{w0} - \frac{8}{r}\right]S_{w_{1,\,n+1}} \\ &\qquad + \left[\left(5+2R_1\Delta X\right)S_{w_{1,\,n+1/2}} + S_{w_{2,\,n+1/2}} - 2S_{w0}\right]S_{w_{2,\,n+1}} \\ &= \left[\left(11-2R_1\Delta X\right)S_{w_{1,\,n+1/2}} - S_{w_{2,\,n+1/2}} + 2S_{w0} - \frac{8}{r}\right]S_{w_{1,\,n}} \\ &\qquad + \left[-\left(5+2R_1\Delta X\right)S_{w_{1,\,n+1/2}} - S_{w_{2,\,n+1/2}} + 2S_{w0}\left(T\right)\right]S_{w_{2,\,n}} \\ &\qquad - 4S_{w0}\left[\left(3-2R_1\Delta X\right)S_{w_{1,\,n+1/2}} - S_{w_{2,\,n+1/2}} + 2S_{w0}\right] \end{aligned} \tag{4.48}$$

with

$$S_{w_{1,\,n+1/2}} = S_{w_{1,\,n}} + \frac{r}{2}\left[\begin{array}{r} S_{w_{1,n}}\cdot\left(S_{w_{2,n}} - 3S_{w_{1,n}} + 2S_{w0}\right) + 0.25\left(S_{w_{2,n}} + S_{w_{1,n}} - 2S_{w0}\right)^2 \\ +0.5\Delta X\left(R_1 S_{w_{1,n}}\right)\left(S_{w_{2,n}} + S_{w_{1,n}} - 2S_{w0}\right) \end{array}\right] \tag{4.48.1}$$

For $i=R$:

$$\begin{aligned} &\left[\left(5-2R_1\Delta X\right)S_{w_{R,\,n+1/2}} + S_{w_{R-1,\,n+1/2}} - 2S_{w1}\right]S_{w_{R-1,\,n+1}} \\ &\qquad + \left[-\left(11+2R_1\Delta X\right)S_{w_{R,\,n+1/2}} + S_{w_{R-1,\,n+1/2}} - 2S_{w1} - \frac{8}{r}\right]S_{w_{R,\,n+1}} \\ &= \left[-\left(5-2R_1\Delta X\right)S_{w_{R,\,n+1/2}} - S_{w_{R-1,\,n+1/2}} + 2S_{w1}\right]S_{w_{R-1,\,n}} \\ &\qquad + \left[\left(11+2R_1\Delta X\right)S_{w_{R,\,n+1/2}} - S_{w_{R-1,\,n+1/2}} + 2S_{w1} - \frac{8}{r}\right]S_{w_{R,\,n}} \\ &\qquad - 4S_{w1}\left[\left(3+2R_1\Delta X\right)S_{w_{R,\,n+1/2}} - S_{w_{R-1,\,n+1/2}} + 2S_{w1}\right] \end{aligned} \tag{4.49}$$

with

$$S_{w_{R,\,n+1/2}} = S_{w_{R,\,n}} + \frac{r}{2}\left[\begin{array}{r} S_{w_{R,n}}\cdot\left(S_{w_{R-1,n}} - 3S_{w_{R,n}} + 2S_{w1}\right) + 0.25\left(2S_{w1} - S_{w_{R,n}} - S_{w_{R-1,n}}\right)^2 \\ +0.5\Delta X\left(R_1 S_{w_{R,n}}\right)\left(2S_{w1} - S_{w_{R,n}} - S_{w_{R-1,n}}\right) \end{array}\right] \tag{4.49.1}$$

V. NUMERICAL RESULTS AND DISCUSSION

Table 4.1 Saturation of water $S_w(X,T)$ for Instability phenomenon in Heterogeneous porous medium

X	$T=0.001$	$T=0.05$	$T=0.10$	$T=0.20$	$T=0.30$	$T=0.4$
0	1	1	1	1	1	1
0.1	0.00193	0.79719	0.85501	0.89455	0.91169	0.92180
0.2	0.00000	0.55695	0.69264	0.78411	0.82317	0.84598
0.3	0.00000	0.29315	0.51355	0.66446	0.72836	0.76540
0.4	0.00000	0.04933	0.32127	0.53593	0.62732	0.67999
0.5	0.00000	0.00000	0.12802	0.39940	0.52023	0.58974
0.6	0.00000	0.00000	0.00172	0.25713	0.40756	0.49455
0.7	0.00000	0.00000	0.00000	0.11582	0.29038	0.39393
0.8	0.00000	0.00000	0.00000	0.00741	0.17128	0.28573
0.9	0.00000	0.00000	0.00000	0.00000	0.05873	0.15936
1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

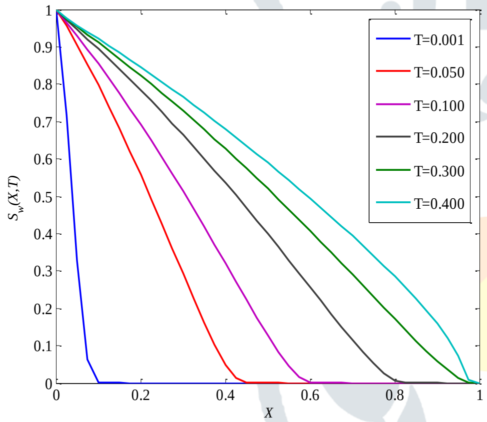

Figure 4.3 Water saturation profile in instability pheno. in Heterogeneous porous medium for $R_1 = 0.01$

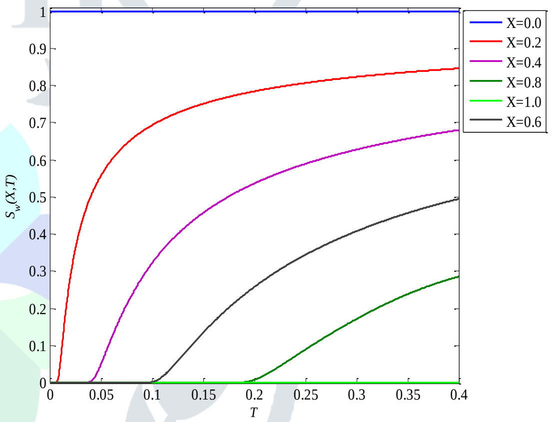

Figure 4.4 Water saturation profile for different distances in Heterogeneous. porous medium for $R_1 = 0.01$

The numerical solution for the governing equation (4.43) representing Instability phenomenon in heterogeneous porous medium is obtained using Crank-Nicolson finite difference scheme given by the equations (4.47), (4.48) and (4.49). The term R_1 appears in the equation (4.43), is referred as a heterogeneity coefficient for the governing equation, if $R_1 = 0$ it describes instability phenomenon in the homogeneous porous medium. The numerical value for the saturation of water for various values of time are shown in Table 4.1 together with its graphical representation shown in the Figures 4.3 and 4.4 for $R_1 = 0.01$. It is observed that the saturation of water increases with time and decreases with the space variable X and hence, the physical fact of the problem is preserved in the case of heterogeneous porous medium and noticeable amount of oil can be displaced.

The influence of heterogeneity coefficient R_1 on the water saturation profile is shown in Figure 4.5, it is observed that after the distance $X = 0.9$ there is sudden fall in the nature of the water saturation profiles for the different values of R_1.The graphical representation given in the Figure 4.3 and Figure 4.4 it is found that for the time $T > 0.3220$ the numerical values of saturation of water are not physically consistent in the heterogeneous porous medium. The porosity is varying in the heterogeneous porous medium, its corresponding effect with the variable porosity is observed in Table 4.2 by considering various values of R_1. By increasing the values of R_1 the saturation of water decreases and consequently will affect on the oil recovery. From the obtained numerical results for the saturation of the water, the oil recovery factor is calculated and it is found that the oil recovery grows with time which is found to be the same in the standard relationship of the experimental setup. Thus, the obtained numerical results are reliable as it is found that the oil is recovered largely which is shown in the Figure 4.6 where oil recovery factor is calculated and it is nearly 58%. From the obtained numerical results, it is found that oil can be recovered to a larger extent. Hence, the numerical results are efficient and reliable.

Table 4.2 Effect of varying heterogeneity for different R_1

X	$R_1 = 0.4$	$R_1 = 0.9$	$R_1 = 1.5$
0	1	1	1
0.1	0.9028	0.8908	0.8753
0.2	0.8050	0.7810	0.7511
0.3	0.7027	0.6695	0.6293
0.4	0.5960	0.5563	0.5096
0.5	0.4852	0.4417	0.3921
0.6	0.3708	0.3263	0.2771
0.7	0.2542	0.2117	0.1664
0.8	0.1385	0.1015	0.0649
0.9	0.0349	0.0129	0.0011
1	0	0	0

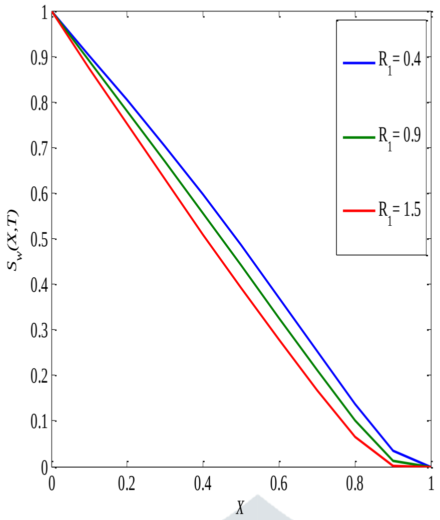

Figure 4.5 Influence of heterogeneity coefficient R_1 on water saturation profiles

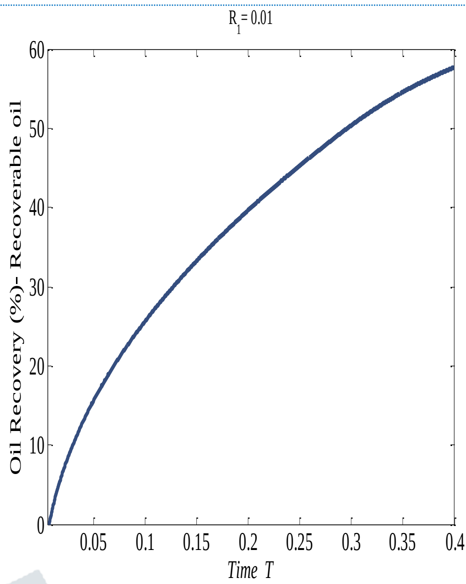

Figure 4.6 Oil recovery factor in (%) in instability phenomenon for $R_1 = 0.01$

In the present paper, the problem of double phase flow through porous media is considered in the oil water displacement. The mathematical formulation for the instability phenomenon in heterogeneous porous medium leads into a non-linear partial differential equation. Crank-Nicolson finite difference scheme is successfully implemented to determine the saturation of water. The numerical results are further simulated for finding the oil recovery. The numerical results behave physically well and the oil recovery shown for the heterogeneous porous medium serves as an evidence to claim that the applied finite difference scheme is reliable and can be extended for the other non-linear equations.

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