THE COMMENCEMENT OF DOUBLE DIFFUSIVE CONVECTION IN A HORIZONTAL ANISOTROPIC POROUS LAYER

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1 Introduction

The problem of double diffusive convection in porous media has attracted considerable interest during the last few decades because of its wide range of applications, from the solidification of binary mixtures to the migration of solutes in water-saturated soils. The other examples include geophysical systems, electro-chemistry, the migration of moisture through air contained in fibrous insulation. A comprehensive review of the literature concerning double diffusive natural convection in a fluid-saturated porous medium may be found in the book by Nield and Bejan (2006). Useful review articles on double diffusive convection in porous media include those by Trevisan and Bejan (1999), Mojtabi and Charrier-Mojtabi (2000, 2005) and Mamou (2002).

The study of the double diffusive generalization of the Horton-Rogers-Lapwood problem was first under taken by Nield (1968) on the basis of linear stability theory for various thermal and solutal boundary conditions. The onset of double diffusive convection in a horizontal porous layer has been investigated by Rudraiah et al. (1982) using a weak non-linear theory. Finite amplitude double diffusive convection near the threshold of both stationary and oscillatory instabilities in a binary mixture was investigated by Brand and Steinberg (1983). The linear stability analysis of the thermosolutal convection in a sparsely packed porous layer was made by Poulikakos (1986) using the Darcy-Brinkman model. Small amplitude nonlinear solutions in the form of standing and traveling waves and the transition to finite amplitude convection, as predicted by bifurcation theory, were studied by Knobloch (1986). The double diffusive convection in porous media in the presence of Soret and Dufour coefficients has been analyzed by Rudraiah and Malashetty (1986). Murray and Chen (1989) have extended the linear stability theory, considering effects of temperature-dependent viscosity and volumetric expansion coefficients and nonlinear basic salinity profile. Natural convection in porous layer, with two stratifying agencies, heated from below in a square cavity has been investigated numerically by Rosenberg and Spera (1992). Double diffusive fingering convection in a porous medium with horizontally periodic boundary conditions was studied by Chen and Chen (1993). Malashetty (1993) made a linear stability analysis to determine the effects of anisotropic thermo convective currents on the double diffusive convection in a sparsely packed porous medium.

Straughan and Hutter (1999) have investigated the double diffusive convection with Soret effect in a porous layer using Darcy-Brinkman model and derived a priori bounds. An analytical and numerical study of double diffusive convection with parallel flow in a horizontal sparsely packed porous layer under the influence of constant heat and mass flux was performed using a Brinkman model by Amahmid et al. (1999). A double diffusive bifurcation phenomenon was studied by Mamou and Vasseur (1999) using linear and nonlinear stability analyses with uniform flux and uniform temperature boundary conditions.

Bahloul et al. (2003) have carried out an analytical and numerical study of the double diffusive convection in a shallow horizontal porous layer under the influence of Soret effect. Hill (2005) performed linear and nonlinear stability analyses of double diffusive convection in a fluid saturated porous layer with a concentration based internal heat source using Darcy's law. Double diffusive natural convection within a multilayer anisotropic porous medium is studied numerically and analytically by Bennacer et al. (2005). Mansour et al. (2006) have investigated the multiplicity of solutions induced by thermosolutal convection in a square porous cavity heated from below and subjected to horizontal concentration gradient in the presence of Soret effect.

The study of double diffusive convection in a rotating porous media is motivated both theoretically and by its practical applications in engineering. Some of the important areas of applications in engineering include the food and chemical process, solidification and centrifugal casting of metals, rotating machinery, petroleum industry, biomechanics and geophysical problems. There are only few studies available on double diffusive convection in the presence of rotation. Chakrabarti and Gupta (1981) have analyzed the nonlinear thermohaline convection in a rotating porous medium. The effect of rotation on linear and non-linear double diffusive convection in a sparsely packed porous medium was studied by Rudraiah et al. (1986). The Lyapunov direct method is applied to study the non-linear conditional stability problem of a rotating doubly diffusive convection in a sparsely packed porous layer by Guo and Kaloni (1995). The nonlinear stability of the conduction-diffusion solution of a fluid mixture heated and salted from below and saturating a porous medium in the presence of rotation is studied by Lombordo and Mulone (2002) using Lyapunov direct method.

Anisotropy is generally a consequence of preferential orientation or asymmetric geometry of porous matrix or fibers and is in fact encountered in numerous systems in industry and nature. Anisotropy is particularly important in a geological context, since sedimentary rocks generally have a layered structure; the permeability in the vertical direction is often much less than in the horizontal direction. Anisotropy can also be a characteristic of artificial porous materials like pelletting used in chemical engineering process and fiber material used in insulating purpose. The review of research on convective flow through anisotropic porous media has been well documented by McKibbin 1985, 1992) and Storesletten (1998, 2004). Castinel and Combarnous (1974) have conducted an experimental and theoretical investigation on the Rayleigh-Benard convection in an anisotropic porous medium. Epherre (1977) extended the stability analysis to a porous medium with anisotropy in thermal diffusivity also. A theoretical analysis of nonlinear thermal convection in an anisotropic porous medium was performed by Kvernvold and Tyvand (1979). Nilsen and Storesletten (1990) have studied the problem of natural convection in both isotropic and anisotropic porous channels. Tyvand and Storesletten (1991) investigated the problem concerning the onset of convection in an anisotropic porous layer in which the principal axes were obliquely oriented to the gravity vector. Recently many authors have studied the effect of anisotropy and/or rotation on the onset of convection in a horizontal porous layer (see e.g. Govender; 2006, 2007, Govender and Vadasz; 2007 and Malashetty and Swamy; 2007). There appears to be only couple of studies available on the double diffusive convection in an anisotropic porous medium with rotation (Patil et al., 1989) and without rotation (Tyvand; 1980). Although the problem of double diffusive convection in isotropis porous medium has been investigated extensively, very little attention has been devoted to the study of double diffusive convection in a rotating porous medium including the mechanical and thermal anisotropic effects. The objective of the present study is therefore to investigate the combined effect of rotation and anisotropy on the double diffusive convection in a horizontal porous layer using linear and nonlinear analyses.

2 Mathematical Formulation

Consider a fluid saturated anisotropic porous layer of infinite horizontal extent confined between parallel, stress-free planes at $z = 0$ and $z = d$ subject to rotation and maintained at constant temperatures $T_0 + \Delta T$ and T_0 with solute concentration $S_0 + \Delta S$ and S_0 respectively. A Cartesian frame of reference is chosen with *x*- and *y*-axes at the lower boundary plane and *z*-axis directed vertically upwards in the gravity field. The axis of rotation is assumed to coincide with the *z*-axis. The porous medium is assumed to possess horizontal isotropy in both mechanical and thermal properties. The extended Darcy law, which includes the time derivative and the Coriolis term, is employed as a momentum equation and Boussinesq approximation is applied to account for the effects of density variations. With these assumptions the basic governing equations may be written as

$$\nabla \cdot \mathbf{q} = 0, \quad (1)$$

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \nu\, K \cdot \mathbf{q} = -\frac{1}{\rho_0}\nabla p + \frac{\rho}{\rho_0}\mathbf{g}, \quad (2)$$

$$\frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla)T = \nabla \cdot (\boldsymbol{\kappa}_{\mathbf{T}} \cdot \nabla T), \quad (3)$$

$$\frac{\partial S}{\partial t} + (\mathbf{q} \cdot \nabla)S = \kappa_S \nabla^2 S, \quad (4)$$

$$\rho = \rho_0[1 - \beta_T(T - T_0) + \beta_S(S - S_0)], \quad (5)$$

where $\varepsilon, \rho, K, \boldsymbol{\kappa}_{\mathbf{T}}$ and κ_S represent porosity, density, permeability tensor, effective thermal diffusivity tensor and solute diffusivity respectively, β_T and β_S , are the thermal expansion coefficient and solute expansion coefficient and $\mathbf{q}$ and $\boldsymbol{\Omega}$ denote the velocity of fluid and angular velocity of rotation respectively. The components of the thermal diffusivity tensor are written in terms of porosity and appropriate thermal diffusivities of the fluid and solid states as $\kappa_{Ti} = \varepsilon(\kappa_{Ti})_f + (1-\varepsilon)(\kappa_{Ti})_s$ with $i = x, z$. The justification for the inclusion of the time derivative term in the Darcy equation is discussed in detail by Vadasz (1998). The boundary conditions in only the *z*-direction are required for solving the Eqs. (1)-(5) and are given by

$$w = T = S = 0 \text{ at } z = 0 \text{ and } z = 1. \quad (6)$$

The basic state of the fluid is assumed to be quiescent. The quantities of the basic state are given by,

$$\mathbf{q}_b = (0, 0, 0),\ T_b = T(z),\ S_b = S(z), p_b = p(z),\ \rho_b = \rho(z), \tag{7}$$

which satisfy the equations

$$\frac{dp_b}{dz} = \rho_b g,\ \frac{d^2 T_b}{dz^2} = 0,\ \frac{d^2 S_b}{dz^2} = 0, \rho_b = \rho_0 \left[1 - \beta_T \left(T_b - T_0\right) + \beta_S \left(S_b - S_0\right)\right]. \tag{8}$$

On the basic state we superpose small perturbations around the basic solutions in the form

$$\mathbf{q} = \mathbf{q}_b + \mathbf{q}',\ T = T_b(z) + T', S = S_b(z) + S',\ p = p_b(z) + p',\ \rho = \rho_b(z) + \rho', \tag{9}$$

where the primes indicate perturbations. Substituting (9) into the Eqs. (1) to (5) using basic state equations (8), and the transformations

$$\begin{aligned} &(x, y, z) = d\left(x^*, y^*, z^*\right),\ t = \frac{d^2}{\kappa_{Tz}} t^*,\ p' = \frac{\mu \kappa_{Tz}}{K_z} p^*, \\ &(u', v', w') = \frac{\kappa_{Tz}}{d}(u, v, w),\ T' = (\Delta T) T^*, S' = (\Delta S) S^* \end{aligned} \tag{10}$$

to render the resulting equations dimensionless, we obtain (after dropping the asterisks for simplicity),

$$\nabla \cdot \mathbf{q} = 0, \tag{11}$$

$$\frac{\partial \mathbf{q}}{\partial t_1} + \mathbf{q}_a = -\nabla p + Ra\, T\, \mathbf{k} - Rs\, S\, \mathbf{k} \tag{12}$$

$$Va\, \frac{\partial T}{\partial t_1} + (\mathbf{q} \cdot \nabla) T - w = \left[\eta \nabla_h^2 + \frac{\partial^2}{\partial z^2}\right] T, \tag{13}$$

$$Va\, \frac{\partial S}{\partial t_1} + (\mathbf{q} \cdot \nabla) S - w = \tau \nabla^2 S \tag{14}$$

where $Ta = \left(2\Omega K_z / \varepsilon \nu\right)^2$, Taylor number, $Ra = \beta_T\, g\, \Delta T d K_z / \nu \kappa_{Tz}$, Rayleigh number, $Rs = \beta_S\, g\, \Delta T d K_z / \nu \kappa_{Tz}$, solute Rayleigh number, $Va = \varepsilon\, Pr / Da$, Vadasz number, $\mathbf{q}_a = \left(\frac{1}{\xi} u,\ \frac{1}{\xi} v,\ w\right)$, the anisotropy modified velocity vector, $\tau = \kappa_S / \kappa_{Tz}$, diffusivity ratio and $Pr = \nu / \kappa_{Tz}$, $Da = K_z / d^2$, $\xi = K_x / K_z$, $\eta = \kappa_{Tx} / \kappa_{Tz}$ being the Prandtl number, Darcy number, mechanical and thermal anisotropy parameters respectively. Further $t_1 = Va\, t$ is the rescaled time and $\nabla_h^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$ is the horizontal Laplacian.

We now eliminate the pressure from (12) by applying the curl operator on it which yield an equation for vorticity, defined as $\boldsymbol{\omega}_1 = \nabla \times \mathbf{q}$, in the form

$$\frac{\partial \boldsymbol{\omega}_1}{\partial t_1} + \frac{1}{\xi}\boldsymbol{\omega}_{1a} = Ra_T\left[\frac{\partial T}{\partial y}\mathbf{i} - \frac{\partial T}{\partial x}\mathbf{j}\right] - Ra_S\left[\frac{\partial S}{\partial y}\mathbf{i} - \frac{\partial S}{\partial x}\mathbf{j}\right], \tag{15}$$

where $\boldsymbol{\omega}_{1a} = \nabla \times \mathbf{q}_a$, denotes the anisotropy modified vorticity vector.

It is important to note that the vertical component of Eq. (15) is independent of temperature. Once again by applying the curl on Eq. (15) one can get the equation

$$\frac{\partial}{\partial t_1}\left(\nabla^2 \mathbf{q}\right) + \mathbf{Q} + Ra\left[\frac{\partial^2 T}{\partial x \partial z}\mathbf{i} + \frac{\partial^2 T}{\partial y \partial z}\mathbf{j} - \nabla_h^2 T\,\mathbf{k}\right] - Rs\left[\frac{\partial^2 S}{\partial x \partial z}\mathbf{i} + \frac{\partial^2 S}{\partial y \partial z}\mathbf{j} - \nabla_h^2 S\,\mathbf{k}\right] = 0 \tag{16}$$

with $\mathbf{Q} = (Q_1,\ Q_2,\ Q_3)$,

where $Q_1 = \left[\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\right]u + \left(1 - \frac{1}{\xi}\right)\frac{\partial^2 v}{\partial x \partial y}$, $Q_2 = \left[\frac{\partial^2}{\partial y^2} + \frac{1}{\xi}\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2}\right)\right]v + \left(1 - \frac{1}{\xi}\right)\frac{\partial^2 u}{\partial x \partial y}$ and

$$Q_3 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w.$$

3 Linear stability analysis

In this section, we perform the linear stability analysis, which is useful in the local nonlinear stability analysis discussed in the next section. For this, we linearize the Eqs. (13)- (16) and eliminating all variables except the vertical component of velocity, to obtain

$$\left\{\left[\frac{\partial}{\partial t_1}\left(\frac{\partial}{\partial t_1} + \frac{1}{\xi}\right)\nabla^2 + \left(\frac{\partial}{\partial t_1} + \frac{1}{\xi}\right)\left(\nabla_h^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\right]\left(Va\frac{\partial}{\partial t_1} - \eta\nabla_h^2 - \frac{\partial^2}{\partial z^2}\right)\left(Va\frac{\partial}{\partial t_1} - \tau\nabla^2\right) - Ra\left(Va\frac{\partial}{\partial t_1} - \tau\nabla^2\right)\left(\frac{\partial}{\partial t_1} + \frac{1}{\xi}\right)\nabla_h^2 + Rs\left(Va\frac{\partial}{\partial t_1} - \eta\nabla_h^2 - \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial}{\partial t_1} + \frac{1}{\xi}\right)\nabla_h^2\right\}w = 0. \tag{17}$$

The boundary conditions in terms of w are given by

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial^4 w}{\partial z^4} = 0 \text{ at } z = 0 \text{ and } z = 1. \tag{18}$$

We assume the normal mode solution in the form

$$w = W(z)\exp\left[i(lx+my)+\omega t_1\right], \tag{19}$$

where l and m are the x- and y- components of wavenumber and ω is the growth rate and in general a complex quantity $(\omega = \omega_r + i\omega_i)$. Using (19) in Eq. (17) one can obtain an ordinary differential equation for $W(z)$ as follows

$$\Big\{\Big[\left(\omega+\xi^{-1}\right)\left\{\omega\left(D^2-a^2\right)+\left(\xi^{-1}D^2-a^2\right)\right\}\Big]\left(D^2-\eta a^2-Va\,\omega\right)\left(\tau D^2-\tau a^2-Va\,\omega\right)$$
$$+Ra\left(\tau D^2-\tau a^2-Va\,\omega\right)\left(\omega+\xi^{-1}\right)a^2-Rs\left(\tau D^2-\eta a^2-Va\,\omega\right)\left(\omega+\xi^{-1}\right)a^2\Big\}W = 0\ . \tag{20}$$

The boundary conditions (18) reduce to

$$W = \frac{d^2W}{dz^2} = \frac{d^4W}{dz^4} = 0 \ \text{at}\ z=0 \ \text{and}\ z=1. \tag{21}$$

Here $a^2 = (l^2+m^2)$ is the horizontal wavenumber and $D \equiv d/dz$. The solution of the above boundary value problem should be a periodic wave of the form $W(z) = A_n \sin(n\pi z)$ which minimizes the Rayleigh number when n = 1, indicating that $W(z) = A_1 \sin(\pi z)$ is the eigenfunction for marginal stability. Substituting this into Eq. (20) and rescaling the parameters in the form

$$R_T = Ra/\pi^2\,, R_S = Rs/\pi^2\,,\ \alpha_1 = a^2/\pi^2 \ \text{and}\ \gamma_2 = Va/\pi^2 \tag{22}$$

yields an expression for the scaled Rayleigh number

$$R_T = \frac{R_S(\omega\gamma_2+\eta\alpha_1+1)}{(\omega\gamma_2+\tau\alpha_1+\tau)} + \frac{\left[1+\eta\alpha_1+\gamma_2\,\omega\right]\left[\left(\omega+\xi^{-1}\right)\left\{\omega\left(1+\alpha_1\right)+\left(\xi^{-1}+\alpha_1\right)\right\}\right]}{\alpha_1\left(\omega+\xi^{-1}\right)}. \tag{23}$$

For single component rotating anisotropic porous layer, i.e. when $R_S = 0$, Eq. (23) takes the form

$$R_T = \frac{\left[1+\eta\alpha_1+\gamma_2\,\omega\right]\left[\left(\omega+\xi^{-1}\right)\left\{\omega\left(1+\alpha_1\right)+\left(\xi^{-1}+\alpha_1\right)\right\}\right]}{\alpha_1\left(\omega+\xi^{-1}\right)} \tag{24}$$

This coincides with the results of Malashetty and Swamy (2007). Further when $\xi = \eta = 1$ Eq. (24) gives

$$R_T = \frac{(1+\alpha_1+\gamma_2\,\omega)\left[(\omega+1)^2(1+\alpha_1)\right]}{\alpha_1(1+\omega)} \tag{25}$$

which is exactly the one given by Vadasz (1998) for the case of isotropic porous layer.

3.1 Stationary state

If ω is real, then marginal stability occurs when $\omega = 0$. Then Eq. (23) gives the Rayleigh number R_1^{st} for the onset of stationary convection, in the form

$$R_T^{st} = \frac{R_S}{\tau}\left(\frac{1+\eta\alpha_1}{1+\alpha_1}\right) + \frac{1}{\alpha_1}\left(\xi^{-1}+\alpha_1\right)\left(1+\eta\alpha_1\right) + \frac{\xi}{\alpha_1}\left(1+\eta\alpha_1\right) \quad . \tag{26}$$

The first term in the expression for stationary Rayleigh number represents the effect of second diffusing component and the third term represents the contribution of rotation.

In case of single component system with rotation, $R_S = 0$, equation (26) gives

$$R_T^{st} = \frac{1}{\alpha_1}\left(\xi^{-1}+\alpha_1\right)\left(1+\eta\alpha_1\right) \tag{27}$$

When we put $\xi = \eta = 1$ in Eq. (27) we recover

$$R_T^{st} = \frac{(1+\alpha_1)}{\alpha_1}[1+\alpha_1] \tag{28}$$

the result of Vadasz (1998) for the case of a rotating isotropic porous layer.

Eq. (27) gives the critical stationary Rayleigh number corresponding to the critical wavenumber $\alpha_1 = \alpha_{1c}$ where α_1 satisfies the equation

$$\eta\alpha_1^4 + 2\eta\alpha_1^3 + \left(\frac{R_S}{\tau}(\eta-1) - \frac{1}{\xi} + \eta\right)\alpha_1^2 - \left(\frac{2}{\xi}\right)\alpha_1 - \frac{1}{\xi} = 0 \tag{29}$$

3.2 Oscillatory state

It is well known that the oscillatory motions are possible only if some additional constraints like rotation, salinity gradient and magnetic field are present. For the oscillatory mode ω must be represented as $\omega = \omega_r + i\omega_i$. At the marginal state $\omega_r = 0$ and $\omega_i \neq 0$. Substituting $\omega = i\omega_i$ into the Eq. (23) and clearing the complex quantities from the denominator, we obtain

$$R_T^{osc} = \frac{R_S\left\{(1+\alpha_1)(1+\alpha_1\eta)\tau+\gamma_2^2\omega_i^2\right\}}{(1+\alpha_1)^2\tau^2+\gamma_2^2\omega_i^2} + \frac{1+\alpha_1(\eta+\xi+\alpha_1\eta\xi)-\gamma_2\xi\omega_i^2(1+\alpha_1)}{\alpha_1\xi}+i\omega_i N \qquad (30)$$

where

$$N = \frac{-1}{\xi^2}\Big[R_S\alpha_1\gamma_2\left\{1+\alpha_1\eta-(1+\alpha_1)\tau\right\}\left(1+\xi^2\omega_i^2\right)\Big] + \gamma_2^2\omega_i^2\Big\}\Big]+\frac{1}{\xi^3}\Big[\Big\{\gamma_2+\left\{1+\alpha_1(1+\gamma_2+\eta+\alpha_1\eta)\right\}\xi\Big\}\Big\{(1+\alpha_1)^2\tau^2+\gamma_2^2\omega_i^2\Big\}\left(1+\xi^2\omega_i^2\right)\Big] \qquad (31)$$

The physical quantity R_T^{osc} must be real so that either $\omega_i = 0$ or $N = 0$. For oscillatory instability $\omega_i \neq 0$, (30) requires $N = 0$. The vanishing of N provides a dispersion relation of the form

$$\Delta_1(\omega_i^2)^2+\Delta_2\omega_i^2+\Delta_3 = 0. \qquad (32)$$

Where

$$\Delta_1 = \frac{\gamma_2^2}{\xi}\Big[\gamma_2+\left\{1+\alpha_1(1+\gamma_2+\eta+\alpha_1\eta)\right\}\xi\Big] \qquad (33)$$

$$\Delta_2 = \frac{1}{\xi^3}\Big[\gamma_2\Big\{\gamma_2^2+\gamma_2\left\{1+\alpha_1(1+\gamma_2+\eta+\alpha_1\eta)\right\}\xi-(R_S\alpha_1)(1+\alpha_1\eta)\xi^3\Big\} + R_S\alpha_1(1+\alpha_1)\gamma_2\xi^3\tau+(1+\alpha_1)^2\xi^2\Big[\gamma_2+\left\{1+\alpha_1(1+\gamma_2+\eta+\alpha_1\eta)\right\}\xi\Big]\tau^2\Big] \qquad (34)$$

$$\Delta_3 = \frac{1}{\xi^3}\Big[(1+\alpha_1)^2\Big[(1+\alpha_1\eta)\xi(1+\alpha_1)+\gamma_2\left\{1+\xi(\alpha_1)\right\}\Big]\tau^2 + R_S\alpha_1\gamma_2\xi\left\{-1+\tau+\alpha_1(-\eta+\tau)\right\}\Big] \qquad (35)$$

Since (32) is a quadratic in ω_i^2, it can give rise to more than one positive value of ω_i^2 for fixed $\alpha_1^2, R_S, Ta, \eta, \xi$ and τ. This has important implications on the linear stability of double-diffusive rotating porous layer. Thus in case Eq. (32) has two real positive roots then there exist two oscillatory neutral solutions. From Descartes' rule of signs in order for (32) to have two positive real roots, it is necessary that $\Delta_2 < 0$ and $\Delta_3 > 0$. We find the oscillatory neutral solutions from Eq. (30). It proceeds as follows: First determine the number of positive solutions of Eq. (32). If there are none, then no oscillatory instability is possible. If there are two, then the minimum (over α_1^2) of Eq. (30) with ω_i^2 given by Eq. (32) gives the oscillatory neutral Rayleigh number R_T^{osc}

corresponding to the critical wavenumber α_{1c} and the critical frequency of the oscillation ω_i^2. The analytical expression for oscillatory Rayleigh number given by Eq. (30) is evaluated at $\alpha_1 = \alpha_{1c}$ and $\omega^2 = \omega_c^2$ for various values of the physical parameters in order to know their effects on the onset of oscillatory convection.

In the next section we perform a nonlinear stability analysis and express heat and mass transfer by conduction and convection and observe the effect of rotation, anisotropy and the solute Rayleigh number on these parameters.

4. Weak nonlinear theory

In this section we consider the nonlinear analysis using a truncated representation of Fourier series with two terms. Although the linear stability analysis is sufficient for obtaining the stability condition of the motionless solution and the corresponding eigenfunctions describing qualitatively the convective flow, it cannot provide information about the values of the convection amplitudes, nor regarding the rate of heat and mass transfer. To obtain this additional information, we perform the nonlinear analysis, which is useful to understand the physical mechanism with minimum amount of mathematical analysis and is a step forward towards understanding full nonlinear problem.

For simplicity of analysis, we confine ourselves to the two-dimensional rolls, so that all the physical quantities are independent of y. We eliminate pressure from Eq. (12) and introduce stream function such that $u = \partial\psi/\partial z,\ \ w = -\partial\psi/\partial x$ into the Eqs. (12)-(14) and setting $\frac{\partial}{\partial t_1} = 0$ (for the steady state) to obtain

$$\left[\left(\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\right]\psi + Ra\frac{\partial T}{\partial x} - Rs\frac{\partial S}{\partial x} = 0, \tag{36}$$

$$\left(\eta\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial z^2}\right)T - \frac{\partial\psi}{\partial z}\frac{\partial T}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial z} - \frac{\partial\psi}{\partial x} = 0, \tag{37}$$

$$\left[\tau\nabla^2\right]S - \frac{\partial\psi}{\partial z}\frac{\partial S}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial S}{\partial z} - \frac{\partial\psi}{\partial x} = 0 \tag{38}$$

A minimal double Fourier series which describes the finite amplitude steady convection is given by

$$\psi = A\sin(ax)\sin(\pi z), \tag{39}$$

$$T = B\cos(ax)\sin(\pi z) + C\sin(2\pi z), \tag{40}$$

$$S = D\cos(ax)\sin(\pi z) + E\sin(2\pi z) \tag{41}$$

where the amplitudes *A*, *B*, *C*, *D* and *E* are constants and are to be determined from the dynamics of the system. Substituting Eqs. (39)-(41) into Eqs. (36)-(38) and equating the coefficients of like terms we obtain the following nonlinear system of equations

$$\left[\left(a^2 + \frac{\pi^2}{\xi}\right)\right] A + a\,Ra\,B - a\,Rs\,D = 0, \tag{42}$$

$$a\,A + \left(\eta a^2 + \pi^2\right) B + \pi\, a\, AC = 0, \tag{43}$$

$$8\pi^2 C - \pi\, a\, AB = 0, \tag{44}$$

$$aA + +\tau\left(a^2 + \pi^2\right) D + \pi a\, AE = 0, \tag{45}$$

$$8\pi^2 E - \pi\, a\, AD = 0. \tag{46}$$

The steady state state solutions are useful because they predict that a finite amplitude solution to the system is possible for subcritical values of the Rayleigh number and that the minimum values of Ra_T for which a steady state solution is possible lies below the critical values for instability to either a marginal state or an overstable infinitesimal perturbation. Elimination of all amplitudes, except for A, yields

$$A\left\{\left[\left(a^2 + \frac{\pi^2}{\xi}\right)\right] - a^2 Ra\left[\left(\eta a^2 + \pi^2\right) + a^2\left(\frac{A^2}{8}\right)\right]^{-1} + a^2 Rs\left[\left(a^2 + \pi^2\right)\tau + a^2\left(\frac{A^2}{8}\right)\right]^{-1}\right\} = 0 \tag{47}$$

The solution $A = 0$ corresponds to pure conduction, which we know to be a possible solution though it is unstable when Ra is sufficiently large. The remaining solutions are given by

$$\left(\frac{A^2}{8}\right)^2 + a_1\left(\frac{A^2}{8}\right) + a_0 = 0. \tag{48}$$

where,

$$a_1 = \beta\left[\pi^4\left(1+\tau\right) + \alpha_1^2 \xi\left\{-Ra + Rs + \alpha_1^2\left(\eta+\tau\right)\right\} + + \pi\alpha_1^2\left\{\eta + \tau + \xi\left(1+\tau\right)\right\}\right],$$

$$a_0 = \frac{\beta}{\alpha_1^2}\Big[Rs\,\alpha_1^2\left(\pi^2+\alpha_1^2\eta\right)\xi + \left(\pi^2+\alpha_1^2\right)\left\{-Ra\,\alpha_1^2\xi+\alpha_1^4\eta\xi + \right.$$

$$\left. + \pi^4 + \pi^2\alpha_1^2\left(\eta+\xi\right)\right\}\tau\Big]$$

and $\beta = \dfrac{1}{\alpha_1^2\left\{\alpha_1^2\xi+\pi^2\right\}}$.

Once we know the amplitude, we can find the heat and mass transfer.

In the study of convection in porous medium, the quantification of heat and mass transports are important. This is because onset of convection, as Rayleigh number is increased, is more readily detected by its effect on the heat and mass transports. If $\overline{H}$ and $\overline{J}$ are the rate of heat transport per unit area and the rate of mass transport per unit area then

$$\overline{H} = -\kappa_{Tz}\left\langle \frac{\partial T_{total}}{\partial z}\right\rangle_{z=0}, \quad \overline{J} = -\kappa_{sz}\left\langle \frac{\partial S_{total}}{\partial z}\right\rangle_{z=0} \tag{49}$$

where the angular bracket corresponds to a horizontal average and

$$T_{total} = T_0 - \Delta T\,\frac{z}{d} + T(x,z,t) \text{ and } S_{total} = S_0 - \Delta S\,\frac{z}{d} + S(x,z,t). \tag{50}$$

Substituting Eq. (40)- (41) into (50) and using the resultant equations in Eq. (49), we get

$$\overline{H} = \frac{\kappa_{Tz}\,\Delta T}{d}\left(1-2\pi C\right) \text{ and } \overline{J} = \frac{\kappa_{sz}\,\Delta S}{d}\left(1-2\pi E\right). \tag{51}$$

The thermal Nusselt number Nu and the Sherwood number Sh are defined by

$$Nu = \frac{\overline{H}}{\kappa_{Tz}\,\Delta T/d} = 1-2\pi C \text{ and } Sh = \frac{\overline{J}}{\kappa_{sz}\,\Delta S/d} = 1-2\pi E. \tag{52}$$

Writing C and E in terms of A, and substituting into Eq. (52), we get

$$Nu = 1 + \frac{2a^2\left(A^2/8\right)}{\left(\eta\, a^2+\pi^2\right)+a^2\left(A^2/8\right)} \tag{53}$$

and

$$Sh = 1 + \frac{2a^2\left(A^2/8\right)}{\tau\left(a^2+\pi^2\right)+a^2\left(A^2/8\right)} \tag{54}$$

The second term on the right-hand side of Eqs. (53) and (54) represent the convective contribution to heat and mass transport respectively. It is obvious that $Nu = Sh = 1$ for all $Ra \leq Ra_c^{St}$, indicating that the convection heat and mass transfer branches off from the conductive heat/ mass transfer line at the critical value of the Rayleigh number. It is important to note that our finite amplitude analysis is valid for Rayleigh number around the convection threshold. Therefore, the thermal Nusselt number and the Sherwood number in the present study are limited by an upper bound value of 3. Better results can only be obtained by including a greater number of terms in the Fourier series representation, which allows the variation of wave number as the value of Rayleigh number varies.

5. Results and discussion

The neutral stability curves in the $R_T - \alpha_1$ plane for various parameter values are shown in Figs. 1-5. From these figures it is clear that the neutral curves are connected in a topological sense. This connectedness allows the linear stability criteria to be expressed in terms of the critical Rayleigh number, $R_{T,c}$, below which the system is stable and unstable above. The points where the overstable solutions branch off from the stationary convection can be easily identified from these figures. Also we observe that for smaller values of the wavenumber each curve is a margin of the oscillatory instability and at some fixed wavenumber α_1 depending on the other parameters the overstability disappears and the curve forms the margin of stationary convection.

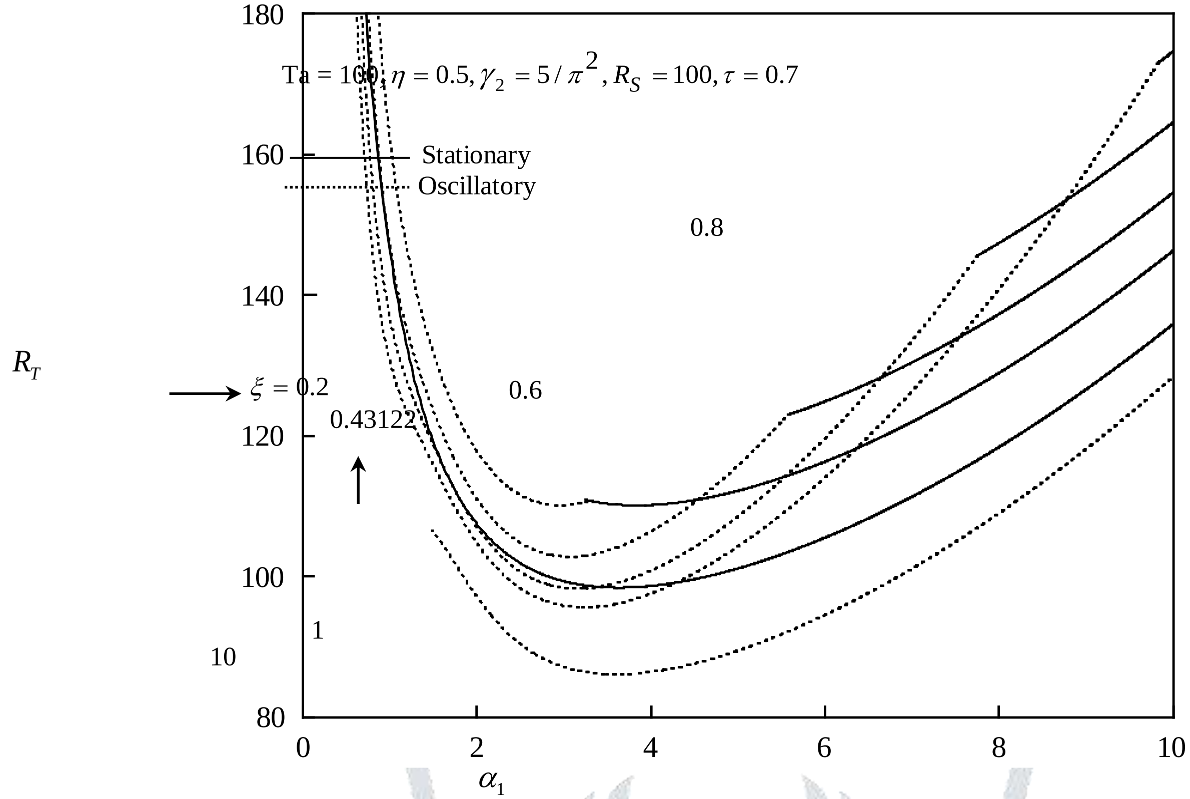


Fig.1 Neutral Stability curves for different values of mechanical anisotropy parameter ξ

The effect of mechanical anisotropy parameter ξ for the fixed values of $\eta = 0.5,$ $Ta = 100$, $\gamma_2 = 5/\pi^2$, $R_S = 100$ and $\tau = 0.7$ on the marginal stability curves is shown in Fig. 1 . It can be observed that an increase in ξ decreases the minimum of the Rayleigh number for oscillatory state. That is the effect of increasing the ratio of permeability ξ is to advance the onset of oscillatory convection. Further an important question is whether, under the critical conditions for the onset of instability, the instability manifests itself as stationary convection or as oscillatory convection. It is interesting to note that there is a critical value $\xi = \xi^*$ (e.g., for a fixed values of $Ta = 100$, $\eta = 0.5$, $\gamma_2 = 5/\pi^2$, $R_S = 100, \tau = 0.7$, we find $\xi^* = 0.43122$ see Fig.1) such that for $\xi < \xi^*$ the instability manifested as stationary convection and for $\xi \geq \xi^*$, the onset of instability

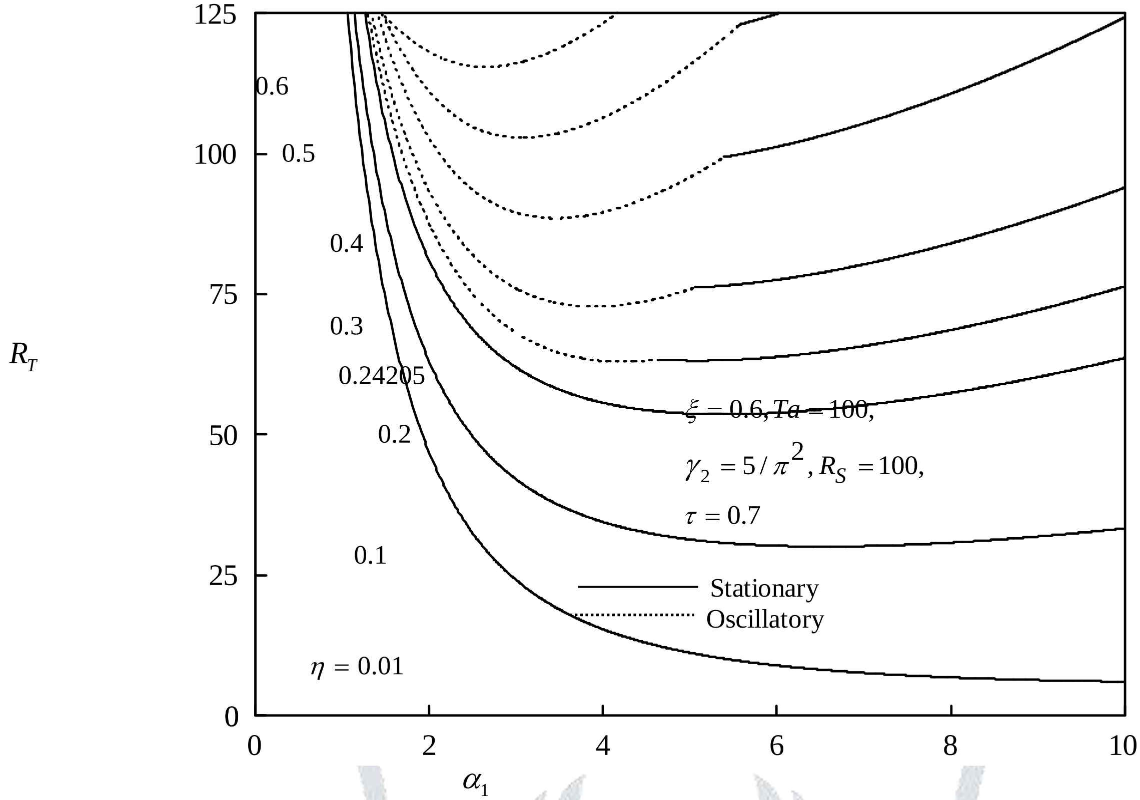


Fig. 2 Neutral Stability curves for different values of thermal anisotropy parameter η

manifests as oscillatory convection. An increase in the value of ξ $(=K_x/K_z)$ can be interpreted as follows: Let us keep the vertical permeability K_z(or the horizontal permeability K_x) fixed and vary the horizontal permeability K_x (or vertical permeability K_z). Then an increased horizontal permeability decreases the critical Rayleigh number for oscillatory mode. Thus, the effect of mechanical anisotropy is to allow the onset of oscillatory convection instead of stationary convection.

Fig. 2 indicates the effect of thermal anisotropy parameter η on the neutral curves for the fixed values of $\xi = 0.6$, $Ta = 100$, $\gamma_2 = 5/\pi^2$, $R_S = 100$ and $\tau = 0.7$. It is observed that critical value of Rayleigh number increases with η, indicating that the effect of thermal anisotropy parameter η is to inhibit the onset of convection. We also find from Fig. 2 that the onset of instability manifests as stationary convection for very small values of the thermal anisotropy parameter η. However as η increases, instability sets in as oscillatory mode. Similar to the mechanical anisotropy parameter, the thermal anisotropy parameter has a critical value $\eta = \eta^*$ such that for $\eta > \eta^*$ the stability sets in via oscillatory mode when other parameters are fixed. (e.g. $\xi = 0.6$, $Ta = 100$, $\gamma_2 = 5/\pi^2$, $R_S = 100$ and $\tau = 0.7$ we find $\eta^* = 0.24205$).

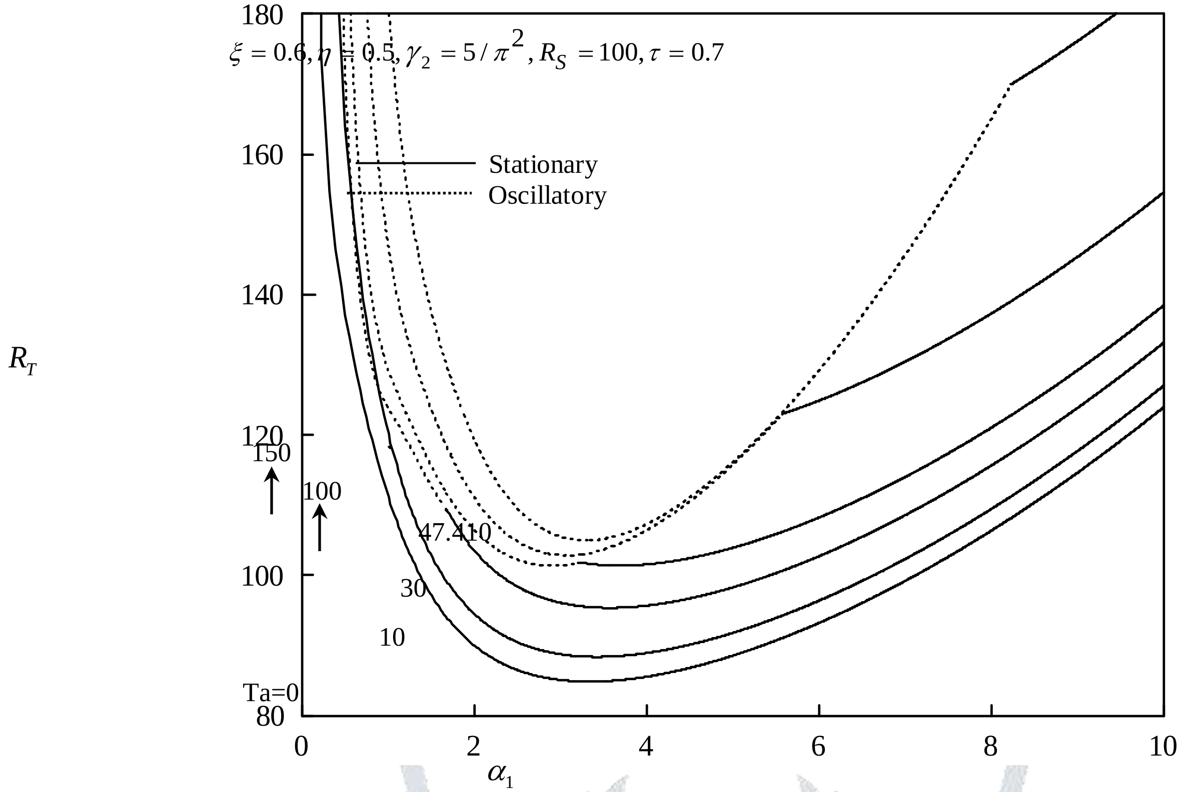


Fig. 3 Neutral Stability curves for different values Taylor number *Ta*

Fig. 3 depicts the effect of rotation on the neutral curves for fixed values $\xi = 0.6$, $\eta = 0.5$, R_S=100, $\tau = 0.7$ and $\gamma_2 = 5/\pi^2$. We find that the effect of increasing *Ta* is to increase the critical value of the Rayleigh number and the corresponding wavenumber implying that the rotation has a stabilizing effect on the double diffusive convection in porous medium. This can be explained as follows: rotation acts so as to suppress vertical motion, and hence thermal convection, by restricting the motion to the horizontal plane. Further this figure also indicates that for small *Ta* the instability manifests as stationary convection while as *Ta* is increased, the instability sets in as oscillatory convection. It is interesting to note that there is a critical value $Ta = Ta*$ (e.g., for a fixed values of $\xi = 0.6$, $\eta = 0.5$, $\gamma_2 = 5/\pi^2$, $R_S = 100$, $\tau = 0.7$, $Ta* = 47.410$ see Fig. 3) such that for $Ta < Ta*$ the instability manifested as stationary convection and for $Ta \geq Ta*$, the onset of instability manifests as oscillatory convection. Thus the effect of rotation is to allow the onset of oscillatory convection instead of stationary convection.

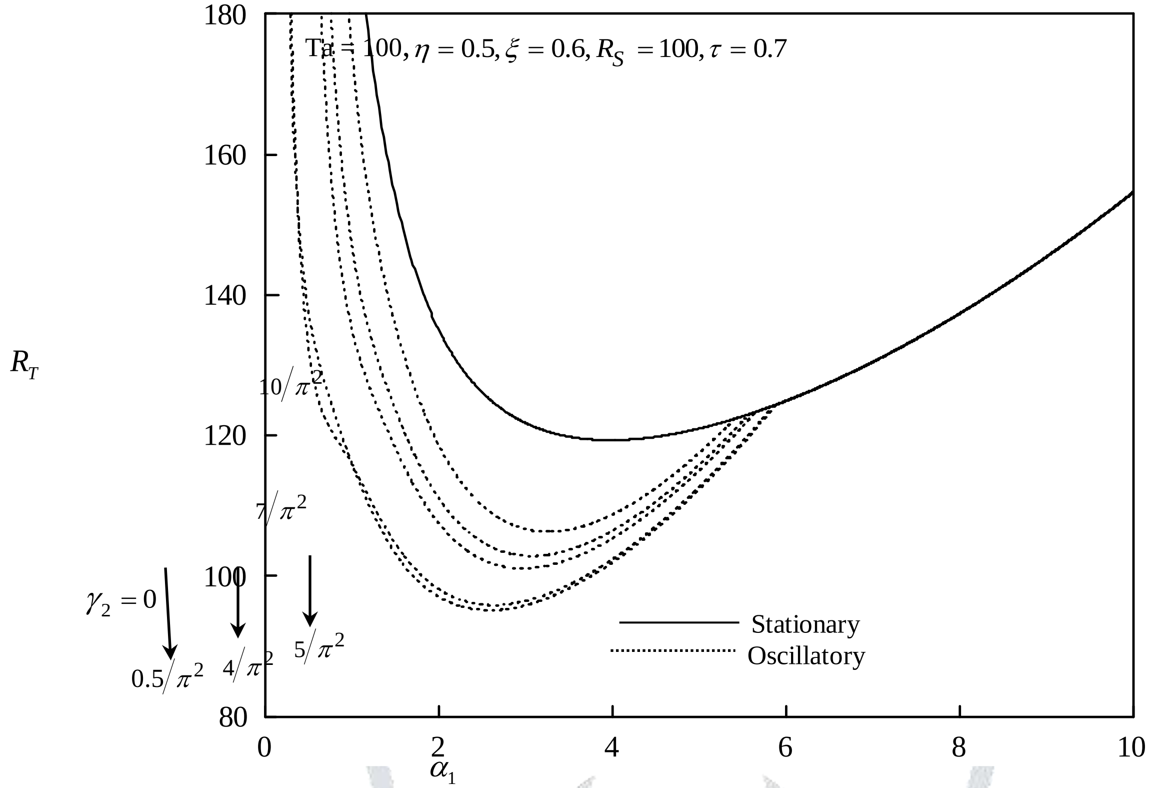


Fig. 4 Neutral Stability curves for different values of scaled Vadasz number γ_2

The neutral stability curves corresponding to $Ta=100, \xi=0.6, \eta=0.5, R_S=100, \tau=0.7$ and for different values of scaled Vadasz number γ_2 are presented in Fig. 4 The points where the overstable solutions branch off from the stationary convection curves can be identified clearly. From this figure it is evident that the characteristic curve for $\gamma_2=0$ provide the lower limit for all other curves. The neutral curves corresponding to different values of γ_2 lies between the curve for $\gamma_2=0$ and the stationary convection curve when the other

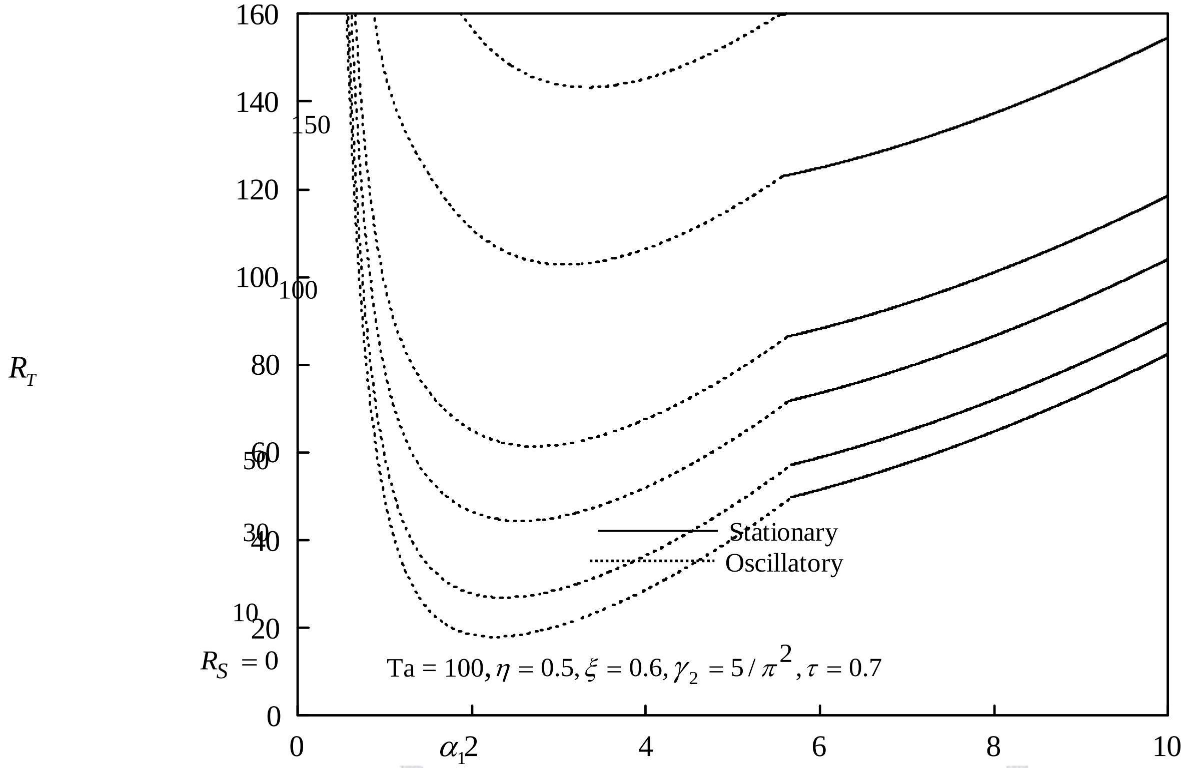


Fig. 5 Neutral Stability curves for different values of scaled solute Rayleigh number R_S

parameters are fixed. Fig. 5 indicates the effect of scaled solute Rayleigh number R_S on the neutral curves for the fixed values $\xi = 0.6,\ \eta = 0.5\ \ Ta = 100, R_S = 100,\ \tau = 0.7$ and $\gamma_2 = 5/\pi^2$. It is observed that critical value of Rayleigh number increases with R_S, indicating that the effect of solute Rayleigh number R_S is to inhibit the onset of convection. We also find from Fig. 5 that the onset of instability manifests as stationary convection for very small values of the solute Rayleigh number R_S. However as R_S increases, instability sets in as oscillatory mode.

The behavior of the stationary and oscillatory critical Rayleigh number as a function of the mechanical anisotropy parameter for different values of Taylor number is shown in Fig. 6. In the absence of rotation i.e., when $Ta = 0$, an increased mechanical anisotropy parameter reduces the stationary critical Rayleigh number. This is the classical result of Epherre (1977). However, in the presence of rotation, it is interesting to note that,

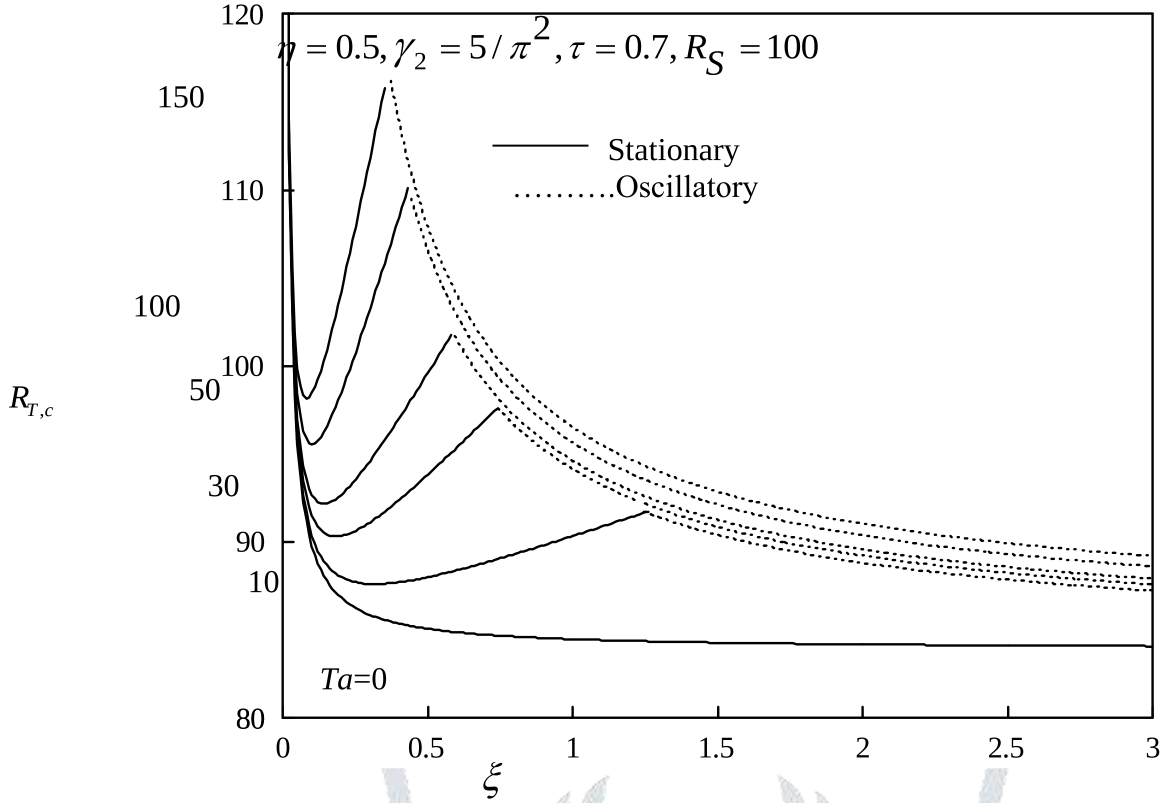
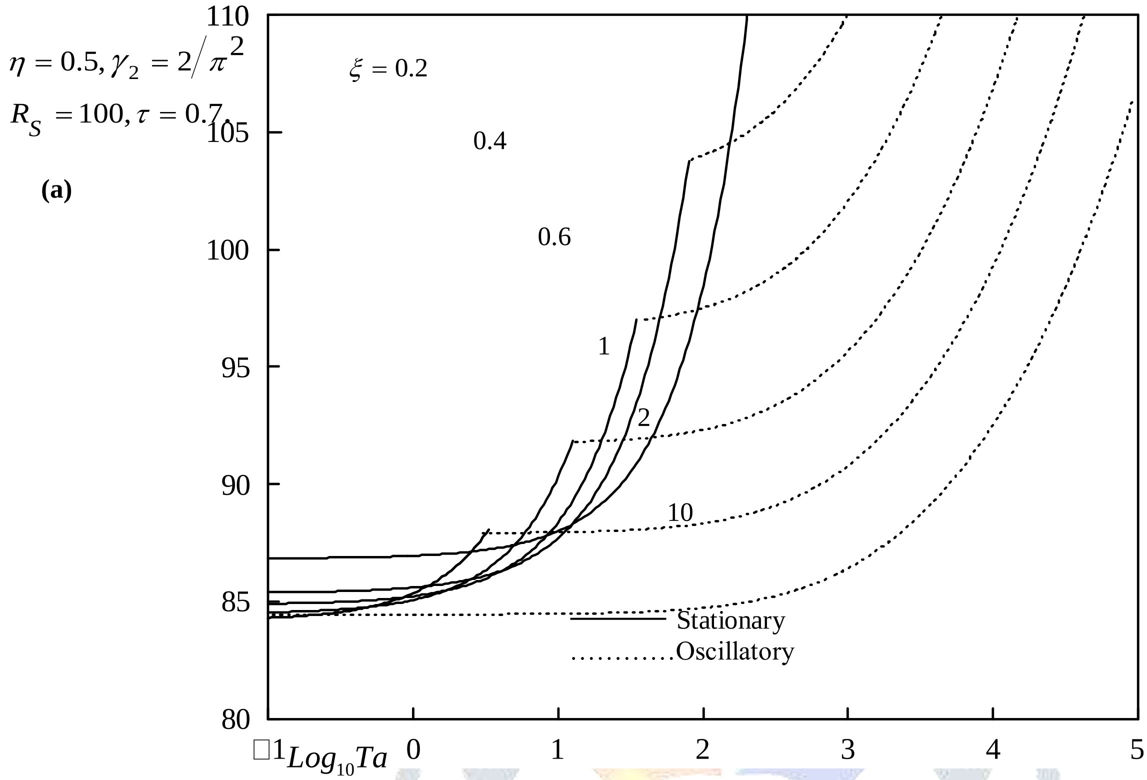


Fig. 6 Variation of scaled critical Rayleigh number with mechanical anisotropy parameter ξ for different values of Taylor number *Ta*.

the stationary critical Rayleigh number decreases to its minimum value with increasing ξ up to a certain value $\xi = \xi_c$ and as ξ is increased further beyond ξ_c, the critical Rayleigh number for stationary mode increases. Further the effect of mechanical anisotropy parameter on the stationary critical Rayleigh number is significant for large Taylor number. The critical Rayleigh number for oscillatory mode however decreases monotonically with increasing ξ. This figure also indicates the stabilizing effect of the rotation

The variation of the critical Rayleigh R_{Tc} with Taylor number *Ta* for different values of ξ, η, γ and R_S is shown in Figs. 7-8. We observe from these figures that the critical Rayleigh number increases with increase in *Ta* indicating that the effect of rotation is to inhibit the onset of thermal convection and it is in agreement with the corresponding problem of isotropic porous layer (Vadasz;1998) and pure fluid layer (Chandrashekhar;1981). However, this effect is not significant for the smaller values of the *Ta*. From each of these figures it is also observed that the convection first sets in as a stationary mode when the Taylor number is small. As the Taylor number is increased further, and when $Ta > Ta^*$ (critical value) which depends on the other parameters such as ξ, η, R_S and γ_2 the instability manifests as oscillatory convection. Therefore, the oscillatory mode is the most dangerous mode for the system with moderate and higher values of *Ta*. Further it is interesting to note that for larger values of the mechanical anisotropy parameter ξ, the oscillatory convection exists even for small values of the Taylor number (Fig. 7 (a)).



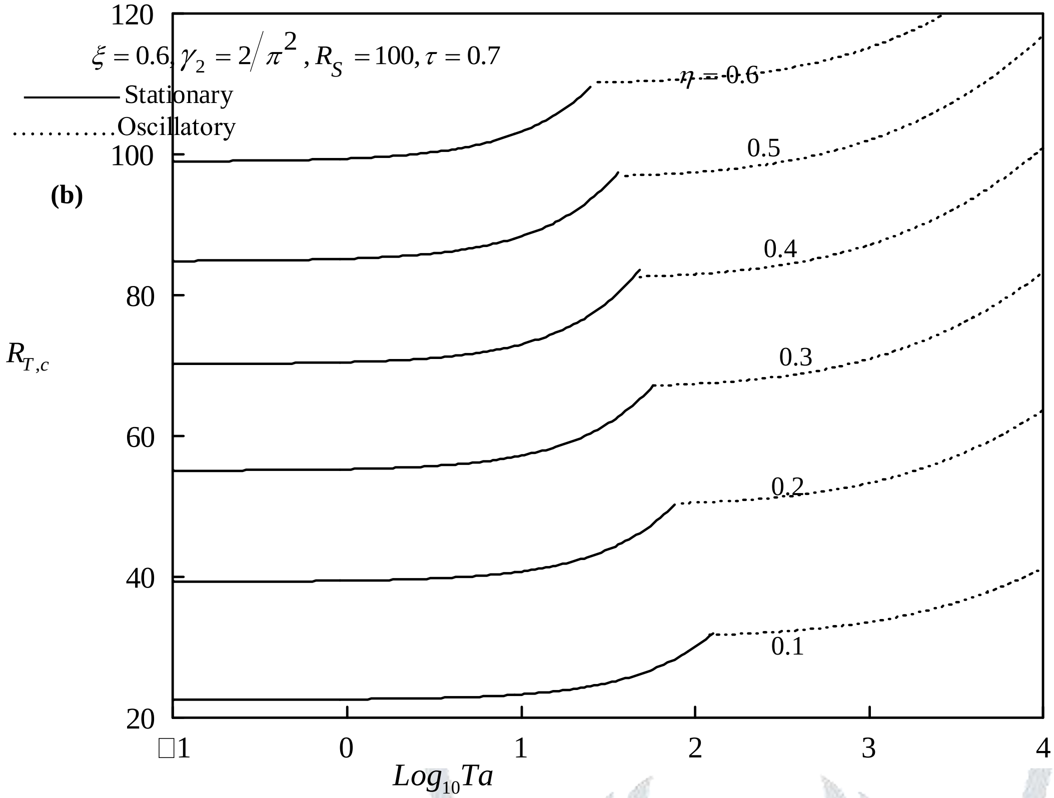
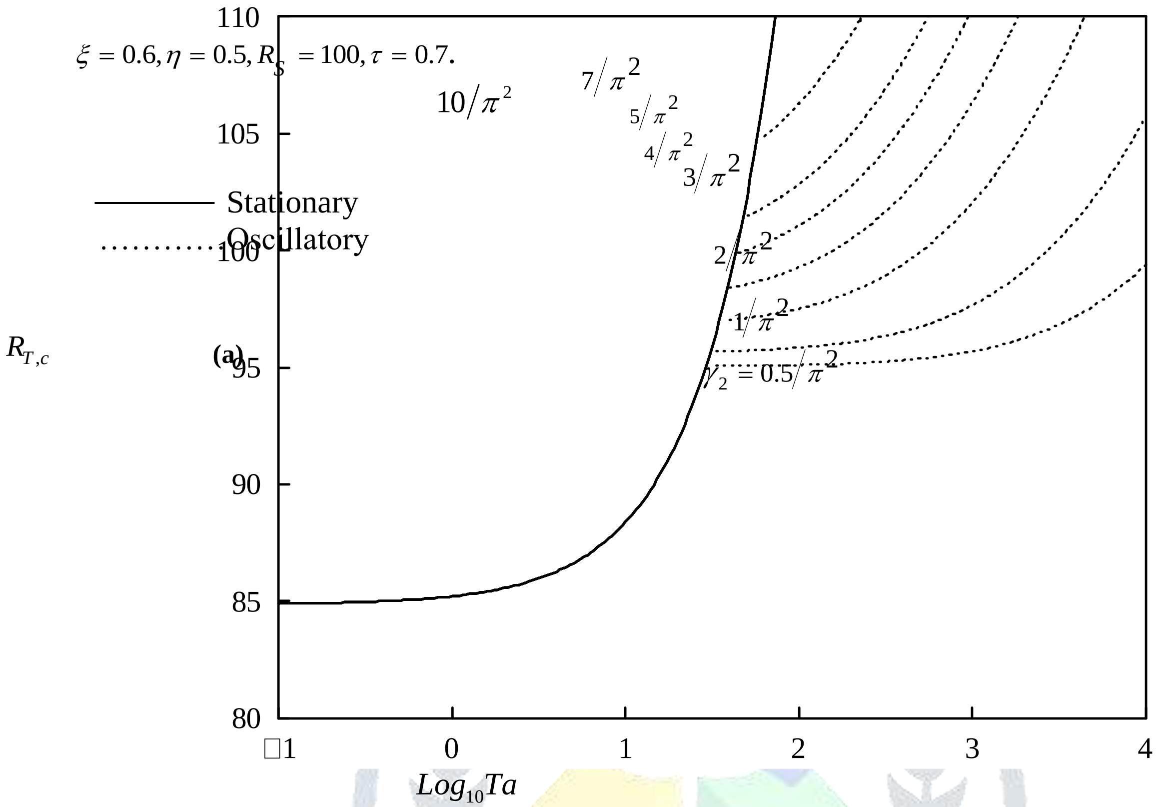


Fig. 7 Variation of scaled critical Rayleigh number with Taylor number for Different values of (a) ξ (b) η

In Fig. 7(a) the variation of R_{Tc} with *Ta* for different values of mechanical anisotropy parameter ξ is shown for the fixed values of $\eta = 0.5$, R_S=100 and $\gamma_2 = 2/\pi^2$. It is important to note that the critical Rayleigh number for the direct mode decreases with increase of ξ for smaller values of *Ta* whereas for the higher values of the *Ta* this trend reverses. The critical Rayleigh number for the overstable mode always decreases with increase in the value of ξ. Further it is important to note that the value of the Taylor number *Ta*, at which the transition from stationary to oscillatory mode takes place decreases with the increase of ξ. Therefore increasing ξ increases the possibility of overstable motions even for small values of the Taylor number.

The effect of thermal anisotropy parameter η on the critical Rayleigh number for stationary and oscillatory modes is shown in Fig. 7(b). We find that the critical Rayleigh number for both the modes increases with increase in η. Further the value of *Ta* at which

the transition from stationary to oscillatory mode occurs is found to decrease with η. The effect of scaled Vadasz number γ_2 and solute Rayleigh number R_S on the onset criteria is shown in Figs. 8 (a-b) respectively. Fig. 8(a) indicates that an increase in the value of γ_2 increases the critical Rayleigh number for the oscillatory mode. We observe from Fig. 8(b) that the effect of R_S is to delay the onset of convection both in stationary and oscillatory modes. Further the value of *Ta* at which the transition from stationary to oscillatory mode occurs is found to increase with both γ_2 and R_S.



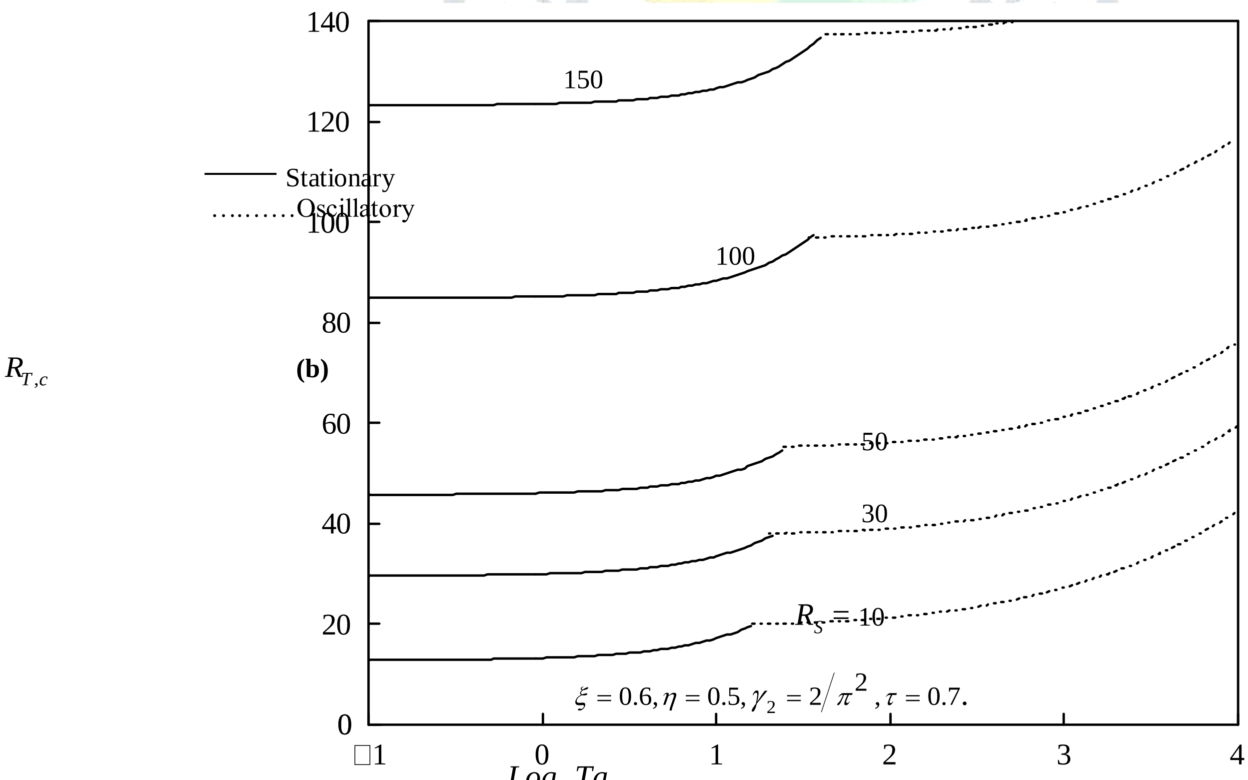
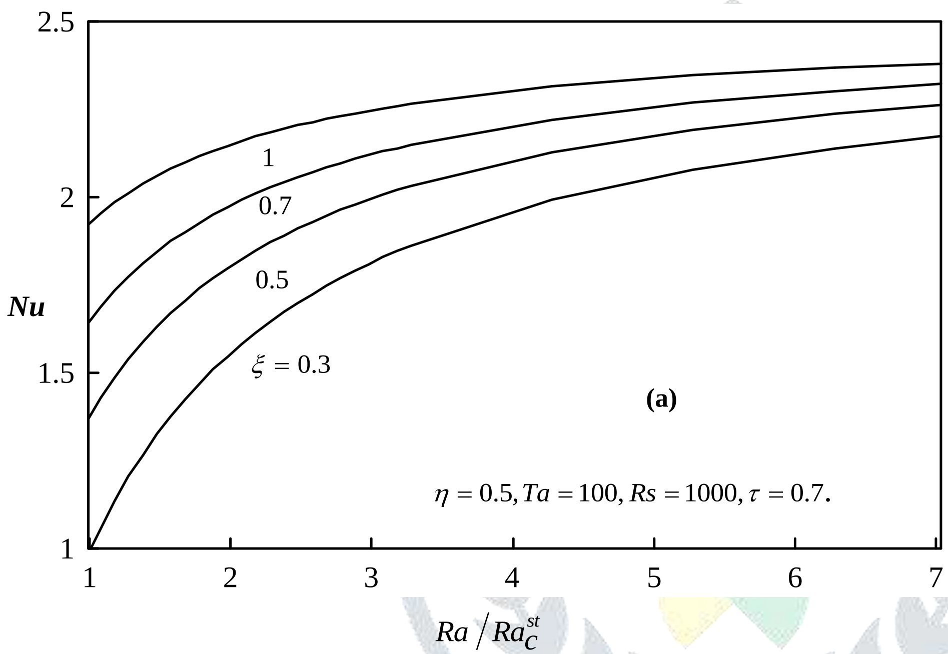


Fig. 8 Variations of scaled critical Rayleigh number with Taylor number for Different values of (a) γ_2 (b) R_S

The quantity of heat and mass transfer across the layer is computed by the thermal Nusselt number and Sherwood number as a function of Rs, ξ, η, Ta and Rs. This is depicted in the Rayleigh-Nusselt number plane through the Figs. 9-12. These figures reveal the quantitative effect of rotation, anisotropy parameters and the solute Rayleigh number on heat and mass transports. We observe that as Ra increases with its critical value, the heat and mass transport increase sharply and as Ra is increased further, they remain almost constant. It is also found that the heat and mass transports increase with increase in ξ and Rs, where as it decreases with the increase in η and Ta.



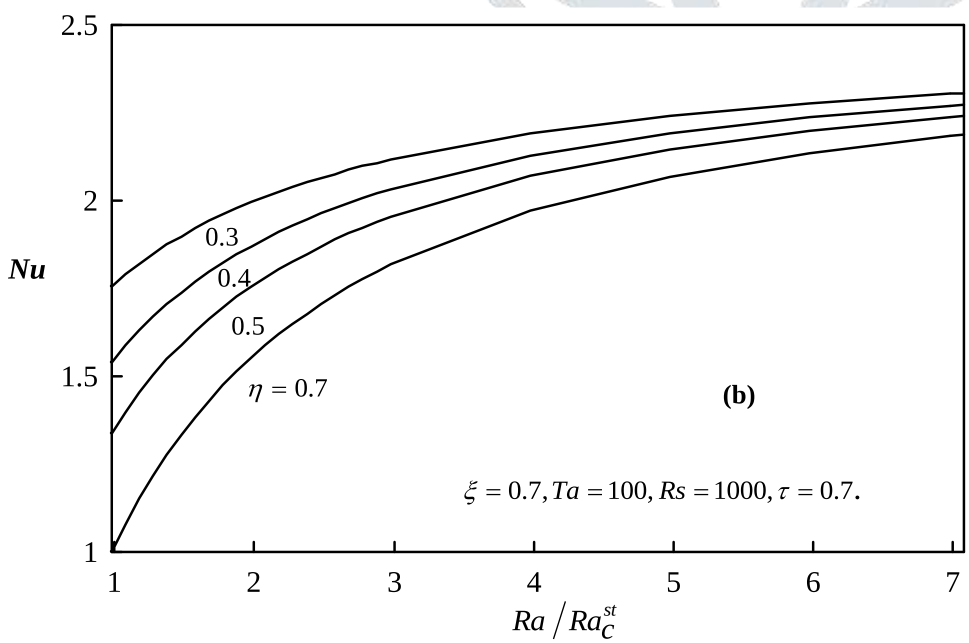
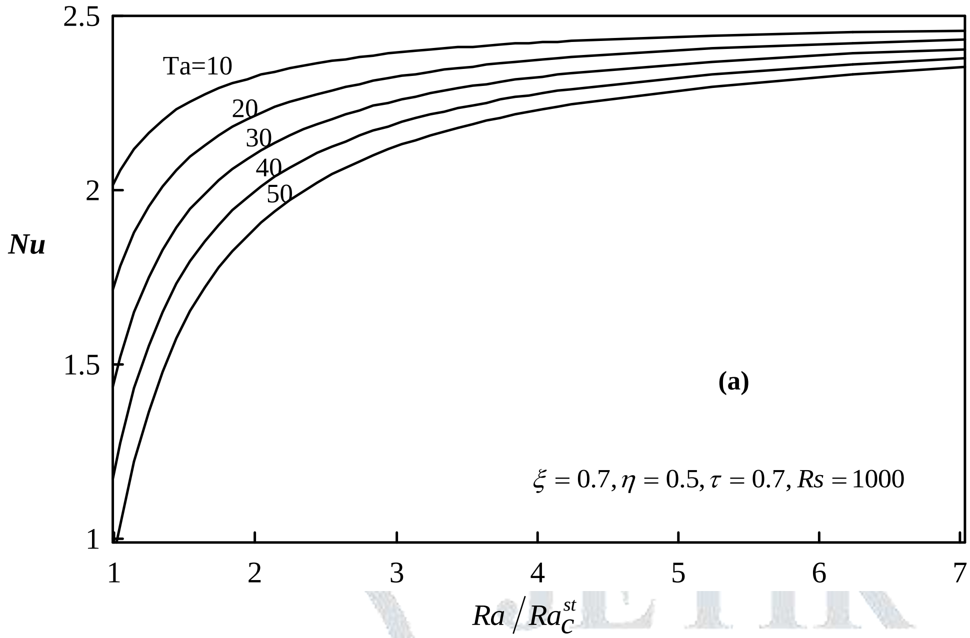


Fig. 9 Variation of thermal Nusselt number *Nu* with Rayleigh number Ra for different values of (a) ξ (b) η



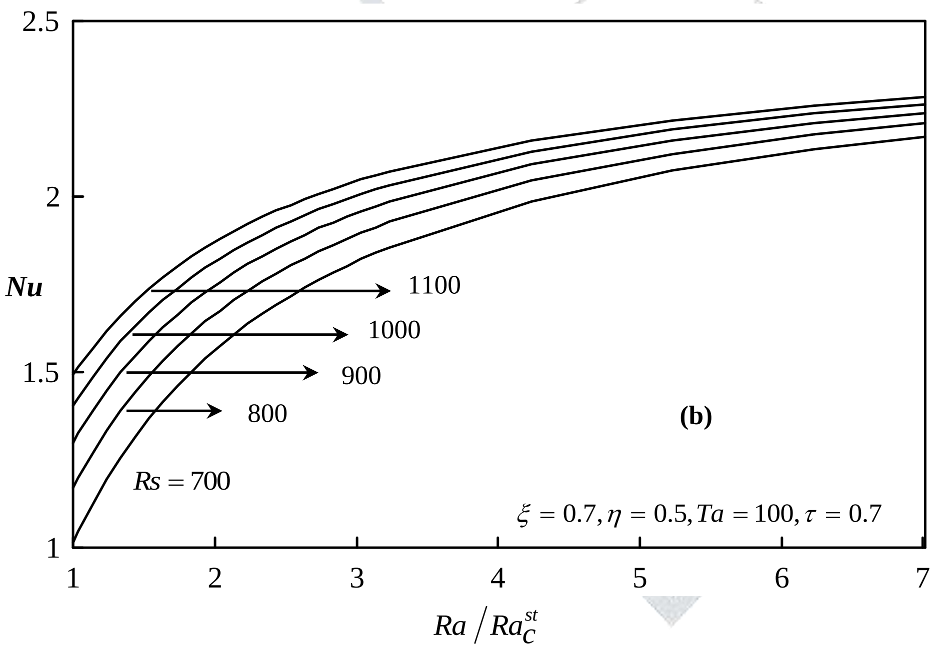


Fig. 10 Variation of thermal Nusselt number *Nu* with Rayleigh number *Ra* for different values of (a) *Ta* (b) *Rs*

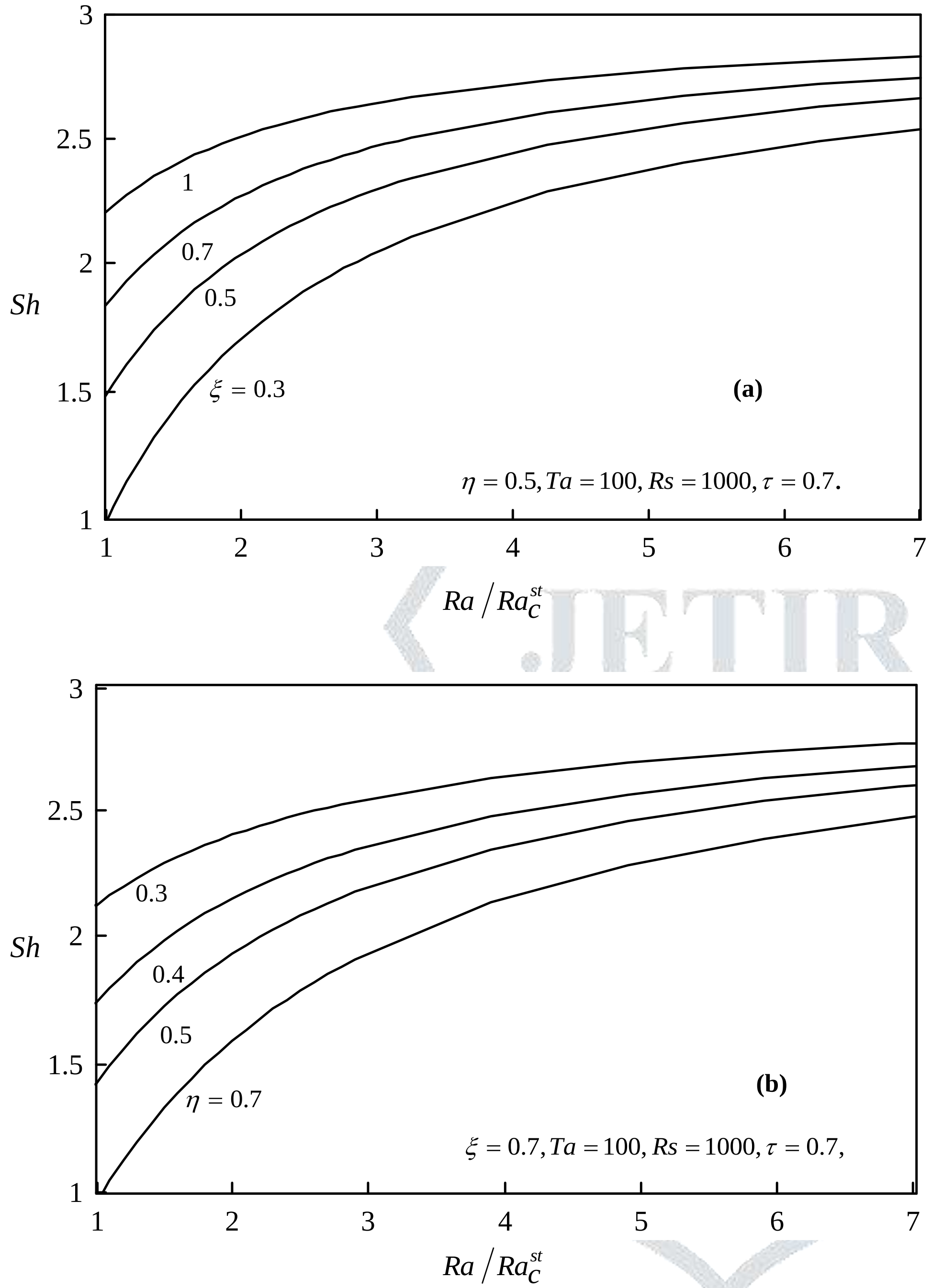
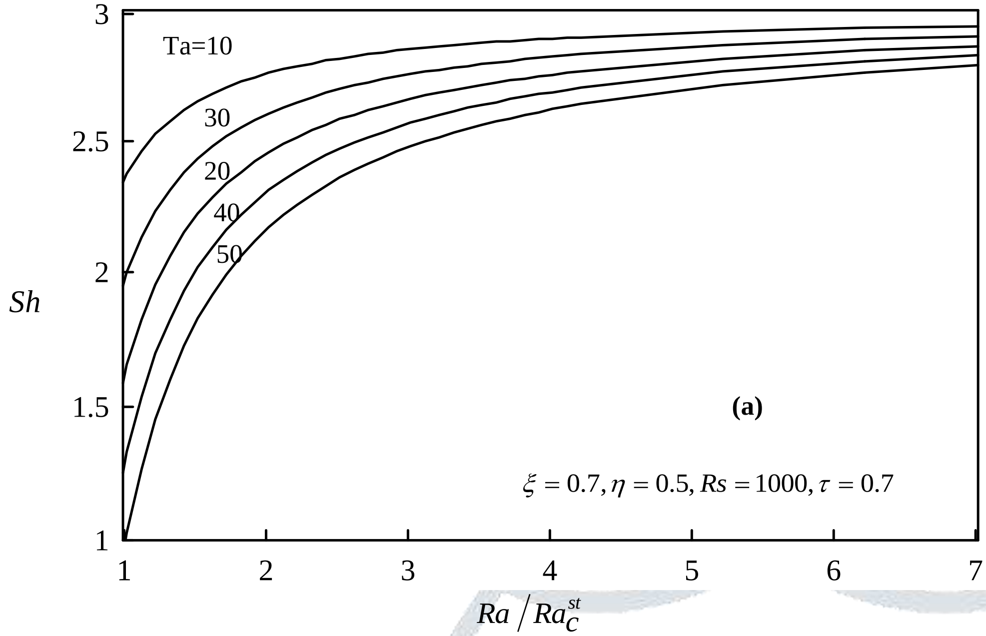


Fig. 11 Variation of Sherwood number *Sh* with Rayleigh number *Ra* for different values of (a) ξ (b) η



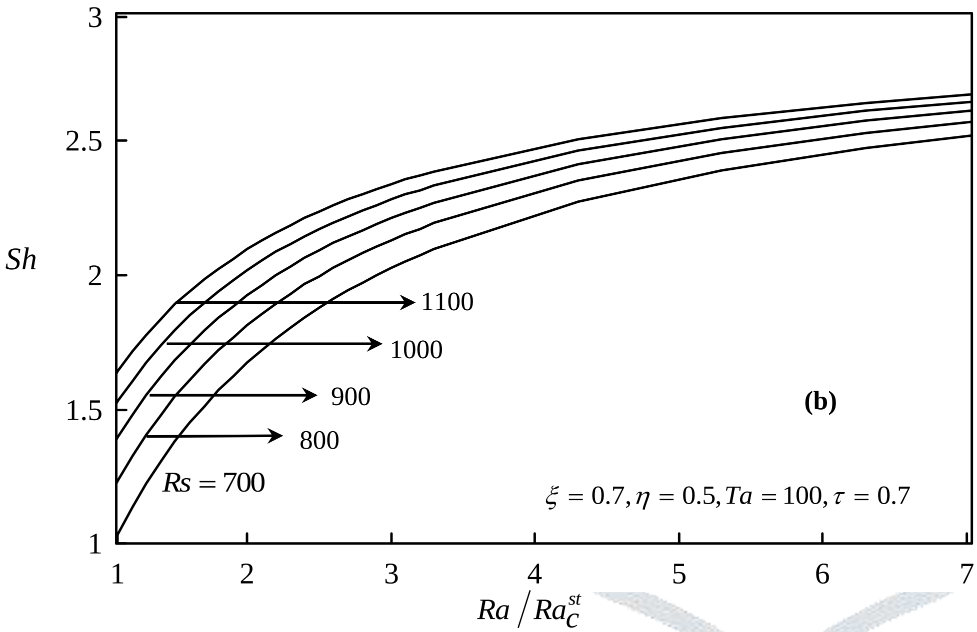


Fig. 12 Variation of Sherwood number *Sh* with Rayleigh number *Ra* for different values of (a) *Ta* (b) *Rs*

This is because the effect of rotation and thermal anisotropy is to inhibit the onset of double diffusive convection.

6 Conclusions

The onset of double diffusive convection in a fluid saturated anisotropic porous layer is investigated using both linear and nonlinear stability analyses. The linear theory provides the criteria for the onset of stationary and oscillatory convection and the nonlinear theory, which is based on the truncated Fourier series method, provides a method to measure the convection amplitudes thereby the rate of heat and mass transfers. The following conclusions are drawn:

1. The oscillatory mode is most favorable for a system with moderate and high values of the Taylor number. However, for large value of the mechanical anisotropy, the oscillatory motions exist even for small values of the Taylor number.

2. The value of Taylor number at which the transition from stationary mode to the oscillatory mode takes place, decreases with increase in the value of the mechanical and thermal anisotropy parameters.

3. The effect of increasing the value of mechanical and thermal anisotropy parameter is to allow the onset of convection to be oscillatory rather than stationary. The effect of mechanical anisotropy parameter is more pronounced compared to that of the thermal anisotropy parameter.

4. The effect of Darcy–Prandtl number is to delay the onset of oscillatory convection.

5. The thermal Nusselt number and solute Nusselt number increase with increase in ξ and Ras whereas it decreases with the increase in η and T a.

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