Average Number of Fuzzy Edge Graceful Labeling on Wheel Graph and Fan Graph

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Abstract: In this paper introduce the concept of average number of fuzzy edge graceful labeling. Average number of Fuzzy edge graceful labeling extends to graphs such as Wheel graph and Fan graph. It is proved that all average number of Wheel graph and average number of Fan graph is in between membership function [0,1].
Keywords: Graceful labeling, Fuzzy edge graceful labeling, Wheel graph, Fan graph.

I. INTRODUCTION

Fuzzy is mathematical framework to exemplify the phenomenon of uncertainty in real life tribulations. Zadeh [1] introduced the fuzzy set as a class of object with a continuum of grades of membership. The fuzzy graph was introduced by Kaufmann [4] in 1973. In contrast of classical crisp sets where a set is defined by either membership or non-membership, the fuzzy approach relates to grade of membership between [0,1]. Fuzzy graphs have many more applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision.

In this paper we discuss when the fuzzy graph has Fuzzy Edge Graceful labeling on average number of Wheel graph and Fuzzy Edge Graceful labeling on average number of Fan graph.

II. PRELIMINARIES AND MAIN RESULTS

Definition: 2.1

Let U and V be two sets. Then ρ is said to be a fuzzy relation from U and V if ρ is a fuzzy set of U × V.

Definition: 2.2

A fuzzy graph G = (σ , μ) is a pair of functions σ : v → [0,1] and μ : v x v → [0,1], where for all u , v ∈ V. We have μ (u , v) ≤ σ(u) ∧ σ(v).

Definition: 2.3

A labeling of a graph is an assignment of values to the vertices and edges of a graph.

Definition: 2.4

A graceful labeling of a graph G with q edges is an injection f : v(G) →{0, 1, 2,…, q} such that when each edge x , y ∈ $E(u) is\ assigned\ the\ lable$ | f(x) – f(y)|, all of the edge lables are distinct.

Definition: 2.5

An edge graceful labeling is defines as, In a graph G, We denote the set of edges by E(G) and the vertices by V(G). Let q be the cardinality of E(G) and p be that of V(G). Once the labeling of edges is given, a vertex u of the graph is labeled by the sum of the labels of edges incident to it, modulo p. ie., V(u) = $\sum E(e) mod$ |V(G)| where V(G) is the label for the vertex and E(e) is the assigned value of an edge incident to u. A graph G is said to the edge graceful if it admits edge graceful labeling.

Definition: 2.6

A graph G = (σ,μ) is said to be a fuzzy labeling graph if σ : V→ [0,1] and μ: V xV→[0,1] is injective such that the membership value of edges and vertices are distinct and μ(u,v) < σ(u) ∧ σ(v) for all u,vϵ V.

Definition: 2.7

In graph theory, a wheel graph w_n is a graph with n vertices (n ≥ 4) formed by connecting a single vertex to all vertices of (n-1) cycle.

Definition: 2.8

In a graph theory, a fan graph $F_{m+,n}$ is defined as the graph join $\overline{k_m}$ + P_n , where $\overline{k_m}$ is the empty graph on m nodes and P_n is the path graph on n nodes.

The case m=1 corresponds to the usual fan graph, while m=2 corresponds to the double fan graph.

III. FUZZY EDGE GRACEFUL LABELING

Definition: 3.1

A Graph G = (σ,μ) (n≥3) said to be a fuzzy edge graceful labeling if σ:V→[0,1] and μ:VxV→[0,1] is bijective and once the membership value of edges is given, the membership value of a vertex is labeled by the sum of the membership values of the edges incident to it. (i.e.) $\sigma(w)= \sum \frac{1}{n}\mu(E(e))$, where n is the number of incident vertex, σ(w) is the average membership value of a vertex w and μ(E(e)) is the membership value of an edge incident to w.

Definition: 3.2

A wheel graph with fuzzy labeling is called a fuzzy wheel graph. A wheel in a fuzzy graph consists of two nodes U and V with |V| =1 with |U| > |, such that $\mu(w,w_i) > 0$, where i=1 to n-1 and $\mu(w_i , w_{i+1}) > 0$ where i = 1 to n-2.

Definition: 3.3

In a fuzzy wheel graph, if all edges are distinct then it is called fuzzy edge graceful wheel graph.

Definition: 3.4

A fan graph with fuzzy labeling is called a fuzzy fan graph. A fan in a fuzzy graph consists of two node sets F and F_n with |F| =1 and $|F_n| > 1$, such that $\mu(F,F_i) > 0$, where i=1 to n and $\mu(F_i , F_{i+1}) > 0$ where i = 1 to n-1.

Definition: 3.5

In a fuzzy fan graph, if all edges are distinct then it is called fuzzy edge graceful fan graph.

Proposition: 3.6

Every fuzzy wheel graph W_n is fuzzy average edge graceful wheel graph.

Proof:

A wheel graph W_n is a graph with n vertices if only $n \geq 3$.

A wheel in a fuzzy graph consists of two node sets w and w_n with |w| = 1 and $|w_n| > 1$ such that $\mu(w ,w_i) > 0$, where i =1 to n-1 and $\mu(w_i , w_{i+1}) > 0$ where i = 1 to n-2.

Here σ:w→[0,1] and μ:w×w→[0,1] is defined by $\sigma(w_i) = \sum \frac{1}{n} \mu(w, w_i)$ where i=1 to n-1.

Example: 3.7

Consider the Wheel graph W_5.

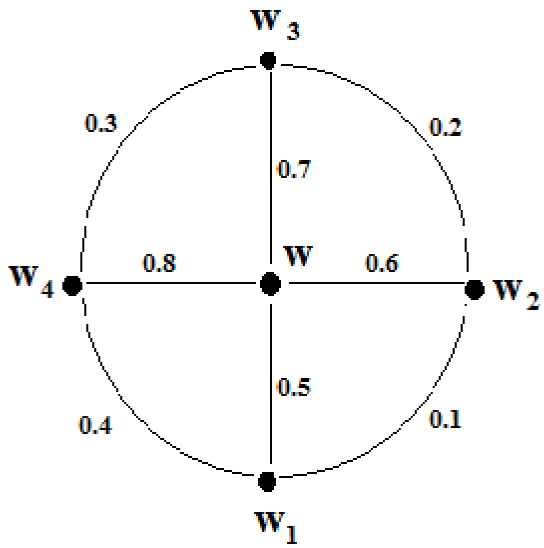
Here σ :V→[0,1] and μ :V×V→[0,1]

$\sigma(w) = \sum \frac{1}{n} \mu(w,w_i)$, where n is the number of incident vertex.

$= \frac{1}{4}(\mu(w,w_1)+ \mu(w,w_2)+ \mu(w,w_3)+ \mu(w,w_4))$

$= \frac{1}{4}$ (0.5+0.6+0.7+0.8)

= 0.7



Fuzzy Edge Graceful wheel graph W_5

$\sigma(w_i) = \sum \frac{1}{n}(\mu(E(e))$, where n is the number of incident vertex.

$\sigma(w_1) = \frac{1}{3}(\mu(w,w_1)+\mu(w_2,w_1)+\mu(w_4,w_1))$

$= \frac{1}{3}(0.5+0.1+0.4)$

$= 0.3$

$\sigma(w_2) = \frac{1}{3}(\mu(w,w_2)+\mu(w_3,w_2)+\mu(w_1,w_2))$

$= \frac{1}{3}(0.6+0.2+0.1)$

$= 0.3$

$\sigma(w_3) = \frac{1}{3}(\mu(w,w_3)+\mu(w_4,w_3)+\mu(w_2,w_3))$

$= \frac{1}{3}(0.7+0.3+0.2)$

$= 0.4$

$\sigma(w_4) = \frac{1}{3}(\mu(w,w_4)+\mu(w_3,w_4)+\mu(w_1,w_4))$

$= \frac{1}{3}(0.8+0.3+0.4)$

$= 0.5$

Since all edge values $\mu(w,w_i) > 0$, where i=1 to n-1 and $\mu(w_i, w_{i+1}) > 0$ where i=1 to n-2 and which are less than the values of the vertices, this wheel graph is fuzzy wheel graph as well it admits fuzzy edge graceful labeling and since all edges are distinct, this average number of Wheel graph is in between membership function [0,1].

According to this condition, we can come to the conclusion that every fuzzy wheel graph w_n is a average number of fuzzy graceful wheel graph.

Proposition: 3.8

Every fuzzy fan graph $F_{1,n}$ is a fuzzy average edge graceful fan graph.

Proof:

A fan graph $F_{1,n}$ is a graph with m=1.

A fan in a fuzzy graph consists of two node sets F and F_n with $|F| = 1$ and $|F_n| > 1$ such that $\mu(F, F_i) > 0$, where i=1 to n and $\mu(u_i, u_{i+1}) > 0$ where i=1 to n-1.

Here $\sigma:F\rightarrow[0,1]$ and $\mu:F\times F\rightarrow[0,1]$ is defined by $\sigma(F_i) = \sum \frac{1}{n}\mu(F,F_i)$ where i=1 to n-1.

Example: 3.9

Consider the fan graph $F_{1,3}$.

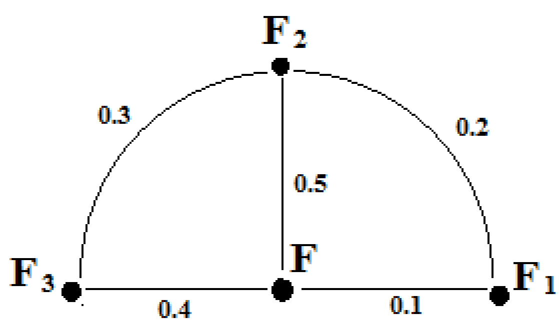
Here $\sigma:F\rightarrow[0,1]$ and $\mu:F\times F\rightarrow[0,1]$

$\sigma(F) = \sum\frac{1}{n}\mu(F,F_i)$,where n is the number of incident vertex.

$= \frac{1}{3}(\mu(F,F_1)+ \mu(F,F_2)+ \mu(F,F_3))$

$= \frac{1}{3}(0.1+0.5+0.4)$

$= 0.3$



Fuzzy Edge Graceful Fan graph $F_{1,3}$

$\sigma(F_i) = \sum\frac{1}{n}(\mu(E(e))$, where n is the number of incident vertex.

$\sigma(F_1) = \frac{1}{2}(\mu(F,F_1)+ \mu(F_2,F_1))$

$= \frac{1}{2}(0.1+0.2)$

$= 0.2$

$\sigma(F_2) = \frac{1}{3}(\mu(F,F_2)+ \mu(F_1,F_2)+ \mu(F_3,F_2))$

$= \frac{1}{3}(0.5+0.2+0.3)$

$= 0.3$

$\sigma(F_3) = \frac{1}{2}(\mu(F,F_3)+ \mu(F_2,F_3))$

$= \frac{1}{2}(0.4+0.3)$

$= 0.4$

Since all edge values $\mu(F,Fi)>0$, where i=1 to n and $\mu(F_i,F_{i+1}) > 0$ where i=1 to n-1 and which are less than the values of the vertices, this fan graph is fuzzy fan graph as well it admits average number of fuzzy edge graceful labeling and since all edges are distinct, this average number of Fan graph is in between membership function [0,1].

According to this condition, we can come to the conclusion that every fuzzy fan graph $F_{1,n}$ is a average number of fuzzy graceful fan graph.

IV. CONCLUSION

In this paper, the concepts of average number of fuzzy edge graceful wheel graph, average number of fuzzy edge graceful fan graph, average number of fuzzy edge graceful friendship graph have been discussed.

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