Reflection of P and SV waves at the free surface of homogenous viscoelastic half-space

Dr.Sandeep Kumari

Department of Mathematics, Govt. College, Mahendergarh, Haryana, India.

Abstract

In this paper, the problem of the reflection of P and SV waves at the free surface of homogeneous viscoelastic half-space media has been solved by a direct method. By using potential method with assumption that the wave equation satisfied by the displacement potentials ϕ and φ but in general, a homogeneous isotropic elastic medium does not remain a isotropic but becomes an anisotropic due to initial stress. The aim of this is to solve the problem by direct method to avoid this ambiguity. The amplitude ratios are obtained in viscoelastic half space.

Keywords: Reflection coefficient, refraction coefficient, Elastic intensity, initial stress, rotation gravity field, magnetic field, anisotropic media.

Introduction

The theory of Rayleigh waves has been widely summarized in the book of Ewing et al. (1957).Biot (1962) investigated the result of gravity on Rayleigh waves. He assumed that gravity creates an initial stresses of hydrostatic nature adapting the same theory of initial stress and using the dynamical equation of motion for the initial compressive stress.

The result of viscosity on the propagation of Rayleigh waves has also been shown a few authors such as Das and Sengupta (1992) but none of them considered the initial stress might be present in the media. But the earth is an initially stressed medium. Hence it should be geophysical interest to see how the initial stress influence the propagation of waves in elastic or a viscoelastic medium.

Das (1995) studied the surface waves in higher order viscoelastic involving time rate of change of strain and stress under the influence of gravity. General equation for the wave velocity was derived. This equation was used to examine various kinds of surface waves including Rayleigh waves, Love waves and Stonley waves. Addy and Chakraborty (2005) showed the result of temperature as well as the initial hydrostatic stress on the propagation of Rayleigh waves in a viscoelastic half space. The authors solved this problem by introducing potential method which is not applicable here.

The present chapter is a sincere effort to study the results on reflection coefficient of P and S waves in a viscoelastic half space using direct method.

Basic Equations

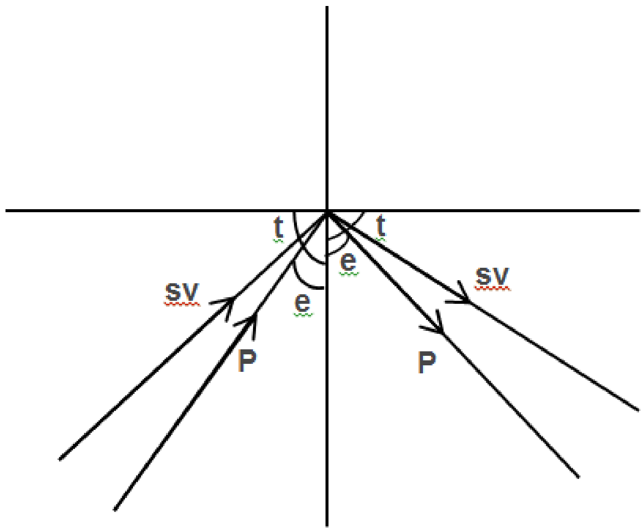

Fig 1 Plane wave at the free surface of viscoelastic half space

Consider a voigt type viscoelastic half –space y≥0, the boundary of which y=0 is free from the traction, incremental stress s_{ij} together with incremental strain e_{ij} are produced in it, which are measured with reference to the axes as showed in Fig (5.1).

The dynamic equations of equilibrium are given by

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} = \rho\left(\frac{\partial^2 u}{\partial t^2}\right) \qquad \text{.......................... (1)}$$

$$\frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} = \rho\left(\frac{\partial^2 v}{\partial t^2}\right).$$

Here s_{11} and s_{22} are the incremental stresses along the x and y axes respectively. $s_{12}$is the incremental shear stress in the xy plane, u and v are the displacement components along x and y axes respectively.

The stress –strain relations in the voigt type viscoelastic half space are given by

$$s_{11} = \left[(\lambda + 2\mu) + (\lambda' + 2\mu')\frac{\partial}{\partial t}\right] e_{xx} + \left[\lambda + \lambda'\frac{\partial}{\partial t}\right] e_{yy}\,, \qquad \text{........................... (2)}$$

$$s_{11} = [(\lambda + 2\mu) + (\lambda' + 2\mu')\frac{\partial}{\partial t}]e_{yy} + [\lambda + \lambda'\frac{\partial}{\partial t}]e_{xx}\,, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$s_{12} = 2\left(\mu + \mu'\frac{\partial}{\partial t}\right)e_{xy}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The incremental strain components are given as

$$e_{xx} = \frac{\partial u}{\partial x}, e_{yy} = \frac{\partial v}{\partial y}, e_{xy} = \frac{1}{2}[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}], \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Equations (1) with the help of equations (2), (3), (4) and (5) change to

$$\left[(\lambda + 2\mu) + (\lambda' + 2\mu')\frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial x^2} + \left[(\lambda + \mu) + (\lambda' + \mu')\frac{\partial}{\partial t}\right]\frac{\partial^2 v}{\partial x \partial y} + \left[\mu + \mu'\frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial y^2} = \rho\left(\frac{\partial^2 u}{\partial t^2}\right), \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\left[(\lambda + 2\mu) + (\lambda' + 2\mu')\frac{\partial}{\partial t}\right]\frac{\partial^2 v}{\partial y^2} + \left[(\lambda + \mu) + (\lambda' + \mu')\frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial x \partial y} + \left[\mu + \mu'\frac{\partial}{\partial t}\right]\frac{\partial^2 v}{\partial x^2} = \rho\left(\frac{\partial^2 v}{\partial t^2}\right). \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Plane wave propagation

For plane waves of circular frequency ω, number k and phase velocity C, incident at angle θ with the Y- axis,

We use direct method to solve the problem, Let

$$u = U\exp(iP_1), \quad v = V\exp(iP_1), \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Where U and V are amplitude factors and

$$P_1 = k[Ct - (x\sin\theta - y\cos\theta)],$$

P_1 is the phase factor associated with the incident waves, similarly for waves reflected at y=0, we assume

$$u = U\exp(iP_2), \quad v = V\exp(iP_2). \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

We use $P_2 = k[Ct - (x\sin\theta + y\cos\theta)]$, is the phase factor associated with the reflected waves.

Using equation (8) in equations (6) and (7), we get

$U[\{(\lambda+2\mu)+(\lambda'+2\mu')(ikC)\}\sin^2\theta+\{(\mu+\mu')ikC\}\cos^2\theta-\rho C^2]+V[\{(\lambda+\mu)+(\lambda'+\mu')(ikC)\}\sin\theta\cos\theta]=0.$ .... (10)

And

$U[(\lambda+\mu)+(\lambda'+\mu')(ikC)]\sin\theta\cos\theta+V[\{(\lambda+2\mu)+(\lambda'+2\mu')(ikC)\}\cos^2\theta+\{(\mu+\mu')(ikC)\}\sin^2\theta-\rho C^2]=0.$ .... (11)

Equations (10) and (11) can be written as

$$-[A-\rho C^2]U\pm BV=0, \qquad \text{........................ (12)}$$

$$\pm UB-[D-\rho C^2]V=0. \qquad \text{.............................. (13)}$$

Here upper and the lower symbol corresponds to the incident waves and the reflected waves respectively and A, B and D are given by

$A=[\{(\lambda+2\mu)+(\lambda'+2\mu')(ikC)\}\sin^2\theta+\{(\mu+\mu')ikC\}\cos^2\theta],$ .... (14)

$B=[\{(\lambda+\mu)+(\lambda'+\mu')(ikC)\}\sin\theta\cos\theta],$ .... (15)

$D=[\{(\lambda+2\mu)+(\lambda'+2\mu')(ikC)\}\cos^2\theta+\{(\mu+\mu')(ikC)\}\sin^2\theta].$ .... (16)

The set of equation (12) and (13) has non -trivial solution if and only if

$$\begin{vmatrix}-(A-\rho C^2) & \pm B \\ \pm B & -(D-\rho C^2)\end{vmatrix}=0. \qquad \text{.................. (17)}$$

On expansion, we get

$$2\rho C^2=(A+D)\pm\sqrt{(A-D)^2+4\beta^2}\,, \qquad \text{.................. (18)}$$

From equation (18), it is clear that, in general, in this two dimensional model of homogeneous viscoelastic medium there exist two type of plane waves whose velocities depends on the angle of incident. Let $C_P'(\theta) and C_s'(\theta)$ be two values of C linked with the upper and lower symbols correspondingly, in equation (18).

Consider a semi-infinite medium inhabiting the region y≥0. For the total displacement field (u, v, 0) in such a medium, we may assume

$$u=U_{i_1}\exp(i\theta_1)+U_{i_2}\exp(i\theta_2)+U_{r_1}\exp(i\theta_3)+U_{r_2}\exp(i\theta_4),$$

$$v = V_{i_1}\exp(i\theta_1) + V_{i_2}\exp(i\theta_2) + V_{r_1}\exp(i\theta_3) + V_{r_2}\exp(i\theta_4). \qquad \ldots\ldots\ldots\ldots (19)$$

Where

$$\theta_1 = \theta_1(x,y) = \left(\frac{\omega}{C_1}\right)[C_1 t - (x \sin e - y \cos e)],$$

$$\theta_2 = \theta_2(x,y) = \left(\frac{\omega}{C_2}\right)[C_2 t - (x \sin t - y \cos t)],$$

$$\theta_3 = \theta_3(x,y) = \left(\frac{\omega}{C_1}\right)[C_1 t - (x \sin e - y \cos e)],$$

$$\theta_4 = \theta_4(x,y) = \left(\frac{\omega}{C_2}\right)[C_2 t - (x \sin t - y \cos t)], \qquad \ldots\ldots\ldots\ldots (20)$$

These phase factor are linked with the incident quasi-P and quasi-SV waves and the reflected quasi-P and quasi-SV waves, Here e is the angle between incident and reflected quasi-P waves and y-axis and t is the angle between incident and reflected quasi-SV waves with the y-axis.

(U_{i_1}, V_{i_1}) is the amplitude factors associated with the incident quasi-P, (U_{i_2}, V_{i_2}) is the amplitude factors associated with the reflected quasi-P waves and (U_{r_1}, V_{r_1}) is the amplitude factors associated with the incident quasi-S $(U_{r_1}, V_{r_1})$is the amplitude factors associated with the reflected quasi-S waves correspondingly.

Meanwhile the displacements given in equation (2) need to fulfill the equations of motion (6) and (7), we have

$$-[A(e) - \rho C^2]U_{i_1} + B(e)V_{i_1} = 0,$$

$$-[A(e) - \rho C^2]U_{r_1} - B(e)V_{r_1} = 0,$$

$$-[A(t) - \rho C^2]U_{i_2} - B(t)V_{i_2} = 0,$$

$$-[A(t) - \rho C^2]U_{r_2} - B(t)V_{r_2} = 0. \qquad \ldots\ldots\ldots\ldots (21)$$

It may be noted that we can obtain another set of similar equations corresponding to equations (13) due to consistency condition (18). Equation (21) may be written as

$$U_{i_1} = F_1 V_{i_1}, U_{r_1} = -F_1 V_{r_1}, U_{r_2} = -F_2 V_{r_2}, U_{i_2} = -F_2 V_{i_2}.$$

$$\ldots\ldots\ldots\ldots (22)$$

Where

$$F_1 = \frac{B(e)}{A(e) - \rho C^2}, F_2 = \frac{B(t)}{A(t) - \rho C^2} \qquad \ldots\ldots\ldots\ldots (23)$$

Boundary Conditions

At y=0, there is no stress i.e.

$$s_{12} = 0,\ \ s_{22} = 0, at\ y = 0. \qquad \text{.................................. (24)}$$

Using equations (3) and (4) in equation (6), we have

$$\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right] = 0, \qquad \text{........................ (25)}$$

And

$$\left(\lambda + \lambda' \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x} + \left[(\lambda + 2\mu) + (\lambda' + 2\mu')\frac{\partial}{\partial t}\right]\frac{\partial v}{\partial y} = 0. \qquad \text{.......................... (26)}$$

From equations (22), (25) and (26), we get

$$\left[\frac{(F_1 \cos e - \sin e)}{C_1}\right]V_{i_1} + \left[\frac{(F_2 \cos t - \sin t)}{C_2}\right]V_{i_2} + \left[\frac{(F_1 \cos e - \sin e)}{c_1}\right]V_{r_1} + \left[\frac{(F_2 \cos t - \sin t)}{c_2}\right]V_{r_1} = 0,$$

$$[-F_1 \sin e\{\lambda + \lambda' i\omega\} + \cos e\,\{(\lambda + 2\mu) + (\lambda' + 2\mu')i\omega\}]V_{i_1} + [-F_2 \sin t\{\lambda + \lambda' i\omega\} + \cos t\,\{(\lambda + 2\mu) + (\lambda' + 2\mu')i\omega\}]V_{i_2} + [F_1 \sin e\{\lambda + \lambda' i\omega\} - \cos e\,\{(\lambda + 2\mu) + (\lambda' + 2\mu')i\omega\}]V_{r_1} + [F_1 \sin t\{\lambda + \lambda' i\omega\} - \cos t\,\{(\lambda + 2\mu) + (\lambda' + 2\mu')i\omega\}]V_{r_2} = 0.$$

.... (27)

$$a_1 V_{i_1} + a_2 V_{i_2} + a_1 V_{r_1} + a_2 V_{r_2} = 0, \qquad \text{................. (28)}$$

$$b_1 V_{i_1} + b_2 V_{i_2} - b_1 V_{r_1} + b_2 V_{r_2} = 0.$$

Where

$$a_1 = \frac{[F_1 \cos e - \sin e]}{c_1},$$

$$a_2 = \frac{[F_2 \cos t - \sin t]}{c_2},$$

$$b_1 = \left[-\{\lambda + \lambda' i\omega\}F_{1\,\sin e + \{(\lambda+2\mu)+(\lambda'+2\mu')i\omega\}\cos e}\right],$$

$$b_2 = \left[\{\lambda + \lambda' i\omega\}F_{2\,\sin t - \{(\lambda+2\mu)+(\lambda'+2\mu')i\omega\}\cos t}\right], \qquad \text{......................... (29)}$$

$$C_1 = C_1(e), C_2 = C_2(t). \qquad \text{.......................... (30)}$$

Here we have put up the results

$$\theta_1(x,0) = \theta_3(x,0) \qquad \text{.............................. (31)}$$

Since equations (30) and (31) should be true for all the values of x, we have

$$\theta_2(x,0) = \theta_4(x,0) and\ \theta_2(x,0) = \theta_4(x,0)$$

This on using equation (28) , gives

$$\frac{\sin e}{C_1} = \frac{\sin t}{C_2}, \qquad \text{........................ (32)}$$

It is the kind of Snell's Law for the homogenous viscoelastic materials.

Quasi – P waves incident

Put $U_{i_2} = V_{i_2} = 0$ in equation (19), the total displacement field can be written as

$$u = U_{i_1}\exp(i\theta_1) + U_{r_1}\exp(i\theta_3) + U_{r_2}\exp(i\theta_4),$$

$$v = V_{i_1}\exp(i\theta_1) + V_{r_1}\exp(i\theta_3) + V_{r_2}\exp(i\theta_4). \qquad \text{........................ (33)}$$

Equation (28) also reduces to

$$a_1 V_{i_1} + a_1 V_{r_1} + a_2 V_{r_2} = 0,$$

$$b_1 V_{i_1} - b_1 V_{r_1} + b_2 V_{r_2} = 0. \qquad \text{.......................... (34)}$$

Where a_1, a_2, b_1 and b_2 are defined in equation (29) solving equation (34), we get

$$\frac{V_{r_1}}{V_{i_1}} = \frac{\Delta_1}{\Delta}, \ \frac{V_{r_2}}{V_{i_1}} = \frac{\Delta_2}{\Delta}, \qquad \text{........................ (35)}$$

$$\frac{U_{r_1}}{U_{i_1}} = \frac{-V_{r_1}}{V_{i_1}} = \frac{-\Delta_1}{\Delta}, \ \frac{U_{r_2}}{U_{i_1}} = \frac{-F_2 V_{r_2}}{F_1 V_{i_1}} = \frac{-F_2\Delta_2}{F_1\Delta}. \qquad \text{.......................... (36)}$$

Where

$$\Delta = \begin{vmatrix} a_1 & a_2 \\ -b_1 & b_2 \end{vmatrix} = a_1 b_2 + a_2 b_1 \qquad \text{.............................. (37)}$$

Δ_1 and Δ_2 are obtained from Δ on replacing the elements of its 1st and 2nd columns by $-a_1$ *and* $-b_1$ respectively. On simplification we get

$$\frac{V_{r_1}}{V_{i_1}} = \frac{a_2 b_1 - a_2 b_1}{\Delta}, \qquad \frac{V_{r_2}}{V_{i_1}} = \frac{-2a_1 b_1}{\Delta} \qquad \text{................................ (38)}$$

$$\frac{U_{r_1}}{U_{i_1}} = \frac{a_1 b_2 - a_2 b_1}{\Delta}, \frac{U_{r_2}}{U_{i_1}} = \frac{-F_2(2a_1 b_1)}{F_1(a_1 b_2 + a_2 b_1)}. \qquad \text{................................ (39)}$$

Quasi – S waves incident

Put $U_{i_1} = V_{i_2} = 0$ in equation (19), the total displacement field can be written as

$$u = U_{i_2}\exp(i\theta_2) + U_{r_1}\exp(i\theta_3) + U_{r_2}\exp(i\theta_4),$$

$$v = V_{i_1}\exp(i\theta_1) + V_{r_1}\exp(i\theta_3) + V_{r_2}\exp(i\theta_4). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Equation (28) can be reduced to

$$a_2V_{i_2} + a_1V_{r_1} + a_2V_{r_2} = 0,$$

$$b_2V_{i_2} - b_1V_{r_1} + b_2V_{r_2} = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

Solving equation (41), we get

$$\frac{V_{r_1}}{V_{i_2}} = \frac{\Delta_3}{\Delta}, \quad \frac{V_{r_2}}{V_{i_2}} = \frac{\Delta_4}{\Delta}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

and

$$\frac{U_{r_1}}{U_{i_1}} = \frac{-F_1\Delta_4}{F_2\Delta}, \frac{U_{r_2}}{U_{i_2}} = -\frac{\Delta_6}{\Delta}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

Where Δ is given by equation (37) and Δ_3 *and* Δ_4 are obtained from Δ on replacing the elements and its 1st and 2nd by $-b_2$ *and* $-a_2$ respectively.

Equations (42) and (43) give the reflection coefficient when quasi-S waves are incident at a free surface of infinite homogenous viscoelastic semi-infinite medium.

Particular case

If the medium is a homogenous, isotropic elastic, then put $\lambda' = \mu' = 0$ using equations (14), (16), (18), (23) and (29), we get

$$C_1 = C_1(e) = \alpha, \quad C_2 = C_2(t) = \beta,$$

$$F_1 = -\tan e\,, \quad F_2 = \cot t, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

$$a_1 = \frac{-2\sin e}{\alpha}, \qquad a_2 = \frac{\cos 2t}{\beta},$$

$$b_1 = \frac{-\mu\alpha\cos 2f}{\beta^2\cos e}, \quad b_2 = \frac{2\mu\cos f}{\beta},$$

Where $\alpha = \sqrt{\frac{\lambda+2\mu}{\rho}}$ and $\beta = \sqrt{\frac{\mu}{\rho}}$ are the usual P and S-wave velocities.

Using equation (44), we may express equation (39) equivalent to equation (4.30).