ROUGH FUZZY-IDEALS OF Γ-NEAR-RINGS

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Abstract

In this manuscript we introduce the notion of rough fuzzy sub Γ-near-ring, rough fuzzy ideals, rough fuzzy bi-ideals and in Γ-near-rings along few characterizations. We also study some of its properties.

Keywords: Γ-near-rings, Rough fuzzy sub Γ-near-ring Rough fuzzy ideals, rough fuzzy bi-ideals and

1 INTRODUCTION

Γ-near-ring and the ideal theory of Γ-near-ring were introduced by Bh. Sathyanaranan[9]. For basic terminology in near-ring we refer to Pilz[7] and in Γ-near-ring.

L.A.Zadeh[12] introduced the notion of a fuzzy sets, and it is now a rigorous area of research with manifold applications ranging from engineering and computer science to medical diagnosis and social behaviour studies. Rosenfeld[8], applied the notion of fuzzy sets to algebra and introduced the notion of fuzzy subgroups. Jun et.al.,[4] defined fuzzy ideals in Γ-near-rings. Meenakumari and Tamizhl Chelvan[5] discussed fuzzy bi-ideals in Γ-near-ring.

Pawlak introduced the notion of rough set in his paper[6]. Rough set theory, a new mathematical approach to deal with in exact, uncertain or vague. Knowledge has recently received wide attention on the research areas in both of the real-life applications and theory itself. It has found practical applications in many areas such as knowledge discovery machine learning, data analysis, approximate classification, conflict analysis and so on. Many researchers studied the algebraic approach of rough sets in different algebraic structures such as [1,2,3,,10,11].

In this manuscript we introduce the notion of rough fuzzy ideals, rough fuzzy bi-ideals and rough fuzzy quasi-ideals in Γ-near-rings along few characterizations. We also study some of its properties.

2 PRELIMINARIES

We first recall some basic concepts for the sake of completeness. Recall from[], that a non empty set N with two binary operations + and • multiplication is called a near-ring, if it satisfies the following axioms.
(i) $(N,+)$ is a group; (ii) $(N,\bullet)$ is a semigroup; (iii) $(n_1+n_2)\circ n_3 = n_1\circ n_3 + n_2\circ n_3$, for all $n_1, n_2, n_3 \in N$.

Definition 2.1. [9] A Γ-near-ring is a triple where $(\mathrm{M},+,\Gamma)$ where

i) $(\mathrm{M},+)$ is a group
ii) Γ is non empty set of binary operators on M such that for $\alpha \in \Gamma$, $(\mathrm{M},+,\alpha)$ is a near-ring
iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x,y,z \in \mathrm{M}$ and $\alpha,\beta \in \Gamma$.

In Γ-near-ring, $0\gamma x = 0$ and $(-x)\gamma y = -x\gamma y$, but in general $x\gamma 0 \neq 0$ for some $x \in \mathrm{M}, \gamma \in \Gamma$. More precisely the above Γ-near-ring is right Γ-near-ring. $M_0 = \{n \in M \,/\, n\gamma 0 = 0\}$ is called the zero-symmetric part of M and $M = \{n \in \mathrm{M} \,/\, n\gamma 0 = n\} = \{n \in \mathrm{M} \,/\, n\gamma n' = n \; for\; all\; n' \in \mathrm{M}\}$ is called the constant part of M. M is called zero-symmetric if $\mathrm{M} = \mathrm{M}_0$ and M is called constant if $\mathrm{M} = \mathrm{M}_c$.

It is easy to see that relation $a \equiv b(mod\ A)$ is an equivalence relation. Therefore, when $U = \mathrm{M}$ and θ is the above equivalence relation, we use the pair (M, A) instead of the approximation space (U,θ).

Let M be a nonempty subset of a Γ-near-ring . A mapping $\mu: M \to [0,1]$ is called a fuzzy subset of M. If μ is a fuzzy subset of M, for $t \in [0,1]$ then the set $\mu_t = \{x \in M \mid \mu(x) \geq t\}$ is called a level subset of M with respect to a fuzzy subset of μ, A fuzzy subset $\mu: M \to [0,1]$ is a non empty fuzzy subset if μ is not a constant function. For any two fuzzy subsets μ and λ of M, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$ for all $x \in M$.

Let A be a non empty subset M. The characteristic function of M is defined by

$$\chi_A(x) = \begin{cases} 1, & if\ x \in A: \\ 0, & if\ x \notin A. \end{cases}$$

Let μ and γ be two fuzzy subsets of Γ-Semigroup M and $x,y,z \in M, \alpha \in \Gamma$. We define

$$\mu\circ\gamma(x) = \begin{cases} sup_{x=y\alpha z}, min\{\mu(y),\gamma(z)\}: \\ \ 0, \qquad otherwise. \end{cases}$$

$\mu \cap \gamma(x) = min\{\mu(x),\gamma(x)\}$, for all $x \in M$.

Definition 2.2.[4] A fuzzy set μ in a Γ-near-ring M is called a fuzzy sub Γ-near-ring of Mif

i) $\mu(x-y) \geq \mu(x) \wedge \mu(y)$
ii) $\mu(x\alpha y) \geq \mu(x) \wedge \mu(y)$, for all $x, y, \in$ Ň, $\alpha, \in \Gamma$.

Definition 2.3.[4].A fuzzy set μ in a Γ-near-ring M is called a *fuzzy left(resp. right) ideal* of M if

i) $\mu(x-y) \geq \mu(x) \wedge \mu(y)$
ii) $\mu(y+x-y) \geq \mu(x)$
iii) $\mu(a\alpha(x+b)-a\alpha b) \geq \mu(x)$ (resp. $\mu(x\alpha a) \geq \mu(a)$ for all $x, y, a, b \in$ M, $\alpha \in \Gamma$.

Definition 2.4.[5] A fuzzy set μ in a Γ-near-ring M is called a *fuzzy bi-ideal* of M if

i) $\mu(x-y) \geq \mu(x) \wedge \mu(y)$
ii) $\mu(x\alpha y\beta z) \geq \mu(x) \wedge \mu(z)$, for all $x, y, z \in$ M, $\alpha, \beta \in \Gamma$.

Definition 2.5.[5] A fuzzy bi-ideal μ in a Γ-near-ring M is said to be *normal* if $\mu(0) = 1$.

Theorem 2.6[4]. *Let μ be a fuzzy subset of M. Then μ is a fuzzy ideal of M if and only if each level subset μ_t, $t \in [0,1]$ be a ideal of M.*

Theorem 2.7[4]. *Let μ be a fuzzy subset of R. Then μ is a fuzzy ideal of M if and only if for all $t \in [0,1]$, if $\mu_t^M \neq \emptyset$ then μ_t^M be a ideal of M.*

Definition 2.8.[3]Let μ be a fuzzy subset of M. Let $\overline{\rho}(\mu)$ and $\underline{\rho}(\mu)$ be the fuzzy subsets of M defined by $\overline{\rho}(\mu)(x) = \bigvee_{a\in[x]_\rho} \mu(a)$ and $\underline{\rho}(\mu) = \bigwedge_{a\in[x]_\rho} \mu(a)$ are called, respectively, the *ρ-upper* and *ρ-lower approximations* of the fuzzy set μ. $\rho(\mu) = (\underline{\rho}(\mu), \overline{\rho}(\mu))$ is called a *rough fuzzy set* with respect to ρ if $\underline{\rho}(\mu) \neq \overline{\rho}(\mu)$.

3. ROUGH FUZZY IDEALS IN Γ-NEAR-RINGS

In this section we introduce the notion of rough fuzzy ideal rough fuzzy bi-ideals and rough fuzzy quasi-ideals in Γ-near-rings M and study some of its properties. Throughout this paper M denotes the Γ-near-rings, unless otherwise specified.

Definition 3.1. *Let* ρ be a congruence relation on M. A fuzzy sub Γ-near-ring μ of M is called an *upper* (resp. *lower*) *rough fuzzy sub Γ-near-ring* of M if $\overline{\rho}(\mu)$ (resp. $\underline{\rho}(\mu)$) is a fuzzy sub Γ-near-ring of M.

Definition 3.2. *Let* ρ be a congruence relation on M. A fuzzy ideal μ of M is called an upper (resp. lower) rough fuzzy ideal of M if $\overline{\rho}(\mu)$ (resp. $\underline{\rho}(\mu)$) is a fuzzy ideal of M.

Definition 3.3. *Let* ρ be a congruence relation on M. A fuzzy bi-ideal μ of M is called an *upper* (resp. *lower*) *rough fuzzy bi-ideal* of M if $\overline{\rho}(\mu)$ (resp. $\underline{\rho}(\mu)$) is a fuzzy bi-ideal of M.

Theorem 3.4. *Let ρ be a congruence relation on M and let μ be a fuzzy subset of M. If μ is a fuzzy sub Γ-near-ring of M, then $\overline{\rho}(\mu)$ is a fuzzy sub Γ-near-ring of M.*

Proof: Let μ be a fuzzy sub Γ-near-ring of M. Let $x, y \in M$. Then $\mu(x-y) \geq \mu(x) \wedge \mu(y)$ and $\mu(x\gamma y) \geq \mu(x) \wedge \mu(y)$, We have

$$\begin{aligned}
\overline{\rho}(\mu)(x-y) &= \bigvee_{m\in[x-y]_\rho} \mu(m) \\
&= \bigvee_{m\in[x]_\rho+[-y]_\rho} \mu(m) \\
&= \bigvee_{p\in[x]_\rho,\ q\in[-y]_\rho} \mu(p\gamma q) \\
&\geq \left(\bigvee_{p\in[x]_\rho} \mu(p)\right) \wedge \left(\bigvee_{q\in[-y]_\rho} \mu(q)\right) \\
&= \overline{\rho}(\mu)(x) \wedge \overline{\rho}(\mu)(-y). \\
&\geq \overline{\rho}(\mu)(x) \wedge \overline{\rho}(\mu)(y).
\end{aligned}$$

Then $\overline{\rho}(\mu)(x-y) \geq \overline{\rho}(\mu)(x) \wedge \overline{\rho}(\mu)(y)$.

$$\begin{aligned}
\overline{\rho}(\mu)(x\gamma y) &= \bigvee_{m\in[x\gamma y]_\rho} \mu(m) \\
&= \bigvee_{m\in[x]_\rho \Gamma [y]_\rho} \mu(m) \\
&= \bigvee_{p\in[x]_\rho,\ q\in[y]_\rho} \mu(p\gamma q) \\
&\geq \left(\bigvee_{p\in[x]_\rho} \mu(p)\right) \wedge \left(\bigvee_{q\in[y]_\rho} \mu(q)\right) \\
&= \overline{\rho}(\mu)(x) \wedge \overline{\rho}(\mu)(y).
\end{aligned}$$

Then $\overline{\rho}(\mu)(x\gamma y) \geq \overline{\rho}(\mu)(x) \wedge \overline{\rho}(\mu)(y)$.
Therefore $\overline{\rho}(\mu)$ is a fuzzy sub Γ-near-ring of M.

Theorem 3.5. *Let ρ be a congruence relation on M and let μ be a fuzzy subset of M. If μ is a fuzzy sub Γ-near-ring of M, then $\underline{\rho}(\mu)$ is a fuzzy sub Γ-near-ring of M.*

Proof: Let μ be a fuzzy sub Γ-near-ring of M. Let $x, y, z \in M$. Then $\mu(x-y) \geq \mu(x) \wedge \mu(y)$ and $\mu(x\gamma y) \geq \mu(x) \wedge \mu(y)$, We have

$$\begin{aligned}
\underline{\rho}(\mu)(x-y) &= \wedge_{m \in [x-y]_\rho} \mu(m) \\
&= \wedge_{m \in [x]_\rho + [-y]_\rho} \mu(m) \\
&= \wedge_{p \in [x]_\rho,\ q \in [-y]_\rho} \mu(p\gamma q) \\
&\geq \left(\wedge_{p \in [x]_\rho}\ \mu(p)\right) \wedge \left(\wedge_{q \in [-y]_\rho} \mu(q)\right) \\
&= \underline{\rho}(\mu)(x) \wedge \underline{\rho}(\mu)(-y) \\
&\geq \underline{\rho}(\mu)(x) \wedge \underline{\rho}(\mu)(y)
\end{aligned}$$

Then $\underline{\rho}(\mu)(x-y) \geq \underline{\rho}(\mu)(x) \wedge \underline{\rho}(\mu)(y)$.

(ii)
$$\begin{aligned}
\underline{\rho}(\mu)(x\gamma y) &= \wedge_{m \in [x\gamma y]_\rho} \mu(m) \\
&= \wedge_{m p\gamma q \in [x]_\rho \Gamma [y]_\rho} \mu(m) \\
&= \wedge_{p \in [x]_\rho,\ q \in [y]_\rho} \mu(p\gamma q) \\
&\geq \left(\wedge_{p \in [x]_\rho}\ \mu(p)\right) \wedge \left(\wedge_{q \in [y]_\rho} \mu(q)\right) \\
&= \underline{\rho}(\mu)(x) \wedge \underline{\rho}(\mu)(y).
\end{aligned}$$

Then $\underline{\rho}(\mu)(x\gamma y) \geq \underline{\rho}(\mu)(x) \wedge \underline{\rho}(\mu)(y)$.

Therefore $\underline{\rho}(\mu)$ is a fuzzy sub Γ-near-ring of M.

Theorem3.6: *Let ρ be a congruence relation on M and let μ be a fuzzy subset of M. If μ is a fuzzy bi-ideal of M, then $\overline{\rho}(\mu)$ is a fuzzy bi-ideal of M.*

Proof. Let μ be a fuzzy bi-ideal of M. Let $x, y, z \in M$ and and $\alpha, \beta \in \Gamma$. Then $\mu(x-y) \geq \mu(x) \wedge \mu(y)$ and $\mu(x\alpha y\beta z) \geq \mu(x) \wedge \mu(y)$.

Obviously we have $\overline{\rho}(\mu)(x-y) \geq \overline{\rho}(\mu)(x) \wedge \overline{\rho}(\mu)(y)$.

Consider

$$\begin{aligned}
\overline{\rho}(\mu)(x\alpha y\beta z) &= \vee_{a \in [x\alpha y\beta z]_\rho} \mu(a) \\
&= \vee_{a \in [x]_\rho \Gamma [y]_\rho \Gamma [z]_\rho} \mu(a) \\
&= \vee_{p \in [x]_\rho, q \in [y]_\rho, r \in [z]_\rho} \mu(p\alpha q\beta r) \\
&\geq \left(\vee_{p \in [x]_\rho} \mu(p)\right) \wedge \left(\vee_{r \in [z]_\rho} \mu(r)\right) \\
&= \overline{\rho}(\mu)(x) \wedge \overline{\rho}(\mu)(z)
\end{aligned}$$

Then $\overline{\rho}(\mu)(x\alpha y\beta z) \geq \overline{\rho}(\mu)(x) \wedge \overline{\rho}(\mu)(z)$.

Therefore $\overline{\rho}(\mu)$ is a fuzzy bi-ideal of M.

Theorem3.7: *Let ρ be a congruence relation on M and let μ be a fuzzy subset of M. If μ is a fuzzy bi-ideal of M, then $\underline{\rho}(\mu)$ is a fuzzy bi-ideal of M.*

Proof. Let μ be a fuzzy bi-ideal of M. Let $x, y, z \in M$ and and $\alpha, \beta \in \Gamma$. Then $\mu(x-y) \geq \mu(x) \wedge \mu(y)$ and $\mu(x\alpha y\beta z) \geq \mu(x) \wedge \mu(y)$.

Obviously we have $\underline{\rho}(\mu)(x-y) \geq \underline{\rho}(\mu)(x) \wedge \underline{\rho}(\mu)(y)$.

$$\begin{aligned}
\underline{\rho}(\mu)(x\alpha y\beta z) &= \wedge_{a \in [x\alpha y\beta z]_\rho} \mu(a) \\
&= \wedge_{a \in [x]_\rho \Gamma [y]_\rho \Gamma [z]_\rho} \mu(a) \\
&= \wedge_{p \in [x]_\rho, q \in [y]_\rho, r \in [z]_\rho} \mu(p\alpha q\beta r) \\
&\geq \left(\wedge_{p \in [x]_\rho} \mu(p)\right) \wedge \left(\wedge_{r \in [z]_\rho} \mu(r)\right) \\
&= \underline{\rho}(\mu)(x) \wedge \underline{\rho}(\mu)(z)
\end{aligned}$$

Then $\underline{\rho}(\mu)(x\alpha y\beta z) \geq \underline{\rho}(\mu)(x) \wedge \underline{\rho}(\mu)(z)$.

Therefore $\underline{\rho}(\mu)$ is a fuzzy bi-ideal of M.

Lemma 3.8: *Let ρ be a congruence relation on M. If μ is a fuzzy subset of M. and $t \in [0,1]$, then*

i) $\left(\underline{\rho}(\mu)\right)_t = \underline{\rho}(\mu_t)$,

ii) $\left(\overline{\rho}(\mu)\right)_t^M = \overline{\rho}(\mu_t^M)$

Proof:

(i) $x \in \left(\underline{\rho}(\mu)\right)_t \Leftrightarrow \underline{\rho}(\mu)(x) \geq t$

$$\Leftrightarrow \wedge_{y\in[x]_\rho} \mu(y) \geq t$$
$$\Leftrightarrow \forall y \in [x]_\rho, \mu(y) \geq t$$
$$\Leftrightarrow [x]_\rho \subseteq \mu_t$$
$$\Leftrightarrow x \in \underline{\rho}(\mu_t)$$

Therefore $\left(\underline{\rho}(\mu)\right)_t = \underline{\rho}(\mu_t)$.

(ii) $x \in \left(\overline{\rho}(\mu)\right)_t^M (x) \Leftrightarrow \overline{\rho}(\mu) > t$

$$\Leftrightarrow \vee_{y\in[x]_\rho} \mu(y) > t$$
$$\Leftrightarrow \exists y \in [x]_\rho, \mu(y) \geq t$$
$$\Leftrightarrow [x]_\rho \cap \mu_t^M \neq \emptyset$$
$$\Leftrightarrow x \in \overline{\rho}(\mu_t^M).$$

Therefore $\left(\overline{\rho}(\mu)\right)_t^M = \overline{\rho}(\mu_t^M)$.

Theorem3.9: *Let ρ be a congruence relation on M and let μ be a fuzzy subset of M. If μ is a fuzzy ideal of M, then $\overline{\rho}(\mu)$ is a fuzzy ideal of M.*

Proof. Let μ is a fuzzy ideal of M. By Theorem 2.7, for all $t \in [0,1]$, $\mu_t^M \neq \emptyset$, then μ_t^M is a ideal of M. Thus for all $t \in [0,1]$, $\underline{\rho}(\mu_t^M)$ is a ideal of M. From this and by Lemma 3.8(ii) for all $t \in [0,1]$, $\left(\overline{\rho}(\mu)\right)_t^M$ is a ideal of M. By Theorem 2.7 $\overline{\rho}(\mu)$ is a fuzzy ideal of M.

Theorem 3.10*:* *Let ρ be a congruence relation on M and let μ be a fuzzy subset of M. If μ is a fuzzy ideal of M, then $\underline{\rho}(\mu)$ is a fuzzy ideal of M.*

Proof. Let μ is a fuzzy ideal of M. By Theorem 2.6, for all $t \in [0,1]$, if $\mu_t \neq \emptyset$, then μ_t is a ideal of M. Obviously for all $t \in [0,1]$, $\underline{\rho}(\mu_t)$ is a ideal of M. From this and by Lemma3.8(i) for all $t \in [0,1]$, $\left(\underline{\rho}(\mu)\right)_t$ is a ideal of M. By Theorem 2.6 $\underline{\rho}(\mu)$ is a fuzzy ideal of M.

Corollary 3.11 *Let ρ be a congruence relation on M and let μ be a fuzzy subset of M. If μ is a fuzzy ideal of M, then $\rho(\mu)$ is a rough fuzzy ideal of M.*

Proof. It follows from Theorem 3.9 and Theorem 3.10 .

Theorem 3.12. *Let ρ be a congruence relation on M. If μ* and *λ are fuzzy ideals of M, then $\rho(\mu) \cap \rho(\lambda)$ is a rough fuzzy ideal of M.*

Proof. Let μ and *λ are fuzzy ideals of M*. By Theorem 3.9 and Theorem 3.10 $\overline{\rho}(\mu), \overline{\rho}(\lambda)$, and $\underline{\rho}(\mu), \underline{\rho}(\lambda)$ are fuzzy ideals of M. Then

$$\overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(x-y) = \min\{\overline{\rho}(\mu)(x-y), \overline{\rho}(\lambda)(x-y)\}$$
$$= min\{min\{\overline{\rho}(\mu)(x), \overline{\rho}(\mu)(y)\}, min\{\overline{\rho}(\lambda)(x), \overline{\rho}(\lambda)(y)\}\},$$
since $\overline{\rho}(\mu), \overline{\rho}(\lambda)$ are ideals of M.
$$= min\{min\{\overline{\rho}(\mu)(x), \overline{\rho}(\lambda)(x)\}, min\{\overline{\rho}(\mu)(y), \overline{\rho}(\lambda)(y)\}\}$$
$$= min\{\overline{\rho}(\mu)(x) \wedge \overline{\rho}(\lambda)(x), \ \overline{\rho}(\mu)(y) \wedge \overline{\rho}(\lambda)(y)\}$$
$$= min\{\overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(x), \ \overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(x)\}.$$

Then $\overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(x-y) \geq min\{\overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(x), \ \overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(x)\}$.

$$\overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(y+x-y) = \min\{\overline{\rho}(\mu)(y+x-y), \overline{\rho}(\lambda)(y+x-y)\}$$
$$\geq \min\{\overline{\rho}(\mu)(x), \overline{\rho}(\lambda)(x)\},$$ since $\overline{\rho}(\mu), \overline{\rho}(\lambda)$ are ideals of M
$$\geq \overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(x).$$

Then $\overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(y+x-y) \geq \overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(x)$.

$$\begin{aligned}\overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(a\alpha(x+b) - a\alpha b) &= \min\{\overline{\rho}(\mu)(a\alpha(x+b) - a\alpha b), \overline{\rho}(\lambda)(a\alpha(x+b) - a\alpha b)\} \\ &\geq \min\{\overline{\rho}(\mu)(x), \overline{\rho}(\lambda)(x)\}, \text{ since } \overline{\rho}(\mu), \overline{\rho}(\lambda) \text{ are ideals of } M \\ &\geq \overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(x).\end{aligned}$$

Hence $\overline{\rho}(\mu) \cap \overline{\rho}(\lambda)$ is a fuzzy ideal of M.

Similarly we prove $\underline{\rho}(\mu) \cap \underline{\rho}(\lambda)$ is a fuzzy ideal of M.

Therefore $\rho(\mu) \cap \rho(\lambda)$ is a rough fuzzy ideal of M.

Theorem 3.12. *Let ρ be a congruence relation on M. For any fuzzy right ideal μ* and *fuzzy left ideal λ of M, then $\rho(\mu) \cap \rho(\lambda) = \rho(\mu) \circ \rho(\lambda)$.*

Proof. Let μ be a fuzzy right ideal and λ be a fuzzy left ideal *of* M. By Theorem 3.9 and Theorem 3.10 $\overline{\rho}(\mu), \underline{\rho}(\mu)$ are fuzzy right ideals and $\overline{\rho}(\lambda)$, $\underline{\rho}(\lambda)$ are fuzzy left ideals of M. Then for any $x, y, z \in M, \alpha \in \Gamma$,

$$\begin{aligned}\overline{\rho}(\mu) \circ \overline{\rho}(\lambda)(x) &= sup_{x=y\alpha z}\{min\{\overline{\rho}(\mu)(y), \overline{\rho}(\lambda)(z)\}\} \\ &\leq \min\left\{sup_{x=y\alpha z}\{\overline{\rho}(\mu)(y), \overline{\rho}(\lambda)(z)\}\right\} \\ &\leq \min\left\{sup_{x=y\alpha z}\{\overline{\rho}(\mu)(y\alpha z), \overline{\rho}(\lambda)(y\alpha z)\}\right\} \\ &= min\left\{sup_{x=y\alpha z}\left\{\vee_{p\in[y\alpha z]_\rho}\mu(p), \vee_{p\in[y\alpha z]_\rho}\lambda(p)\right\}\right\} \\ &= \min\left\{\left\{\vee_{p\in[x]_\rho}\mu(p), \vee_{p\in[x]_\rho}\lambda(p)\right\}\right\} \\ &= \min\{\overline{\rho}(\mu)(x), \overline{\rho}(\lambda)(x)\} \\ &= \overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(x).\end{aligned}$$

Then $\overline{\rho}(\mu) \circ \overline{\rho}(\lambda) \subseteq \overline{\rho}(\mu) \cap \overline{\rho}(\lambda)$. (1)

Again consider

$$\begin{aligned}\overline{\rho}(\mu) \cap \overline{\rho}(\lambda)(x) &= \min\{\overline{\rho}(\mu)(x), \overline{\rho}(\lambda)(x)\}. \\ &\leq min\left\{sup_{x=y\alpha z}\{\overline{\rho}(\lambda)(y), \overline{\rho}(\mu)(z)\}\right\} \\ &\leq sup_{x=y\alpha z}\{min\{\overline{\rho}(\lambda)(y), \overline{\rho}(\mu)(z)\}\} \\ &= \overline{\rho}(\lambda) \circ \overline{\rho}(\mu).\end{aligned}$$

Then $\overline{\rho}(\mu) \cap \overline{\rho}(\lambda) \subseteq \overline{\rho}(\lambda) \circ \overline{\rho}(\mu)$. (2) From (1) and (2) we have $\overline{\rho}(\mu) \circ \overline{\rho}(\lambda) = \overline{\rho}(\mu) \cap \overline{\rho}(\lambda)$.

Similarly we prove $\underline{\rho}(\mu) \cap \underline{\rho}(\lambda) = \underline{\rho}(\mu) \circ \underline{\rho}(\lambda)$.

Therefore $\rho(\mu) \cap \rho(\lambda) = \rho(\mu) \circ \rho(\lambda)$.

4.CONCLUSION

The main objective of this paper is to concentrate our study to the application of rough set theory in fuzzy ideals with respect to Γ-near-ring. In this manuscript we introduce the notion of rough fuzzy ideals, rough fuzzy bi-ideals and rough fuzzy quasi-ideals in Γ-near-rings along few characterizations. We hope that to extend this work characterize rough fuzzy ideals in rings also.

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