SOLUTION OF A MULTI OBJECTIVE STOCHASTIC INVENTORY MODEL WITH FUZZY COST COMPONENTS: AN INTUITIONISTIC FUZZY GEOMETRIC PROGRAMMING TECHNIQUE

SOUMEN BANERJEE
Gangarampur College, Gangarampur
(A Government Sponsored College)
Dakshin Dinajpur 733124

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Abstract:
Paknejad et al.'s model is considered in this paper. Itemwise multiobjective models for both exponential and uniform lead-time demand are taken and the results are compared numerically both in Intuitionistic fuzzy optimization and intuitionistic fuzzy geometric programming techniques. Objective of this paper is to establish that intuitionistic fuzzy geometric programming method is better than usual intuitionistic fuzzy optimization technique as expected annual cost of this inventory model is more minimized in case of intuitionistic fuzzy geometric programming method. As a single objective stochastic inventory model where the lead-time demand follows normal distribution and with varying defective rate, expected annual cost is also measured. Finally the model considers for fuzzy cost components, which make the model more realistic, and numerical values for uniform, exponential lead time demand are compared.

1. **Introduction**

Geometric Programming (GP) is an effective method to solve a non-linear programming problem. It has certain advantages over the other optimization methods.

Here, the advantages are that is usually much simpler to work with the dual than primal. Degree of Difficulty plays a significant role for solving a non-linear programming problem by GP method. Since late 1960, GP has been known and used in various fields (like OR, Engineering Sciences etc.). Duffin, Petersen and Zener (1966) discussed the basic theories with engineering applications in their books. Another famous book on GP and its application appeared in Beightler and Philips (1976). There are many references on application and the methods of GP in the survey papers (like Eckar (1980), Beightler et.al. (1979), Zener (1971). Hariri et. al. (1997) discussed the multi-item production lot-size inventory model with varying order cost under a restriction Jung and Klain (2001) developed single item inventory problems and solved by GP method. Ata Fragany and Wakeel (2003) considered some inventory problems solved by GP technique. Zadeh (1965) first gave the concept of fuzzy set theory. Later on Bellman and Zadeh (1970) used the fuzzy set theory to the decision making problem Tanaka (1974) introduced the objective as fuzzy goal over the α-cut of a fuzzy constraint set and Zimmerman (1978) gave the concept to an inventory and production problem. Banerjee and Roy (2008) discussed the single and multi-objective stochastic inventory model in fuzzy environment. Cao (1993) and his recent book (2002) discussed fuzzy geometric programming with zero degree of difficulty. Das et. al. (2000) developed a multi-item inventory model with quantity dependent inventory costs and demand dependent unit cost under imprecise objective function and constraint and solved by GP technique. Roy and Maiti (1997) solved single objective fuzzy EOQ model by GP technique. Recently Mondal et. al. (2005) developed a multi-objective inventory model and solved it by GP method. A multi objective fuzzy economic production quantity model is solved using GP approach by Islam and Roy (2004).

Intuitionistic Fuzzy Set (IFS) was introduced by K. Atanassov (1986) and seems to be applicable to real world problems. The concept of IFS can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. Thus it is expected that, IFS can be used to simulate human decision-making process and any activitities requiring human expertise and knowledge that are inevitably imprecise or totally reliable. Here the degree of rejection and satisfaction are considered so that the sum of both values is always less than unity (1986). Atanossov also analyzed Intuitionistic fuzzy sets in a more explicit way. Atanassov(1989) discussed an Open problems in intuitionistic fuzzy sets theory. An Interval valued intuitionistic fuzzy sets was analyzed by Atanassov and Gargov(1999). Atanassov and Kreinovich(1999) implemented Intuitionistic fuzzy interpretation of interval data. The temporal intuitionistic fuzzy sets are discussed also by Atanossov[1999]. Intuitionistic fuzzy soft sets are considered by Maji Biswas and Roy(2001). Nikolova, Nikolov, Cornelis and Deschrijver(2002) presented a Survey of the research on intuitionistic fuzzy sets. Rough intuitionistic fuzzy sets are analyzed by Rizvi, Naqvi and Nadeem(2002). Angelov (1997) implemented the Optimization in an intuitionistic fuzzy environment. He (1995) also contributed in his another two important papers, based on Intuitionistic fuzzy optimization. Pramanik and Roy (2005) solved a vector optimization problem using an Intuitionistic Fuzzy goal programming. A transportation model is solved by Jana and Roy (2007) using multi-objective intuitionistic fuzzy linear programming. Paknejad et al.'s model is considered in this paper. Itemwise multiobjective models for both exponential and uniform lead-time demand are taken and the results are compared numerically both in Intuitionistic fuzzy optimization and intuitionistic fuzzy geometric programming techniques. It is observed that our Intuitionistic Fuzzy Geometric programming always performs better than the Intuitionistic fuzzy optimization technique.

2. Mathematical Model

Multi-item Paknejad et al.'s model (1995) along with the notations and some assumptions will be taken into account throughout the paper. Each lot contains a random number of defectives following binomial distribution. After the arrival purchaser examines the entire lot. An order of size Q is placed as soon as the inventory position reaches the reorder point s. the shortages are allowed and completely backordered. Lead-time is constant and probability distribution of lead-time demand is known. Thus a quality adjusted lot-sizing model is formed as:

EC ($Q_1,Q_2,\dots Q_n, s_1,s_2,\dots s_n$) = setup cost + non-defective item holding cost + stock out cost + defective item holding cost + inspecting cost

$$=\sum_{i=1}^{n}\left(\frac{D_iK_i}{Q_i(1-\theta_i)}+h_i(s_i-\mu_i+\frac{1}{2}(Q_i(1-\theta_i)+\theta_i))+\frac{D_i\pi_i\bar{b}_i(s)}{Q_i(1-\theta_i)}+h_i'\theta_i(Q_i-1)+\frac{D_iv_i}{1-\theta_i}\right) \quad \dots.(1)$$

Where, (for the i^{th} item)

D_i = expected demand per year

Q = lot size

s_i = reorder point

K_i = setup cost

θ_i = defective rate in a lot of size Q, $0 \le \theta \le 1$

h_i = nondefective holding cost per unit per year

h_i' = defective holding cost per unit per year

π_i = shortage cost per unit short

v_i = cost of inspecting a single item in each lot

μ_i = expected demand during lead time

p_i = purchasing price of each product

B = total budget

$\bar{b}_i(s_i)$ = the expected demand short at the end of the cycle

$$\bar{b}_i(s) = \int_{s_i}^{\infty} (x - s_i) f_i(x) dx$$

Where, $f_i(x)$ is the density function of lead-time demand.

EC ($Q_1, Q_2, \ldots Q_n, s_1, s_2, \ldots s_n$) = expected annual cost given that a lot size Q is ordered.

Model I: Multi Objective Stochastic Inventory Model [MOSIM]

In reality, a managerial problem of a responsible organization involves several conflicting objectives to be achieved simultaneously that refer to a situation on which the DM has no control. For this purpose a latest tool is linear or non-linear programming problem with multiple conflicting objectives. So the following model may be considered:

To solve the problem in equation (1) as a MOSIM, it can be reformulated as:

$$MinEC_i(Q_i, s_i) = \frac{D_i K_i}{Q_i(1-\theta_i)} + h_i(s_i - \mu_i + \frac{1}{2}(Q_i(1-\theta_i) + \theta_i)) + \frac{D_i \pi \bar{b}_i(s_i)}{Q_i(1-\theta_i)} + h_i'\theta_i(Q_i - 1) + \frac{D_i v_i}{1-\theta_i}$$

$$Q_i, s_i > 0 \forall i = 1, 2, \ldots\ldots, n. \qquad \ldots.\ (2)$$

Model II: Multi Item Stochastic Model With Fuzzy Cost Components

Stochastic non-linear programming problem with fuzzy cost components considers as:

$$MinEC(Q_1, \ldots\ldots, Q_n, s_1, \ldots\ldots, s_n) =$$

$$\sum_{i=1}^{n} \left(\frac{D_i \tilde{K}_i}{Q_i(1-\theta_i)} + \tilde{h}_i(s_i - \mu_i + \frac{1}{2}(Q_i(1-\theta_i) + \theta_i)) + \frac{D_i \tilde{\pi}_i \bar{b}_i(s)}{Q_i(1-\theta_i)} + \tilde{h}_i'\theta_i(Q_i - 1) + \frac{D_i v_i}{1-\theta_i} \right)$$

$$Q_i, s_i > 0 \forall i = 1, 2, \ldots\ldots, n. \qquad \ldots..(3)$$

Here $\tilde{K}_i, \tilde{\pi}_i, \tilde{h}_i, \tilde{h}_i'$ represents vector of fuzzy parameters involved in the objective function EC. We assume $\tilde{K}_i = (K_i^-, K_i^0, K_i^+)$, $\tilde{\pi}_i = (\pi_i^-, \pi_i^0, \pi_i^+)$, $\tilde{h}_i = (h_i^-, h_i^0, h_i^+)$ and $\tilde{h}_i' = (h_i'^-, h_i'^0, h_i'^+)$, all of which are triangular fuzzy numbers.

3. Stochastic models with Different Distributions

CASE 1: Demand follows Uniform distribution

We assume that lead time demand for the period for the i^{th} item is a random variable which follows uniform distribution and if the decision maker feels that demand values for item I below a_i or above b_i are highly unlikely and values between a_i and b_i are equally likely, then the probability density function $f_i(x)$ are given by:

$$f_i(x) = \begin{cases} \dfrac{1}{b_i - a_i} & \text{if } a_i \le x \le b_i \\ 0 & \text{otherwise} \end{cases} \quad \text{for } i = 1,2,\ldots\ldots, n.$$

So, $\bar{b}_i(s_i) = \dfrac{(b_i - s_i)^2}{2(b_i - a_i)}$ for i=1,2,…,n …. (4)

Where, $\bar{b}_i(s_i)$ are the expected number of shortages per cycle and all these values of $\bar{b}_i(s_i)$ affect all the desired models.

CASE 2: Demand follows Exponential distribution

We assume that lead-time demand for the period for the i^{th} item is a random variable that follows exponential distribution. Then the probability density function $f_i(x)$ are given by:

$f_i(x) = \lambda_i e^{(-\lambda_i x)}$, x>0 for $i = 1,2,\ldots\ldots, n.$

$= 0,$ otherwise

So, $\bar{b}_i(s_i) = \dfrac{e^{-\lambda_i s_i}}{(-\lambda_i)}$ for $i = 1,2,\ldots\ldots, n.$

…. (5)

Where, $\bar{b}_i(s_i)$ are the expected number of shortages per cycle and all these values of $\bar{b}_i(s_i)$ affect all the desired models.

4. Mathematical Analysis

4.1 Formulation of Intuitionistic Fuzzy Optimization [IFO]

When the degree of rejection (non-membership) is defined simultaneously with degree of acceptance (membership) of the objectives and when both of these degrees are not complementary to each other, then IF sets can be used as a more general tool for describing uncertainty.

To maximize the degree of acceptance of IF objectives and constraints and to minimize the degree of rejection of IF objectives and constraints, we can write:

$$\max \mu_i(\bar{X}), \bar{X} \varepsilon R, i = 1,2,\ldots\ldots, K+n$$
$$\min \upsilon_i(\bar{X}), \bar{X} \varepsilon R, i = 1,2,\ldots\ldots, K+n$$

Subject to

$$\upsilon_i(\bar{X}) \ge 0,$$
$$\mu_i(\bar{X}) \ge \upsilon_i(\bar{X})$$
$$\mu_i(\bar{X}) + \upsilon_i(\bar{X}) < 1$$
$$\bar{X} \ge 0$$

Where $\mu_i(\bar{X})$ denotes the degree of membership function of $(\bar{X})$ to the i^{th} IF sets and $\nu_i(\bar{X})$ denotes the degree of non-membership (rejection) of $(\bar{X})$ from the i^{th} IF sets.

4.2 An Intuitionistic Fuzzy Approach for Solving MOIP with Linear Membership and Non-Membership Functions

To define the membership function of MOIM problem, let L_k^{acc} and U_k^{acc} be the lower and upper bounds of the k^{th} objective function. These values are determined as follows: Calculate the individual minimum value of each objective function as a single objective IP subject to the given set of constraints. Let $\bar{X}_1^*, \bar{X}_2^*, ...\bar{X}_k^*$ be the respective optimal solution for the k different objective and evaluate each objective function at all these k optimal solution. It is assumed here that at least two of these solutions are different for which the k^{th} objective function has different bounded values. For each objective, find lower bound (minimum value) L_k^{acc} and the upper bound (maximum value) U_k^{acc}. But in intuitionistic fuzzy optimization (IFO), the degree of rejection (non-membership) and degree of acceptance (membership) are considered so that the sum of both values is less than one. To define membership function of MOIM problem, let L_k^{rej} and U_k^{rej} be the lower and upper bound of the objective function $Z_k(\bar{X})$ where $L_k^{acc} \le L_k^{rej} \le U_k^{rej} \le U_k^{acc}$. These values are defined as follows:

The linear membership function for the objective $Z_k(\bar{X})$ is defined as:

$$\mu_k(Z_k(\bar{X})) = \begin{cases} 1 & \text{if } Z_k(\bar{X}) \le L_k^{acc} \\ \dfrac{U_k^{acc} - Z_k(\bar{X})}{U_k^{acc} - L_k^{acc}} & \text{if } L_k^{acc} \le Z_k(\bar{X}) \le U_k^{acc} \\ 0 & \text{if } Z_k(\bar{X}) \ge U_k^{acc} \end{cases}$$

…. (6)

$$\nu_k(Z_k(\bar{X})) = \begin{cases} 1 & \text{if } Z_k(\bar{X}) \ge U_k^{rej} \\ \dfrac{Z_k(\bar{X}) - L_k^{rej}}{U_k^{rej} - L_k^{rej}} & \text{if } L_k^{rej} \le Z_k(\bar{X}) \le U_k^{rej} \\ 0 & \text{if } Z_k(\bar{X}) \le L_k^{rej} \end{cases}$$

…. (7)

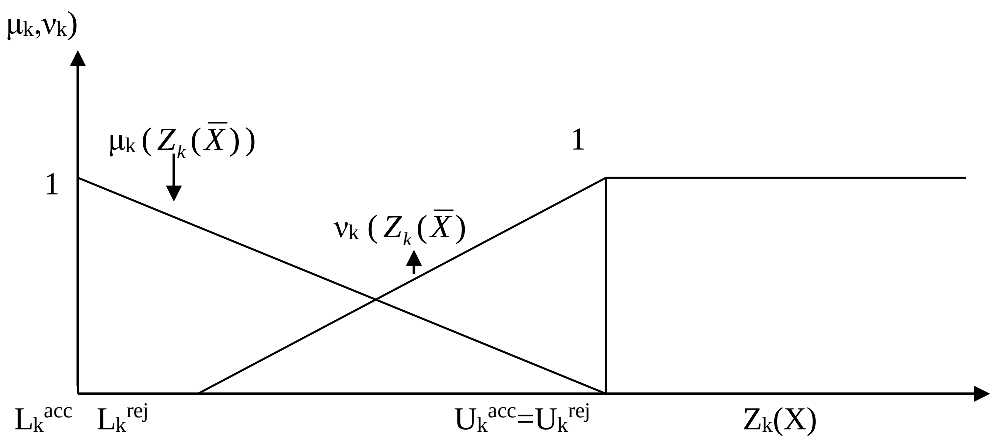

Figure- 1: Membership and non-membership functions of the objective goal

Lemma: In case of minimization problem, the lower bound for non-membership function (rejection)) is always greater than that of the membership function (acceptance).

Now, we take new lower and upper bound for the non-membership function as follows:

$L_k^{rej} = L_k^{acc} + t(U_k^{acc} - L_k^{acc})$ where $0 < t < 1$

$U_k^{rej} = U_k^{acc} + t(U_k^{acc} - L_k^{acc})$ for $t = 0$

Following the fuzzy decision of Bellman-Zadeh (1970) together with linear membership function and non-membership functions of (6) and (7), an intuitionistic fuzzy optimization model of MOIM problem can be written as:

$$\max \mu_k(\overline{X}), \overline{X} \varepsilon R, \mathrm{k} = 1,2,......,\mathrm{K}$$
$$\min \upsilon_k(\overline{X}), \overline{X} \varepsilon R, \mathrm{k} = 1,2,......,\mathrm{K} \quad ....(8)$$

Subject to

$$\upsilon_k(\overline{X}) \geq 0,$$
$$\mu_k(\overline{X}) \geq \upsilon_k(\overline{X})$$
$$\mu_k(\overline{X}) + \upsilon_k(\overline{X}) < 1$$
$$\overline{X} \geq 0$$

The problem of equation (8) can be reduced following Angelov (1997) to the following form:

$$Max \alpha - \beta \quad ....(9)$$

Subject to

$$Z_k(\overline{X}) \leq U_k^{acc} - \alpha(U_k^{acc} - L_k^{acc})$$
$$Z_k(\overline{X}) \leq L_k^{rej} + \beta(U_k^{rej} - L_k^{rej})$$
$$\beta \geq 0$$
$$\alpha \geq \beta$$
$$\alpha + \beta < 1$$
$$\overline{X} \geq 0$$

Then the solution of the MOIM problem is summarized in the following steps:

Step 1. Pick the first objective function and solve it as a single objective IP subject to the constraint, continue the process K-times for K different objective functions. If all the solutions (i.e. $\overline{X}_1^* = \overline{X}_2^* = ... = \overline{X}_k^*)$ $(\mathrm{k} = 1,2,....., \mathrm{K})$ same, then one of them is the optimal compromise solution and go to step 6. Otherwise go to step 2. However, this rarely happens due to the conflicting objective functions.

Then the intuitionistic fuzzy goals take the form

$$Z_k(\overline{X}) \,\tilde{\leq}\, L_k(\overline{X})^*{}_k \; k = 1,2,......., K.,$$

Step 2. To build membership function, goals and tolerances should be determined at first. Using the ideal solutions, obtained in step 1, we find the values of all the objective functions at each ideal solution and construct pay off matrix as follows:

$$\begin{bmatrix} Z_1(\overline{X}_1^*) & Z_2(\overline{X}_1^*) & .... & .... & .... & Z_k(\overline{X}_1^*) \\ Z_1(\overline{X}_2^*) & Z_2(\overline{X}_2^*) & .... & .... & .... & Z_k(\overline{X}_2^*) \\ .... & .... & .... & .... & .... & .... \\ .... & .... & .... & .... & .... & .... \\ .... & .... & .... & .... & .... & .... \\ Z_1(\overline{X}_k^*) & Z_2(\overline{X}_k^*) & .... & .... & .... & Z_k(\overline{X}_k^*) \end{bmatrix}$$

Step 3. From Step 2, we find the upper and lower bounds of each objective for the degree of acceptance and rejection corresponding to the set of solutions as follows:

$$U_k^{acc} = \max_{1 \le r \le k}(Z_k(\overline{X}_r^*)) \quad \text{and} \quad L_k^{acc} = \min_{1 \le r \le k}(Z_k(\overline{X}_r^*))$$

For linear membership functions,

$L_k^{rej} = L_k^{acc} + t(U_k^{acc} - L_k^{acc})$ where $0 < t < 1$

$U_k^{rej} = U_k^{acc} + t(U_k^{acc} - L_k^{acc})$ for $t = 0$

Step 4. Construct the fuzzy programming problem of equation (8) and find its equivalent NLP problem of equation (9).

Step 5. Solve equation (9) by using appropriate mathematical programming algorithm to get an optimal solution and evaluate the K objective functions at these optimal compromise solutions

Step 6. STOP.

5. Geometric Programming Problem

Geometric Programming (GP) can be considered to be an innovative modus operandi to solve a nonlinear problem in comparison with other nonlinear techniques. It was originally developed to design engineering problems. It has become a very popular technique since its inception in solving nonlinear problems. The advantages of this method is that, this technique provides us with a systematic approach for solving a class of nonlinear optimization problems by finding the optimal value of the objective function and then the optimal values of the design variables are derived, also. This method often reduces a complex nonlinear optimization problem to a set of simultaneous equations and this approach is more amenable to the digital computers.

GP is an optimization problem of the form:

$Min\, g_0(t)$ …(10)

subject to

$g_j(t) \le 1$,

j = 1, 2, ………, m.

$h_k(t) = 1$, k=1, 2, ……….., p

$t_i > 0$, i = 1, 2, ………, n

where, $g_j(t)$ (j = 1, 2, ………, m) are posynomial or signomial functions and $h_k(t)$ (k=1, 2, ……….., p) are monomials t_i (i = 1, 2, ………., n) are decision variable vector of n components.

The problem (10) can be written as:

$Min\ g_0(t)$

subject to

$g'_j(t) \le 1,$ j = 1, 2, ………, m.

t > 0, [since $g_j(t) \le 1,\ h_k(t) = 1 \Rightarrow g'_j(t) \le 1$ where $g'_j(t)$(= $g_j(t)/h_k(t)$) be a posynomial (j=1, 2, ………, m ; k=1, 2, ………, p)].

5.1 Posynomial Geometric Programming Problem

A. Primal problem

$Min\ g_0(t)$ …(11)

subject to

$g_j(t) \le 1,$ j = 1, 2, ………, m.

$t_i > 0$, (i =1, 2, …………..,n)

where $g_j(t) = \sum_{k=1}^{N_j} c_{jk} \prod_{i=1}^{n} t_i^{\alpha_{jki}}$

here, $c_{jk} > 0$ and α_{jki} (i=1, 2, ………,n ; k=1, 2, ………, N_j ; j=0, 1, ………,m) are real numbers.

$T = (t_1, t_2, ………., t_n)^T$.

It is a constrained posynomial primal geometric problem (PGP). The number of inequality constraints in the problem (4.4) is m. The number of terms in each posynomial constraint function varies and is denoted by N_j for each j=0, 1, 2, ……, m.

The degree of difficulty (DD) of a GP is defined as (number of terms in a PGP) –(number of variables in PGP)-1.

B. Dual Problem

The dual problem of (4.4) is as follows:

Max $d(w) = \prod_{j=0}^{m} \prod_{k=1}^{N_j} \left(\frac{c_{jk} w_{j0}}{w_{jk}} \right)^{w_{jk}}$

Subject to

$\sum_{k=1}^{N_0} w_{0k} = 1$ (normality condition)

$\sum_{j=0}^{m} \sum_{k=1}^{N_j} \alpha_{jki} w_{jk} = 0,$ (i=1, 2, ………, n) (orthogonality condition)

$w_{j0} = \sum_{k=1}^{N_0} w_{jk} \ge 0,\ w_{jk} \ge 0$, (i=1, 2, ………,n ; k=1, 2, ………, N_j), $w_{00} = 1$.

There are n+1 independent dual constraint equalities and $N=\sum_{j=1}^{m} N_j$ independent dual variables for each term of primal problem. In this case DD=N-n-1.

5.2 Signomial Geometric Programming Problem

A. Primal problem

$$Min\ g_0(t) \quad \ldots(12)$$

subject to

$g_j(t) \le \delta_j$, j = 1, 2, ………, m.

$t_i > 0$, (i =1, 2, …………..,n)

where $g_j(t)=\sum_{k=1}^{N_j}\delta_{jk}c_{jk}\prod_{i=1}^{n} t_i^{\alpha_{jki}}$

here, $c_{jk} > 0$ and α_{jki} $\delta_j = \pm 1$ (j = 2, …..,m)

$\delta_{jk} = \pm 1$ (k=1, 2, ………, N_j ; j= 1, ………,m) are real numbers.

$T=(t_1, t_2, ………., t_n)^T$.

B. Dual Problem

The dual problem of (4.5) is as follows:

$$\text{Max}\ d(w)=\delta_0\left(\prod_{j=0}^{m}\prod_{k=1}^{N_j}\left(\frac{c_{jk}w_{j0}}{w_{jk}}\right)^{\alpha_{jk}w_{jk}}\right)^{\delta_0} \quad \ldots(13)$$

Subject to

$$\sum_{k=1}^{N_0}\delta_{0k}w_{0k}=\delta_0 \quad \text{(normality condition)}$$

$$\sum_{j=0}^{m}\sum_{k=1}^{N_j}\delta_{jk}\alpha_{jki}w_{jk}=0,\ (i=1, 2, \ldots, n) \quad \text{(orthogonality condition)}$$

$\delta_j = \pm 1$ (j = 2, …..,m) $\delta_0 = +1, -1$.

$\delta_{jk} = \pm 1$ (k=1, 2, ………, N_j ; j= 1, ………,m) are real numbers.

$w_{j0}=\delta_j\sum_{k=1}^{N_0}\delta_{jk}w_{jk} \ge 0$, $w_{jk} \ge 0$, (j=1, 2, ………,m ; k=1, 2, ………, N_j), $w_{00}=1$.

5.3 Functional Substitution

When a non-linear programming problem (NLP) is of the following form:

$$Min y(x)=f(x)+(q(x))^n h(x) \quad x>0,\ n>0.$$

Where, $f(x)$, $q(x)$ and $h(x)$ are single or multi-term functionals of posynomial or signomial form. This generalized formulation is not directly solvable using geometric programming; however, under a simple transformation it can be changed into standard geometric programming form. Let $P=q(x)$ and replace the above problem with the following one:

$$Min\bar{y}(x)=f(x)+P^n h(x)$$

subject to

$$P^{-1}(q(x)) \le 1$$

$$x, P > 0.$$

The rationale used in constructing the equivalent problem with an inequality constraint is based on the following logic. Since $y(x)$ is to be minimized, if $q(x)$ is replaced by P, then it is correct to say that $P \ge q(x)$, realizing that in the minimization process P will remain as small as possible. Hence $P = q(x)$ at optimality. Note that $h(x)$ and/or $q(x)$ are permitted to be multiple term expressions and that the optimal (minimizing) solution to $\bar{y}(x)$ is obviously the same as the optimal solution to $y(x)$.

6. Fuzzy Geometric Programming Problem

Multi-objective geometric programming (MOGP) is a special type of a class of MONLP problems. Biswal (1992) and Verma (1990) developed a fuzzy geometric programming technique to solve a MOGP problem. Here, we have discussed a fuzzy geometric programming technique based on max-min and max-convex combination operators to solve a MOGP problem.

To solve the MOGP problem we use the Zimmerman's technique. The procedure consists of the following steps.

Step 1. Solve the MOGP problem as a single GP problem using only one objective at a time and ignoring the others. These solutions are known as ideal solutions. Repeat the process k times for k different objectives. Let $x^1, x^2, \ldots\ldots\ldots, x^k$ be the ideal solutions for the respective objective functions, where

$x^r = (x_1^r, x_2^r, \ldots\ldots\ldots\ldots\ldots\ldots\ldots, x_n^r)$

Step 2. From the ideal solutions of Step1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each solution, the pay-off matrix of size (k x k) can be formulated as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ \ldots \\ x^k \end{array} \begin{array}{cccc} f_1(x) & f_2(x) & \ldots\ldots & f_k(x) \\ \end{array}$$

$$\begin{array}{c} x^1 \\ x^2 \\ \ldots \\ x^k \end{array} \begin{bmatrix} f_1^*(x^1) & f_2(x^1) & \ldots & f_k(x^1) \\ f_1(x^2) & f_2^*(x^2) & \ldots & f_k(x^2) \\ \ldots & \ldots & \ldots & \ldots \\ f_1(x^k) & f_2(x^k) & \ldots & f_k^*(x^k) \end{bmatrix}$$

Step 3. From the Step 2, find the desired goal L_r and worst tolerable value U_r of $f_r(x)$, r = 1, 2, …….., k as follows:

$L_r \le f_r \le U_r$, r = 1, 2, …….., k

Where, $U_r = \max \{f_r(x^1), f_r(x^2), \ldots\ldots, f_r(x^k)\}$

$L_r = \min \{f_r(x^1), f_r(x^2), \ldots\ldots, f_r(x^k)\}$

Step 4. Define a fuzzy linear or non-linear membership function $\mu_r [f_r(x)]$ for the r-th objective function $f_r(x)$, r = 1, 2, …….., k

$\mu_r [f_r(x)] =$ 0 or $\rightarrow 0$ if $f_r(x) \ge U_r$

$\quad = d_r(x)$ if $L_r \le f_r(x) \le U_r$ (r = 1, 2, …… , k)

$= 1$ or $\to 1$ if $f_r(x) \le L_r$

Here $d_r(x)$ is a strictly monotonic decreasing function with respect to $f_r(x)$.

Step 5. At this stage, either a max-min operator or a max-convex combination operator can be used to formulate the corresponding single objective optimization problem.

Through a Max-Min operator

According to Zimmerman (1978) the problem can be solved as:

$$\mu_D(x^*) = Max(Min(\mu_1(f_1(x)), \mu_2(f_2(x)), \ldots\ldots, \mu_k(f_k(x))))$$

subject to

$g_j(x) \le b_j$, j=1, 2, ….., m, $x > 0$

which is equivalent to the following problem as:

Max α …(14)

Subject to

$\alpha \le \mu_r[f_r(x)]$, for r = 1, 2, …….., k

$g_j(x) \le b_j$, j=1, 2, ….., m, $x > 0$

7. Numerical Examples

B. Solution of the model of equation (3)

In case of MOSIM of equation (3), we use the methods to solve it by Intuitionistic fuzzy optimization technique and also by intuitionistic fuzzy geometric programming technique and the following data are considered:

Case1. The lead-time demand follows uniform distribution and thus $\bar{b}_i(s_i)$, the expected demand short at the end of the cycle takes up the value according to the equation (4).

We consider two different sets of data as:

D_1=2700; K_1=12; h_1=0.55; $\theta 1$=0.6; $\mu_1=(a_1+b_1)/2$; ν_1 =0.03; π_1=1; h'_1=0.25; a_1=20; b_1=70; $\mu_1=(a_1+b_1)/2$.

D_2=2750; K_2=10; h_2=0.25; $\theta 2$=0.8; $\mu_2=(a_2+b_2)/2$; ν_2 =0.02; π_2=2; h'_2=0.15; a_2=10; b_2=50; $\mu_2=(a_2+b_2)/2$.

[All the cost related parameters are measured in '$']

Comparison of solutions of FO and IFO (UNIFORM)

METHODS	Q_1	Q_2	s_1	s_2	EC_1 ($)	EC_2 ($)	α^*	β^*
Intuitionistic Fuzzy Optimization	967.6	1136.8	65.371	49.647	516.8335	563.7091	0.76	0.023
Intuitionistic Fuzzy Geometric Programming	988.4	1034.6	68.091	51.097	500.2348	542.9812	0.81	.019

Table - 1

Then from Table- 1, we conclude that Intuitionistic fuzzy Geometric Programming [IFGP] obtains more optimized values of EC_1 and EC_2 than Intuitionistic fuzzy optimization [IFO].

Case2. The lead-time demand follows exponential distribution and thus $\bar{b}_i(s_i)$, the expected demand short at the end of the cycle takes up the value according to the equation (5).

We consider two different sets of data as:

D_1=2700; K_1=8; h_1=1; θ_1=0.4; $\mu_1=1/\lambda_1$; ν_1 =0.03; π_1=1; h'_1=0.25; λ_1=1.

D_2=2750; K_2=10; h_2=1;θ_2=0.7; ν_2 =0.02; π_2=1.1; h'_2=0.15; $\mu_2=1/\lambda_2$; λ_2=1.1

[All the cost related parameters are measured in '$']

Comparison of solutions of FO and IFO (EXPONENTIAL)

METHODS	Q_1	Q_2	s_1	s_2	EC_1 ($)	EC_2 ($)	α^*	β^*
Intuitionistic Fuzzy Optimization	201.73	939.16	3.10	2.28	377.5540	412.6827	0.78564	0.1026
Intuitionistic Fuzzy geometric Programming	199.64	978.23	2.98	2.13	367.0645	401.5634	0.82165	0.0965

Table - 2

Then from Table- 2, we conclude that Intuitionistic fuzzy Geometric Programming [IFGP] obtains more optimized values of EC_1 and EC_2 than Intuitionistic fuzzy optimization [IFO].

Conclusion

Paknejad et al.'s model is considered here. Itemwise multiobjective models for both exponential and uniform lead-time demand are taken and the results are compared numerically both in Intuitionistic fuzzy optimization and intuitionistic fuzzy geometric programming techniques. Objective of this paper is to prove that intuitionistic fuzzy geometric programming always obtains the better value of the objective function than the intuitionistic fuzzy optimazation technique.

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