STUDY OF SOME INTEGRAL EQUATIONS ARISING IN APPLICATIONS OF SCIENCE AND ENGINEERING

Danish-Ul-Islam[1] & Irfan Ul Haq[2]
[1] Research Scholar, [2]Research Scholar
Department of Mathematics
Sant Baba Bhag Singh University, Jalandhar-144030, Punjab India
Emails: danishulislam95@gmail.com , shahirfan3926gmail.com

Abstract: We present the numerical solution of the Fredholm Integral Equations by using the analytic method (Adomian Decomposition Method). To illustrate the accuracy and efficiency of the proposed method (ADM), some numerical examples have been performed. A Fredholm integral equations is solved by ADM which gives us the approximate solution of the problem that tends to the exact solution of the problem.

Keywords: Adomian Decomposition Method, Integral Equations,Fredholm Integral Equations, Numerical Example.

Adomian Decomposition Method

The Adomian Decomposition method (ADM) is very powerful method which considers the approximate solution of a nonlinear equation as an infinite series which actually converges to the exact solution in this paper, ADM is proposed to solve some first order, second order and third order differential equations and integral equations. The Adomian Decomposition method (ADM) was firstly introduced by George Adomain in 1981. This method has been applied to solve differential equations and integral equations of linear and nonlinear problem in Mathematics, Physics, Biology and Chemistry up to know a large number of research paper have been published to show the feasibility of the decomposition method.

Proposed method for solving the Fredholm integral equation.

The type of integral equation in which the limits of the integration are constant, in which a and b are constant are called the Fredholm Integral equations, and is given as

$$u(x) = f(x) + \gamma \int_a^b K(x,t)u(t)dt \quad (1)$$

Where the function and the kernel are given in the advance, and γ is a parameter. In this part,the process of the Adomian decomposition method is used. The Adomian decomposition method involving the decomposing of the unknown function $u(x)$ of any equation into a addition of an infinite number of constituents defined by the decomposition series

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \quad (1.1)$$

Or correspondingly

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots$$

When the constituents $u_n(x), n \geq 0$ will be resolved. The Adomian decomposition method analyze itself with discover the components

$$u_0(x), u_1(x), u_2(x) \dots$$

To organize the recurrence relation, we substitute (1.1) into the Fredholm integral equation (1) to get

$$\sum_{n=0}^{\infty} u_n(x) = f(x) + \int_a^b K(x,t)(\sum_{n=0}^{\infty} u_n(t)dt \quad (4)$$

Or correspondingly

$$u_0(x) + u_1(x) + u_2(x) + \cdots = f(x) + \int_a^b K(x,t)[u_0(t) + u_1(t) + \cdots]dt$$

The zeroth component $u_0(x)$ is spotted by all terms that are not comprises under the integral sign. This signifies that the components $u_n(x), n \geq 0$ of the unknown function $u(x)$are totally resolved by the recurrence relation

$$u_0(x) = F(x), \qquad u_{n+1}(x) = \int_a^b K(x,t)u_n(t)dt, \; n \geq 0$$

Or correspondingly

$$u_0(x) = f(x)$$
$$u_1(x) = \int_a^b K(x,t)u_0(t)dt$$
$$u_2(x) = \int_a^b K(x,t)u_1(t)dt$$
$$u_3(x) = \int_a^b K(x,t)u_2(t)dt$$

And so on the other constituents

Thus the constituents $u_0(x), u_1(x), u_2(x), \dots$ are resolved totally.Thus the solution of the Fredholmintegral equation (1) is easily acquired in a series form by utilizethe series as assumption in (1.1).

Applications Fredholm integral equations:

Example1. Consider the linear Fredholm integral equation given as

$$u(x) = \sin x - x + x\int_0^{\frac{3\pi}{2}} u(t)dt \quad (1.2)$$

Let, $u(x) = \sum_{n=0}^{\infty} u_n(x)$

Then by applying the Adomian decomposition method, equation (1.2) becomes

$$\sum_{n=0}^{\infty} u_n(x) = \sin x - x + x\int_0^{\frac{3\pi}{2}} \sum_{n=0}^{\infty} u_n(t)dt \quad (1.3)$$

To determine the components of u(x), we use the recurrence relation

$$u_0(x) = \sin x - x$$
$$u_{n+1}(x) = x\int_0^{\frac{3\pi}{2}} u_n(t)dt$$

This in turn gives 9+

$$u_0(x) = \sin x - x \quad (1.4)$$

Solving for

$$\begin{aligned} u_1(x) &= x\int_0^{\frac{3\pi}{2}} u_0(t)dt \\ &= x\int_0^{\frac{3\pi}{2}} (\sin t - t)dt \\ &= x[1 - \frac{9\pi^2}{8}] \quad (1.5) \\ u_2(x) &= x\int_0^{\frac{3\pi}{2}} u_1(t)dt \\ &= x\int_0^{\frac{3\pi}{2}} t[1 - \frac{9\pi^2}{8}]dt \end{aligned}$$

$$= \frac{9\pi^2}{8}\left[1 - \frac{9\pi^2}{8}\right] \tag{1.6}$$

Similarly we can get

$$u_3(x) = \frac{9\pi^2}{8}\left[\frac{9\pi^2}{8} - \frac{9\pi^4}{64}\right]$$

$$u_4(x) = \frac{9\pi^2}{8}\left[\frac{9\pi^2}{64} - \frac{9\pi^6}{512}\right] \tag{1.7}$$

And so on

Now by using equation (1.1) we obtain

$$u(x) = sinx - x + x\left[1 - \frac{9\pi^2}{8}\right] + \frac{9\pi^2}{8}x\left[1 - \frac{9\pi^2}{8}\right] + \frac{9\pi^2}{8}\left[\frac{9\pi^2}{8} - \frac{9\pi^4}{64}\right] + \frac{9\pi^2}{8}\left[\frac{9\pi^4}{64} - \frac{9\pi^2}{512}\right] + \cdots$$

$$u(x) = sinx - x + x[1 - \frac{9\pi^2}{8} + \frac{9\pi^2}{8} - \frac{81\pi^4}{64} + \frac{81\pi^4}{64} + \cdots \tag{1.8}$$

thus the solution will be

$$u(x) = sinx - x + x$$

$$u(x) = sinx$$

Example2. Consider the Fredholm Integral equation given as

$$\mathrm{f(x)} = 3 - \frac{4}{3}\mathrm{x} + \int_0^1 \mathrm{xtf(t)dt} \tag{1.9}$$

Let, $u(x) = \sum_{n=0}^{\infty} u_n\,(x)$

Then by applying the Adomian decomposition method, equation (1.9) becomes

$$\sum_{n=0}^{\infty} u_n(x) = 3 - \frac{4}{3}x + \int_0^1 xtf(t)dt$$

To determine the components of u_x, we use the recurrence relation

$$u_0(x) = 3 - \frac{4}{3}x \tag{1.10}$$

$$u_1(x) = \int_0^1 xtu_0(t)dt$$

$$= \int_0^1 3xtdt - \int_0^1 \frac{4}{3}xt^2$$

$$= 3x\left[\frac{t^2}{2}\right]_0^1 - \frac{4}{3}x\left[\frac{t^3}{3}\right]_0^1$$

$$= \frac{3}{2}x - \frac{4}{9}x$$

$$= \frac{19}{18}x \tag{1.11}$$

$$u_2(x) = \int_0^1 (xt)\frac{19}{18}t$$

$$= \frac{19}{18}x\int_0^1 t^2$$

$$= \frac{19}{18}x\left[\frac{t^3}{3}\right]_0^1$$

$$= \frac{19}{54}x \tag{1.12}$$

$$u_3(x) = \int_0^1 (xt)\frac{19}{54}t$$

$$= \frac{19}{54}x\left[\frac{t^2}{2}\right]_0^1$$

$$= \frac{19}{108}x \tag{1.13}$$

And so on

Now by using equation (1.1) we obtain

$$u_nx = 3 - \frac{4}{3}x + \frac{19}{18}x + \frac{19}{54}x + \frac{19}{108}x + \cdots \tag{1.14}$$

thus the solution will be

$$u_x = 3 - \frac{19}{36}x$$

Example3. Consider the Fredholm Integral equation given as

$$f(x) = x + \int_0^1 (xt - x^2)f(t)dt \tag{1.15}$$

Let, $\mathrm{u(x)} = \sum_{\mathrm{n=0}}^{\infty} \mathrm{u_n}\,(\mathrm{x})$

Then by applying the Adomian decomposition method, equation (1.15) becomes

$$\sum_{n=0}^{\infty} \mathrm{u_n}\,(\mathrm{x}) = \mathrm{x} + \int_0^1 \sum_{n=0}^{\infty} \mathrm{u_n}(\mathrm{t})\mathrm{dt}$$

To determine the components of u(x), we use the recurrence relation

$$u_0(x) = x$$
$$u_{n+1}(x) = \int_0^1 u_n t dt$$

This in turn gives

$$u_0(x) = x \tag{1.16}$$

Solving for

$$\begin{aligned} u_1(\mathrm{x}) &= \int_0^1 \mathrm{u_0}\,(\mathrm{t})\mathrm{dt} \\ &= \int_0^1 (xt - x^2)tdt \\ &= x\int_0^1 t^2\,dt - x^2\int_0^1 tdt \\ &= x\left[\frac{t^3}{3}\right]_0^1 - x^2\left[\frac{t^2}{2}\right]_0^1 \\ &= \frac{x}{3} - \frac{x^2}{2} \end{aligned} \tag{1.17}$$

$$\begin{aligned} u_2(x) &= \int_0^1 \mathrm{u_1}\,(\mathrm{t})\mathrm{dt} \\ &= \int_0^1 (xt - x^2)(\frac{t^3}{3} - \frac{t^2}{2})dt \\ &= \int_0^1 \frac{xt^2}{3}dt - \int_0^1 \frac{xt^3}{2}dt - \int_0^1 \frac{tx^2}{3}dt + \int_0^1 \frac{x^2t^2}{2}dt \\ &= \frac{x}{3}\left[\frac{t^3}{3}\right]_0^1 - \frac{x}{2}\left[\frac{t^4}{4}\right]_0^1 - \frac{x^2}{3}\left[\frac{t^2}{2}\right]_0^1 + \frac{x^2}{2}\left[\frac{t^3}{3}\right]_0^1 \\ &= \frac{x}{9} - \frac{x}{8} - \frac{x^2}{6} + \frac{x^2}{6} \\ &= -\frac{x}{72} \end{aligned} \tag{1.18}$$

$$\begin{aligned} u_3(x) &= \int_0^1 \mathrm{u_2}\,(\mathrm{t})\mathrm{dt} \\ &= \int_0^1 (xt - x^2) - \frac{t}{72}dt \\ &= \int_0^1 -\frac{xt^2}{72}dt + \int_0^1 \frac{tx^2}{72}dt \\ &= -\frac{x}{72}\left[\frac{t^3}{3}\right]_0^1 + \frac{x^2}{72}\left[\frac{t^2}{2}\right]_0^1 \\ &= -\frac{x}{216} + \frac{x^2}{144} \end{aligned} \tag{1.19}$$

And so on

Now by using equation (1.1) we obtain

$$u(x) = x + \frac{x}{3} - \frac{x^2}{2} - \frac{x}{72} - \frac{x}{216} + \frac{x^2}{144} + \cdots \tag{1.20}$$

thus the solution will be

$$u(x) = \frac{71}{54}x - \frac{71}{144}x^2$$

Example4. Consider the linear Fredholm integral equation

$$u(x) = 1 + \int_0^1 xu(t)dt \tag{1.21}$$

Let, $u(x) = \sum_{n=0}^{\infty} u_n\, x$

Then by applying the Adomian decomposition equation (1.21) becomes

$$\sum_{n=0}^{\infty} u_n\,(x) = 1 + \int_0^1 x\sum_{n=0}^{\infty} u_n\,(t)dt \tag{1.22}$$

$$u_0(x) + u_1(x) + u_3(x) + \cdots = 1 + \int_0^1 x[u_0 + u_1 + u_2 + \cdots]dt$$

This in turn gives

$$u_0(x) = 1 \tag{1.23}$$

Solving for

$$\begin{aligned} u_1(x) &= \int_0^1 xu_0\,(t)dt \\ &= x \end{aligned} \tag{1.24}$$

$$u_2(x) = \int_0^1 xu_1 tdt$$
$$= \frac{x}{2} \quad (1.25)$$
$$u_3(x) = \int_0^1 xu_2(t)dt$$
$$= \frac{x}{4} \quad (1.26)$$

And so on

Now,

$$u(x) = 1 + x + \frac{x}{2} + \frac{x}{4} + \frac{x}{8} + \cdots$$
$$u(x) = 1 + x + x[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots] \quad (1.27)$$
$$= 1 + x + x[1]$$
$$= 2x + 1$$

Conclusion:

The goal of this paper is to use the Adomian decomposition method for solving the Fredholm integral equations. It is can be clearly seen that the decomposition method for the Fredholm integral equation is equivalent to successive approximation method. Although, the Adomian decomposition method is a very strong and useful device for solving the integral equations

.Following points have been identified while solving the numerical examples:

1.Linear Fredholm integral equation can be solved by this method.

2. It is clear that using the Adomian decomposition method when there is increasing in the 'n' order then there is the decreasing in the error.

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