A Multi-Objective Inventory Model With Deterministic And Probabilistic Constraints: Fuzzy Geometric Programming And Intuitionistic Fuzzy Geometric Programming Approach

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Abstract

A multi-objective inventory model with budgetary and floor space constraints is analyzed by fuzzy geometric programming technique and the result is compared numerically to the solution by intuitionistic fuzzy geometric programming technique. Then the model is changed to a stochastic model under probabilistic constraint. In our discussion the better performance of intuitionistic fuzzy geometric programming technique is established for different objective functions.

1. Introduction

Intuitionistic Fuzzy Set (IFS) was introduced by K. Atanassov (1986) and seems to be applicable to real world problems. The concept of IFS can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. Thus it is expected that, IFS can be used to simulate human decision-making process and any activities requiring human expertise and knowledge that are inevitably imprecise or totally reliable. Here the degree of rejection and satisfaction are considered so that the sum of both values is always less than unity (1986). Atanossov also analyzed Intuitionistic fuzzy sets in a more explicit way. Atanassov (1989) discussed an open problems in intuitionistic fuzzy sets theory. An Interval valued intuitionistic fuzzy sets was analyzed by Atanassov and Gargov (1999). Atanassov and Kreinovich (1999) implemented Intuitionistic fuzzy interpretation of interval data. The temporal intuitionistic fuzzy sets are discussed also by Atanossov (1999). Intuitionistic fuzzy soft sets are considered by Maji, Biswas and Roy (2001). Nikolova, Nikolov, Cornelis and Deschrijver (2002) presented a Survey of the research on intuitionistic fuzzy sets. Rough intuitionistic fuzzy sets are analyzed by Rizvi, Naqvi and Nadeem (2002). Angelov (1997) implemented the Optimization in an intuitionistic fuzzy environment. He (1995) also contributed in his another two important papers, based on Intuitionistic fuzzy optimization. Pramanik and Roy (2005) solved a vector optimization problem using an Intuitionistic Fuzzy goal programming. A transportation model is solved by Jana and Roy (2007) using multi-objective intuitionistic fuzzy linear programming. Mahapatra and Mahapatra (2011) discussed redundancy optimization using intuitionistic fuzzy multi-objective programming

Deterministic optimization problems are well studied, but they are very limited and in many cases they do not represent exactly the real problem (Zimmarmann, 1985). Usually, it is difficult to describe the constraints of an optimization problem by crisp relations (equalities and/or non-equalities) (Zimmarmann, 1983). Practically, a small violation of a given constraint is admissible and it can lead to a more efficient solution of the real problem. Objective formulation represents, in fact, a subjective estimation of a possible effect of a given value of the objective function. In the last two decades optimization problems have been investigated in the sense of fuzzy set theory. Fuzzy optimization formulations are more flexible and allow finding solutions, which are more adequate to the real problem. One of the poorly studied problems in this field is definition of membership degrees. However, the author investigates mainly the transformations and the solution procedures (Sakwa, 1989). On the other hand, fuzzy set theory has been widely developed and various modifications and generalizations have appeared. One of them is the concept of intuitionistic fuzzy (IF) sets. They consider not only the degree of membership to a given set, but also the degree of rejection such that the sum of both values is less than 1 (Atanassov, 1986). Applying this concept it is possible to reformulate the optimization problem by using degrees of rejection of constraints and values of the objective that are non-admissible. The degrees of acceptance and of rejection can be arbitrary (the sum of both have to be less than or equal to 1).

Recent interest in granular computing has focused on fuzzy subsets with non-standard membership grades. Among the most significant of these non-standard fuzzy subsets are intuitionistic fuzzy subsets (Atanassov, 1986, 1989). Here membership is expressed with two values whose sum must be less then one. The difference between this sum and one is called the hesitancy. Generally these values refer a degree of membership and non-membership. In this work we consider the extension to intuitionistic fuzzy subsets of a number of ideas from standard fuzzy subsets. In particular we look at the measure of specificity (Yagor, 1992, 1998). In addition we consider the problem of alternative selection when decision criteria satisfaction is expressed using intuitionistic fuzzy subsets. We also briefly look at the related problem of defuzzification of intuitionistic fuzzy subsets. Before turning to the main body of interest we shall make some brief comment on the semantics associated with intuitionistic fuzzy subsets an issue that is of some interest given the close formal mathematical relationship between intuitionistic fuzzy subsets and interval valued fuzzy subsets. In most cases intuitionistic fuzzy subsets have been used to represent situations in which there exists some uncertainty with respect to knowledge of membership grade. Under this semantics the hesitancy associated with an element is a reflection of the uncertainty about its membership grade. A second, less common, semantics that can be associated with intuitionistic fuzzy subsets is to model situations in which there is a lack of appropriateness of associating a membership grade to an element in a set.1 As an extreme example consider the fuzzy subset corresponding to the predicate happy. If we ask if a rock is happy, this predicate not applicable to a rock. We can use the intuitionistic membership grade (0, 0). Here the hesitancy reflects the lack of applicability of the concept happy to rock. More generally the use of intuitionistic fuzzy allows us to model partial applicability of concept to an object.

Intuitionistic Fuzzy Geometric Programming is a new optimization technique. This method is more effective than Fuzzy Geometric Programming (FGP) in non-linear programming. It has certain advantages over the other optimization methods. Since late 1960, GP has been known and used in various fields (like OR, Engineering Sciences etc.). Duffin, Petersen and Zener (1966) discussed the basic theories with engineering applications in their books. Another famous book on GP and its application appeared in Beightler and Philips (1976). There are many references on application and the methods of GP in the survey papers (like Eckar (1980), Beightler et.al. (1979), Zener (1971). Hariri et al. (1997) discussed the multi-item production lot-size inventory model with varying order cost under a restriction Jung and Klain (2001) developed single item inventory problems and solved by GP method. Ata Fragany and Wakeel (2003) considered some inventory problems solved by GP technique. Zadeh (1965) first gave the concept of fuzzy set theory. Later on Bellman and Zadeh (1970) used the fuzzy set theory to the decision making problem Tanaka (1974) introduced the objective as fuzzy goal over the α-cut of a fuzzy constraint set and Zimmerman (1978) gave the concept to an inventory and production problem. Cao (1993) and his recent book (2002) discussed fuzzy geometric programming with zero degree of difficulty. Das et al. (2000) developed a multi-item inventory model with quantity dependent inventory costs and demand dependent unit cost under imprecise objective function and constraint and solved by GP technique. Roy and Maiti (1997) solved single objective fuzzy EOQ model by GP technique. Recently Mondal et al. (2005) developed a multi-objective inventory model and solved it by GP method. A multi-objective fuzzy economic production quantity model is solved using GP approach by Islam and Roy (2004). Islam and Roy (2007) solved another fuzzy economic production quantity model under space constraint by GP method. Cao (2009) discussed about rough posynomial geometric programming problem. Mahaptra and Roy (2009) analyzed Single and multi container maintenance model by fuzzy geometric programming approach. Shivanian and Khorram (2009) discussed Monomial geometric programming with fuzzy relation inequality constraints with max-product composition. Yousef et-al. (2009) considered some Geometric programming problems with fuzzy parameters. Sadjadi et-al. (2010) discussed a fuzzy pricing and marketing planning model using possibilistic geometric programming approach. Mahapatra and Mahapatra (2011) presented the redundancy optimization by intuitionistic fuzzy multi-objective programming.

In our problem, a stochastic inventory model with deterministic and then with probabilistic constraint is analyzed here. We solve this multi-objective inventory problem with uniform lead-time demand by intuitionistic fuzzy geometric programming technique. We also compare the results solved by Fuzzy Geometric programming technique and it is observed that our Intuitionistic Fuzzy Geometric programming always performs better than the Fuzzy Geometric programming.

2. Mathematical Model

Firstly, we use the following notations to describe the above model:

For the i-th item (i =1, 2, …………,n),

p_i = price per unit item (a decision variable),

Q_i = lot size (a decision variable),

TC(p,Q) = average annual cost,

(p and Q are the vectors of n decision variables p_i (i=1, 2, ……,n) and Q_i (i=1, 2, ……,n) respectively.)

C_{1i} = set up cost per cycle,

C_{2i} = holding cost per unit item,

f_i = floor space available per unit,

n = number of item,

F = available floor space,

B = total budget.

The following assumption are made regarding the above model:

(i) Replenishment is instantaneous,

(ii) No back order is allowed,

(iii) Lead time is zero,

(iv) Demand $D_i(p_i)$ is related to the unit price as:

$D_i = \lambda_i p_i^{-\eta_i}$, where $\lambda_i > 0$ and $0 < \eta_i < 1$ are constants.
...(2.1)

The average total annual cost is minimized here, subject to the limited storage area and fixed budgetary constraints:

$$MinTC(p,Q) = \sum_{i=1}^{n}\left[\lambda_i p_i^{(1-\eta_i)} + \lambda_i C_{1i} p_i^{-\eta_i} Q_i^{-1} + \frac{C_{2i}Q_i}{2}\right]$$
…. (2.2)

Subject to

$$\sum_{i=1}^{n} f_i Q_i \le F$$

$$\sum_{i=1}^{n} p_i Q_i \le B$$

$$p_i, Q_i > 0.$$

(i =1, 2, …………,n)

Model I. Multi-Objective Inventory Model Under Budgetary and Floor Space Constrai*nts*

We model (2.2) as a multi-objective inventory model splitting item-wise objectives as:

$$MinTC_1(p_1,Q_1) = \lambda_1 p_1^{(1-\eta)} + \lambda_1 C_{11} p_1^{-\eta} Q_1^{-1} + \frac{C_{21}Q_1}{2}$$

$$MinTC_2(p_2,Q_2) = \lambda_2 p_2^{(1-\eta)} + \lambda_2 C_{12} p_2^{-\eta} Q_2^{-1} + \frac{C_{22}Q_2}{2}$$

.

.

.

$$MinTC_n(p_n, Q_n) = \lambda_n p_n^{(1-\eta)} + \lambda_n C_{1n} p_n^{-\eta} Q_n^{-1} + \frac{C_{2n} Q_n}{2}$$

…(2.3)

Subject to

$$\sum_{i=1}^{n} f_i Q_i \le F$$

$$\sum_{i=1}^{n} p_i Q_i \le B$$

$$p_i, Q_i > 0.$$

(i =1, 2, …………,n)

Model II. Stochastic Model with Deterministic Storage and Stochastic Budget

In this case p_i's, set up cost, investment cost and holding cost are random parameters. Then the model (2.2) changes to a probabilistic model as:

$$MinTC(\hat{p}, Q) = \sum_{i=1}^{n} \left[\lambda_i \hat{p}_i^{(1-\eta_i)} + \lambda_i \hat{C}_{1i} \hat{p}_i^{-\eta_i} Q_i^{-1} + \frac{\hat{C}_{2i} Q_i}{2} \right]$$

…. (2.4)

Subject to

$$\sum_{i=1}^{n} f_i Q_i \le F$$

$$\sum_{i=1}^{n} \hat{p}_i Q_i \le \hat{B}$$

$$\hat{p}_i, Q_i > 0.$$

(i =1, 2, …………,n)

(Here, '$\wedge$' indicates the randomization of the parameters.)

3. Mathematical Analysis

3.1 Geometric Programming Problem

Geometric Programming (GP) can be considered to be an innovative modus operandi to solve a nonlinear problem in comparison with other nonlinear techniques. It was originally developed to design engineering problems. It has become a very popular technique since its inception in solving nonlinear problems. The advantages of this method is that, this technique provides us with a systematic approach for solving a class of nonlinear optimization problems by finding the optimal value of the objective function and then the optimal values of the design variables are derived, also. This method often reduces a complex nonlinear optimization problem to a set of simultaneous equations and this approach is more amenable to the digital computers.

GP is an optimization problem of the form:

$$Min\ g_0(t)$$

…(3.1)

subject to

$$g_j(t) \le 1,$$

j = 1, 2, ………, m.

$h_k(t) = 1$, k=1, 2, ……….., p

$t_i > 0$, i = 1, 2, ………., n

where, $g_j(t)$ (j = 1, 2, ………, m) are posynomial or signomial functions and $h_k(t)$ (k=1, 2, ……….., p) are monomials t_i (i = 1, 2, ………., n) are decision variable vector of n components.

The problem (3.1) can be written as:

$Min\ g_0(t)$

subject to

$g'_j(t) \le 1$, j = 1, 2, ………, m.

t > 0, [since $g_j(t) \le 1,\ h_k(t) = 1 \Rightarrow g'_j(t) \le 1$ where $g'_j(t)(= g_j(t)/h_k(t))$ be a posynomial (j=1, 2, ………, m ; k=1, 2, ………, p)].

I. Posynomial Geometric Programming Problem

A. Primal problem

$Min\ g_0(t)$ …(3.2)

subject to

$g_j(t) \le 1$, j = 1, 2, ………, m.

$t_i > 0$, (i =1, 2, …………..,n)

where $g_j(t) = \sum_{k=1}^{N_j} c_{jk} \prod_{i=1}^{n} t_i^{\alpha_{jki}}$

here, $c_{jk} > 0$ and α_{jki} (i=1, 2, ………,n ; k=1, 2, ………, N_j ; j=0, 1, ………,m) are real numbers.

$T = (t_1, t_2, \ldots, t_n)^T$.

It is a constrained posynomial primal geometric problem (PGP). The number of inequality constraints in the problem (3.2) is m. The number of terms in each posynomial constraint function varies and is denoted by N_j for each j=0, 1, 2, ……, m.

The degree of difficulty (DD) of a GP is defined as (number of terms in a PGP) –(number of variables in PGP)-1.

B. Dual Problem

The dual problem of (4.4) is as follows:

Max $d(w) = \prod_{j=0}^{m} \prod_{k=1}^{N_j} \left(\frac{c_{jk} w_{j0}}{w_{jk}} \right)^{w_{jk}}$

Subject to

$\sum_{k=1}^{N_0} w_{0k} = 1$ (normality condition)

$\sum_{j=0}^{m} \sum_{k=1}^{N_j} \alpha_{jki} w_{jk} = 0$, (i=1, 2, ………, n) (orthogonality condition)

$w_{j0} = \sum_{k=1}^{N_0} w_{jk} \geq 0$, $w_{jk} \geq 0$, (i=1, 2, ………,n ; k=1, 2, ………, N_j), $w_{00} = 1$.

There are n+1 independent dual constraint equalities and $N = \sum_{j=1}^{m} N_j$ independent dual variables for each term of primal problem. In this case DD=N-n-1.

II. Signomial Geometric Programming Problem

A. Primal problem

$Min\ g_0(t)$
…(3.3) subject to

$g_j(t) \leq \delta_j$, j = 1, 2, ………, m.

$t_i > 0$, (i =1, 2, …………..,n)

where $g_j(t) = \sum_{k=1}^{N_j} \delta_{jk} c_{jk} \prod_{i=1}^{n} t_i^{\alpha_{jki}}$

here, $c_{jk} > 0$ and α_{jki} $\delta_j = \pm 1$ (j = 2, …..,m)

$\delta_{jk} = \pm 1$ (k=1, 2, ………, N_j ; j= 1, ………,m) are real numbers.

T=(t_1, t_2, ………., t_n)^T.

B. Dual Problem

The dual problem of (3.3) is as follows:

$$Maxd(w) = \delta_0 \left(\prod_{j=0}^{m} \prod_{k=1}^{N_j} \left(\frac{c_{jk} w_{j0}}{w_{jk}} \right)^{\alpha_{jk} w_{jk}} \right)^{\delta_0}$$
…(3.4)

Subject to

$\sum_{k=1}^{N_0} \delta_{0k} w_{0k} = \delta_0$ (normality condition)

$\sum_{j=0}^{m} \sum_{k=1}^{N_j} \delta_{jk} \alpha_{jki} w_{jk} = 0$, (i=1, 2, ………, n) (orthogonality condition)

$\delta_j = \pm 1$ (j = 2, …..,m) $\delta_0 = +1, -1$.

$\delta_{jk} = \pm 1$ (k=1, 2, ………, N_j ; j= 1, ………,m) are real numbers.

$w_{j0} = \delta_j \sum_{k=1}^{N_0} \delta_{jk} w_{jk} \geq 0$, $w_{jk} \geq 0$, (j=1, 2, ………,m ; k=1, 2, ………, N_j), $w_{00} = 1$.

3.2 Functional Substitution

When a non-linear programming problem (NLP) is of the following form:

$$Min\, y(x) = f(x) + (q(x))^n h(x) \qquad x > 0, \;\; n > 0.$$

Where, $f(x)$, $q(x)$ and $h(x)$ are single or multi-term functionals of posynomial or signomial form. This generalized formulation is not directly solvable using geometric programming; however, under a simple transformation it can be changed into standard geometric programming form. Let $P = q(x)$ and replace the above problem with the following one:

$$Min\, \bar{y}(x) = f(x) + P^n h(x)$$

subject to

$$P^{-1}(q(x)) \leq 1$$

$$x, P > 0.$$

The rationale used in constructing the equivalent problem with an inequality constraint is based on the following logic. Since $y(x)$ is to be minimized, if $q(x)$ is replaced by P, then it is correct to say that $P \geq q(x)$, realizing that in the minimization process P will remain as small as possible. Hence $P = q(x)$ at optimality. Note that $h(x)$ and/or $q(x)$ are permitted to be multiple term expressions and that the optimal (minimizing) solution to $\bar{y}(x)$ is obviously the same as the optimal solution to $y(x)$.

3.3 Lemma

A stochastic non-linear programming problem is considered as:

Min $f_0(X)$

Subject to

$f'_j(X) \leq c_j$ (j=1, 2, ……….,m)

$X \geq 0.$

So,Min$f_0(X)$
….(3.5)

Subject to

$f_j(X) \leq 0$ (j=1, 2, ……….,m)

$X \geq 0.$

Where, $f_j(X) = f'_j(X) - c_j$

Here X is a vector of N random variables $y_1, y_2, \ldots\ldots, y_n$ and it includes the decision variables $x_1, x_2, \ldots\ldots, x_n$.

Expanding the objective function $f_0(X)$ about the mean value $\bar{y}_i$ of y_i and neglecting the higher order term:

$$f_0(X) = f_0(\bar{X}) + \sum_{i=1}^{N} \left(\frac{\partial f_0}{\partial y_i} \bigg| \bar{X} \right) (y_i - \bar{y}_i) = \xi(X) \text{ (say)} \qquad \ldots(3.6)$$

If y_i (i=1, 2, ……. ,n) follow normal distribution then so does $\xi(X)$. The mean and variance of $\xi(X)$ are given by:

$$\bar{\xi} = \xi(\bar{X})$$
….(3.7)

$$\sigma_{\xi}^{2} = \sum_{i=1}^{N}\left(\frac{\partial f_0}{\partial y_i}\bigg|\overline{X}\right)\sigma_{y_i}$$
….(3.8)

When some of the parameters of the constraints are random in nature then the constraints will be probabilistic and thus, the constraints can be written as:

$P(f_j \leq 0) \geq r_j$ (j=1,2………,m)

Then in the light of the theoretical convention given above, equivalent deterministic constraints are:

$$\bar{f}_j - \phi_j(r_j)\left[\sum_{i=1}^{N}\left(\frac{\partial f_j}{\partial y_i}\bigg|\overline{X}\right)\sigma_{y_i}{}^{2}\right]^{1/2} \leq 0 \quad \text{(j=1,2 , ………,m)} \qquad ….(3.9)$$

where, $\phi_j(r_j)$ is the value of the standard normal variate corresponding to the probability r_j.

4. Intuitionistic Fuzzy Geometric Programming Problem [IFGPP]

Multi-objective geometric programming (MOGP) problem is a special type of a class of MONLP problems. Here an intuitionistic fuzzy geometric programming technique is developed to solve a MOGP problem.

A Multi-Objective Non-Linear Programming (MONLP**)** or Vector Minimization problem (VMP) may be taken in the following form**:**

Min $f(x) = [f_1(x), f_2(x), f_3(x), …. , f_k(x)]^T$

Subject to $x \in X = \{x \in R^n : g_j(x) \leq$ or $=$ or $\geq b_j$ for j = 1, …….., m

and $l_i \leq x_i \leq u_i$ (i = 1, 2, ……. , n)$\}$.

Zimmermann (1978) showed that fuzzy programming technique could be used to solve the multi-objective programming problem.

To solve the MOGP problem we follow the Zimmerman's technique (1978). The procedure consists of the following steps:

Step 1. Pick the first objective function and solve it by geometric programming, as a single objective NLP problem subject to the same constraints. Continue the process K-times for K different objective functions. These K sets of solutions are called ideal solutions. If all the solutions i.e. $\overline{X}_1{}^* = \overline{X}_2{}^* = ... = \overline{X}_k{}^*$ are same, then one of them is the optimal compromise solution and go to step 6. Otherwise go to step 2. (However, this rarely happens due to the conflicting objective functions)

Step 2. To build membership function, goals and tolerances should be determined. Using the ideal solutions, obtained in step 1, we find the values of all the objective functions at each ideal solution and construct pay off matrix as follows:

$$\begin{bmatrix} Z_1(\overline{X}_1{}^*) & Z_2(\overline{X}_1{}^*) & .... & .... & .... & Z_k(\overline{X}_1{}^*) \\ Z_1(\overline{X}_2{}^*) & Z_2(\overline{X}_2{}^*) & .... & .... & .... & Z_k(\overline{X}_2{}^*) \\ .... & .... & .... & .... & .... & .... \\ .... & .... & .... & .... & .... & .... \\ .... & .... & .... & .... & .... & .... \\ Z_1(\overline{X}_k{}^*) & Z_2(\overline{X}_k{}^*) & .... & .... & .... & Z_k(\overline{X}_k{}^*) \end{bmatrix}$$

Step 3. Determine the upper and lower bounds of each objective for the degree of acceptance and rejection corresponding to the set of solutions as follows:

$$U_k{}^{acc} = \max_{1 \leq r \leq k}(Z_k(\overline{X}_r{}^*)) \quad \text{and} \quad L_k{}^{acc} = \min_{1 \leq r \leq k}(Z_k(\overline{X}_r{}^*))$$

For linear membership functions,

$$L_k^{rej} = L_k^{acc} + t(U_k^{acc} - L_k^{acc}) \text{ where } 0 < t < 1$$

$$U_k^{rej} = U_k^{acc} + t(U_k^{acc} - L_k^{acc}) \text{ for } t = 0$$

Then the intuitionistic fuzzy goals take the form

$$Z_k(\bar{X}) \tilde{\leq} L_k(\bar{X})^*{}_k \; k = 1,2,......., K.$$

The linear membership and non-membership function for the Intuitionistic fuzzy goals $Z_k(\bar{X})$ are defined as:

$$\mu_k(Z_k(\bar{X})) = \begin{cases} 1 & \text{if } Z_k(\bar{X}) \leq L_k^{acc} \\ \dfrac{U_k^{acc} - Z_k(\bar{X})}{U_k^{acc} - L_k^{acc}} & \text{if } L_k^{acc} \leq Z_k(\bar{X}) \leq U_k^{acc} \\ 0 & \text{if } Z_k(\bar{X}) \geq U_k^{acc} \end{cases}$$

…. (4.1)

$$\nu_k(Z_k(\bar{X})) = \begin{cases} 1 & \text{if } Z_k(\bar{X}) \geq U_k^{rej} \\ \dfrac{Z_k(\bar{X}) - L_k^{rej}}{U_k^{rej} - L_k^{rej}} & \text{if } L_k^{rej} \leq Z_k(\bar{X}) \leq U_k^{rej} \\ 0 & \text{if } Z_k(\bar{X}) \leq L_k^{rej} \end{cases}$$

…(4.2)

Step 4. When the degree of rejection (non-membership) is defined simultaneously with degree of acceptance (membership) of the objectives and when both of these degrees are not complementary to each other, then IF sets can be used as a more general tool for describing uncertainty.

To maximize the degree of acceptance of IF objectives and to minimize the degree of rejection of IF objectives and following the fuzzy decision of Bellman-Zadeh (1970) and Anglov (1997) together with linear membership and non-membership functions of (6.2) and (6.3), an intuitionistic fuzzy optimization model of NLP problem can be written as:

$$\max \; \mu_k(\bar{X}), \bar{X} \varepsilon R, \; k = 1,2,......,K$$
$$\min \; \upsilon_k(\bar{X}) \text{ [i.e. max } (-\upsilon_k(\bar{X}))], \bar{X} \varepsilon R, \; k = 1,2,......,K$$

....(4.3)

Subject to

$$\upsilon_k(\bar{X}) \geq 0,$$
$$\mu_k(\bar{X}) \geq \upsilon_k(\bar{X})$$
$$\mu_k(\bar{X}) + \upsilon_k(\bar{X}) < 1$$
$$\bar{X} \geq 0$$

Using max-additive operator, problem (4.1) can be reduced as:

$$Max \sum_{k=1}^{K} \left(\mu_k(\bar{X}) - \upsilon_k(\bar{X})\right)$$

….(4.4)

Subject to

$\upsilon_k(\bar{X}) \geq 0,$

$\mu_k(\bar{X}) \geq \upsilon_k(\bar{X})$

$\mu_k(\bar{X}) + \upsilon_k(\bar{X}) < 1$

$\bar{X} \geq 0, \bar{X} \varepsilon R, k = 1, 2, ..., K.$

Step 5. Solve the crisp NLP problem (4.2) by Geometric programming method.

Step 6. STOP.

5. Numerical Examples

For numerical illustration of model (2.3), we consider the following data:

n=2, λ_1=100; λ_2=120; C_{11}=100; C_{12}=120; C_{21}=1; C_{22}=1.5; η_1=0.85; η_2=0.8;

B=12000;f_1=20;f_2=30;F=20000;$U_1^{acc} = U_1^{rej}$ =202.23; L_1^{acc} =197.56; L_1^{rej} =199;
$U_2^{acc} = U_2^{rej}$ =310.67; L_2^{acc} =276.43; L_2^{rej} =280.

[All the cost related parameters are measured in "\$" and area is measured in "m^2" and all the first and second components of the fuzzy and stochastic normal variables indicates respectively, their mean and standard deviations.]

The comparative study of Fuzzy Geometric Programming (FGP) and Intuitionistic Fuzzy Geometric Programming (IFGP) technique of the model (2.3)

Method	TC_1	TC_2	Q_1	Q_2	p_1	p_2	μ^*	ν^*
FGP	197.04	274.27	31.49	42.01	14.96	12.16	0.897	--
IFGP	191.22	270.23	35.45	41.37	14.98	11.08	0.913	0.058

Table - 5.1

From the above Table - 5.1 we conclude that TC_1 and TC_2 both are minimized more in case of IFGP than FGP, for the model (2.3)

For numerical illustration of model (2.4), we consider the following data:

n=2, λ_1=100; λ_2=120; r = 0.95; $\hat{C}_{11}$=(\$100,\$1); $\hat{C}_{12}$=(\$120,\$1.2); $\hat{C}_{21}$=(\$1,\$0.01); $\hat{C}_{22}$=(\$1.5,\$0.015); η_1=0.85; η_2=0.8; $\hat{B}$=(\$12000,\$12); f_1=2 m^2; f_2=3 m^2;

F=15000m^2;$U_1^{acc} = U_1^{rej}$ =450.43; L_1^{acc} =486.36; L_1^{rej} =490; $U_2^{acc} = U_2^{rej}$ =0.7321; L_2^{acc} =0.6143; L_2^{rej} =0.62.

$\hat{p}_i = (\bar{p}_i, 0.01\bar{p}_i)$ for i = 1, 2.

[All the cost related parameters are measured in "\$" and area is measured in "m^2" and all the first and second components of the fuzzy and stochastic normal variables indicates respectively, their mean and standard deviations.]

The comparative study of Fuzzy Geometric Programming (FGP) and Intuitionistic Fuzzy Geometric Programming (IFGP) technique of the model (2.4)

Method	μ_{TC}	σ_{TC}	Q_1	Q_2	p_1	p_2	μ^*	ν^*
FGP	483.05	0.58	25.75	31.16	19.72	24.74	0.96	--
IFGP	482.679	0.57	28.25	34-26	18.89	20.92	0.98	0.071

Table - 5.2

From Table - 5.2, we conclude that μ_{TC} is more minimized in case of Intuitionistic Fuzzy Geometric Programming (IFGP) than Fuzzy Geometric Programming (FGP) method, for the model (2.4).

References

Abou-El-Ata, M.O., Fergany H.A. and E1- Wakeel M.F. (2003). Probabilistic Multi-item Inventory Model With Varying Order Cost Under Two Restrictions: A Geometric Programming Approach, Intl. J. Productions Economics, 83, 223-231.

Angelov P P (1995). Intuitionistic fuzzy optimization, Notes on Intutionistic Fuzzy Sets, 1, 27-33.

Angelov P P (1995). Intuitionistic fuzzy optimization, Notes on Intutionistic Fuzzy Sets, 1(2), 123-129.

Angelov P P (1997). Optimization in an intuitionistic fuzzy environment, Fuzzy sets and systems 86, 299-306.

Atanassov, K. T. (1986). Intuitionistic fuzzy sets, Fuzzy Sets and Systems, Vol.20, No.1, 87–96.

Atanassov, K. T. (1989). More on intuitionistic fuzzy sets, Fuzzy Sets and Systems, 33, 37–459.

Atanassov K, Gargov G (1989). Interval valued intuitionistic fuzzy set, Fuzzy Sets and Systems, Vol. 31, No. 3, 343-349.

Atanassov K (1995), Ideas for intuitionistic fuzzy sets equations, inequalities and optimization, Notes on Intuitionistic Fuzzy Sets I, 17- 24.

Atanassov K, Kreinovich V (1999). Intuitionistic fuzzy interpretation of intetrval data, Notes on Intuitionistic Fuzzy Sets, Vol. 5, No. 1, 1-8.

Atanassov K (1999). Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Berlin.

Bellman, R. E. and Zadeh, L. A.(1970). Decision-making in a fuzzy environment, Management Science, 17, B141-B164.

Beightler, G.S. and Phillips, D.T. (1976). Applied Geometric Programming, Willey, New York.

Beightler, G.S. and Phillips, D.T. and Wild, D.J. (1979). Foundation of optimization, Prentice-Hall, Englewood Cliffs, NJ.

Biswal M.P. (1992), "Fuzzy Programming Technique to solve multi-objective geometric programming problem", Fuzzy Sets and Systems, 51: 67-71.

Cao, B.Y. (1993). Fuzzy geometric programming (I)", Fuzzy Sets and Systems, 53, 135-153.

Cao B Y (2009). Rough posynomial geometric programming, Fuzzy Information and Engineering 1(1), 37-55.

Cao, B.Y. (2002). Fuzzy geometric programming, Kluwer Academic Publishers, London.

Cornelis, C., Atanassov, K. T., & Kerre, E. E. (2003). Intuitionistic fuzzy sets and interval valued fuzzy sets: A critical comparison, Proceedings Third European Conference (pp. 227–235). Germany: Fuzzy Logic Technology (EUSFLAT).

Das, K., Roy T.K. and Maiti, M. (2000). Multi-item inventory model with quantity dependent inventory costs and demand-dependent unit cost under imprecise objective and restrictions: a geometric programming approach, Production Planning and control, 11(8), 781-788.

Dubois, D., & Prade, H. (1985). A note on measures of specificity for fuzzy sets, International Journal of General Systems, 10, 279–283.

Dubois, D., & Prade, H. (1987). The principle of minimum specificity as a basis for evidential reasoning, In B. Bouchon & R. R. Yager (Eds.), Uncertainty in knowledge-based systems (pp. 75–84). Berlin: Springer-Verlag.

Dubois, D., & Prade, H. (1991). Fuzzy sets in approximate reasoning Part I: Inference with possibility distributions, Fuzzy Sets and Systems, 40, 143–202.

Dubois, D., Gottwald, S., Hajek, P., Kacprzyk, J., & Prade, H. (2005). Terminological difficulties in fuzzy set theory—The case of Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 156, 485–491.

Duffin, R.J., Peterson, E.L. and Zener, C. (1966). Geometric Programming Theory and Applications, Willey, New York.

Eckar, J. (1980), Geometric programming methods, computational and applications, SIAM Rev., 22, 338-362.

Hariri, A.M.A and Abou-EI-Ata, M.O. (1997). Multi-item EOQ inventory model with varying holding cost under two restrictions: a geometric programming approach ,Production Planning & Control, 8(5), 608-611.

Islam Saidul and Roy T.K. (2004). An economic production quantity model with flexibility and reliability consideration and demand dependent unit cost with negetive degree of difficulty: Geometric Programming Approach, Tamsui Oxford Journal of Management Science, Taiwan, vol. 20(1), pp. 01-17.

Jung, H. and Klein, C.M. (2001) Optimal inventory Plolicies under decreasing cost functions via geometric programming, European Journal of Operational Research, 132,628-642.

Kaufmann, A. and Gupta, M., (1988). Fuzzy Mathematical Models in Engineering and Management Science, North Holland.

Klir, G. J. (2006). Uncertainty and information, New York: John Wiley & Sons.

Mahapatra G S, Roy T K (2009). Single and multi container maintenance model: A fuzzy geometric programming approach, Journal of Mathematics Research 1(2), 47-60.

Mahapatra G.S. and Mahapatra B.S. (2011), " Redundancy optimization using intuitionistic fuzzy multi-objective programming", Vol.7, No.2, 155-164.

Maji P.K., Biswas R, Roy A R, (2001). Intuitionistic fuzzy soft sets, The Journal of Fuzzy Mathematics, Vol. 9, No. 3, 677-692.

Mandal, N.K., Roy, T.K. and Maiti, M. (2005). Multi-objective Fuzzy EOQ model with three constraints: A geometric programming approach, Fuzzy Sets and Systems, 150(1), 87-106.

Nikolova M., N. Nikolov, C. Cornelis, G.Deschrijver, (2002). Survey of the research on intuitionistic fuzzy sets, Advanced Studies in Contemporary Mathematics, Vol. 4, No. 2, 127-157.

Pramanik, S. and Roy, T.K., (2005). An Intuitionistic Fuzzy goal programming approach to vector optimization problem", Notes on Intutionistic Fuzzy Sets, 11(1), 1-14.

Rizvi S., H.J. Naqvi, D. Nadeem. (2002). Rough intuitionistic fuzzy sets, Proceedings of 6-th Joint Conf. on Information Sciences, Research Triangle Park (North Carolina, USA), March 8-13, 101-104.

Sadjadi S J, Ghazanfari M, Yousefli A (2010). Fuzzy pricing and marketing planning model: A possibilistic geometric programming approach, Expert Systems with Applications 37, 3392-3397.

Sakawa M and Yano H (1989), An interactive fuzzy satisficing method for multiobjective non linear programming problems with fuzzy parameters, Fuzzy Sets and Systems 30, 221- 238.

Shivanian E, Khorram E (2009). Monomial geometric programming with fuzzy relation inequality constraints with max-product composition, Computers and Industrial Engineering 56(4), 1386-1392.

Tanaka, H., Okuda, T. and Asai, K. (1974). On fuzzy mathematical programming, Journal of Cybernetics, 3(4), 37-46.

Tiwari, R.N., Dharmar, S. and Rao, J.R. (1987). Fuzzy goal programming: an additive model, Fuzzy Sets and Systems, 24, 27-34.

Verma R. K. (1990). Fuzzy geometric programming with several objective functions, Fuzzy Sets and Systems, 35, 115-120.

Yager, R. R. (1986). A characterization of the extension principle, Fuzzy Sets and Systems, 18, 205–217.

Yager, R. R. (1992). On the specificity of a possibility distribution, Fuzzy Sets and Systems, 50, 279–292.

Yager, R. R. (1998). On measures of specificity. In O. Kaynak, L. A. Zadeh, B. Turksen, & I. J. Rudas (Eds.), Computational intelligence: Soft computing and fuzzy-neuro integration with applications (pp. 94–113), Berlin: Springer-Verlag.

Yager, R. R., & Kikuchi, S. (2004). On the role of anxiety in decisions under possibilistic uncertainty, IEEE Transactions on Systems, Man and Cybernetics: Part B, 34, 1224–1234.

Yager, R. R. (2007a). Owa aggregation of intuitionistic fuzzy sets, Technical report# MII–2717. New Rochelle, NY: Machine Intelligence Institute, Iona College.

Yager, R. R. (2008). Measures of specificity over continuous spaces under similarity relations, Fuzzy Sets and Systems, 159, 2193–2210.

Yousef S, Badra N, Yazied Abu-El T G (2009). Geometric programming problems with fuzzy parameters and its application to crane load sway, World Applied Sciences Journal 7(1), 94-101.

Zadeh, L. A.(1965). Fuzzy sets, Information and Control, 8, 338-356.

Zadeh, L. (1975). The concept of a linguistic variable and its application to approximate reasoning, Part 1. Information Sciences, 8, 199–249.

Zadeh, L. A. (2005). Toward a generalized theory of uncertainty (GTU)-An outline. Information Sciences, 172, 1–4.

Zener, C. (1971), Engineering design by geometric programming, Wiley.

Zimmerman, H. J.(1976). Description and optimization of fuzzy system, International Journal of General Systems, 2, 209-215.

Zimmermann, H.J. (1978), "Fuzzy linear programming with several objective functions", Fuzzy Sets and Systems, 1, 46-55.

Zimmermann H J (1983), Fuzzy mathematical programming, Comput. Oper. Res. 10, 291- 298.

Zimmermann H J (1985), Fuzzy Sets Theory and its Application (Kluwer, Dordrecht).