OPTIMUM CONSUMPTION EQUILIBRIUM

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Abstract : The solution to the problem of maximizing fuzzy utility function subject to a traditional budget constraints yields an optimum consumption equilibrium. We define a nonfuzzy budget set as a subset in the commodity space bounded by the price line and the consumer's initial endowments. In this paper, we through light on the conditions for obtaining consumption equilibriums. For expositional purpose, we confine the analysis to a simple case of two commodities, labor service and wheat.

Keywords : optimum consumption equilibrium, β-consumption equilibrium point, u-consumption equilibrium point, nonfuzzy budget.

The solution to the problem of maximizing fuzzy utility function subject to a traditional budget constraints yields an optimum consumption equilibrium. We define a nonfuzzy budget set as a subset in the commodity space bounded by the price line and the consumer's initial endowments.

We consider again the simple case of two commodities e.g., labor service and wheat. Let the prices of labor service and wheat be denoted by p_1 and p_2, respectively. Thus, $p_1x_1 + p_2x_2$ represents the amount, net of income earned by supplying labor services the consumers spend on wheat. We assume that the consumer has an initial holdings of wheat x_1^0 and faces an exogeneously determined set of prices. The consumer maximizes his fuzzy utility from consuming wheat and providing labor services subject to the budget constraint, $p_1x_1 + p_2x_2 = p_1x_1^0$. The nonfuzzy budget line (or price line) is the boundary of the nonfuzzy budget set.

What we face here is to seek the 'tangent point' between a fuzzy set and a straight line.

We define a u-consumption equilibrium point as a point $\hat{x} = (x_1, x_2)$ at which the budget line is tangent to the edge of the convex $\hat{\alpha}$ -cut set of R_u. We note that the grade of membership of a u-consumption equilibrium (with respect to the fuzzy consumption equilibrium set) is given by $\hat{\alpha}$ in Fig. 1) Obviously, another value u will induce another weak preference set R_u and the point, at which the convex edge of $\hat{\alpha}$ -cut set of R_u is tangent to budget line, and its membership grade $\hat{\alpha}$ will be changed at the same time. The fuzzy u-consumption equilibrium set is defined by

$$E_u = \sum_u \hat{\alpha}_i / \hat{\chi}_i$$

where every u, u_i, is the value of utility which is associated with the consumption equilibrium point $\hat{\alpha}_i / \hat{x}_i$. E_u is a fuzzy subset of X on the budget line.

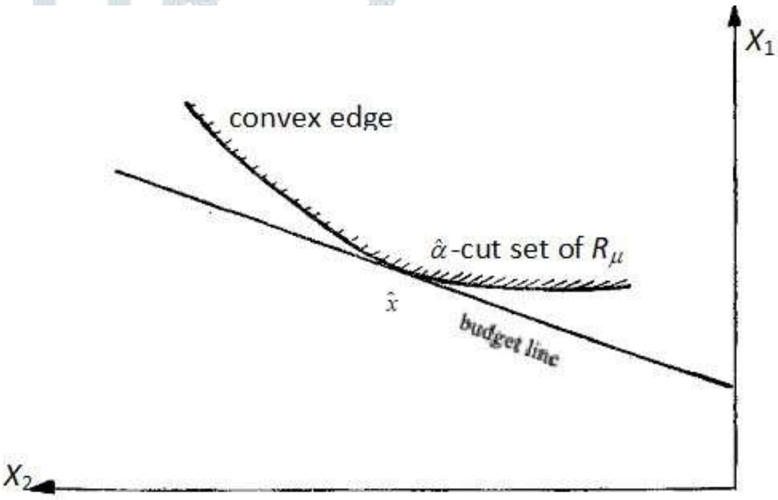

Fig. 1. Definition of the μ-consumption equilibrium point

We define β-consumption equilibrium point as a point $x_i = (x_1, x_2)$ at which the budget line becomes tangent to the edge of convex α -cut set of R_β. Its grade of membership equals to α, $\alpha = \beta\hat{\alpha}$.

We define M-consumption equilibrium set as a fuzzy set, denoted by E_M, of X on the budget line.

$$E_M = \sum_M \sum_\beta \alpha_{ij} / x_{ij}$$

where α_{ij} / x_{ij} is the β-consumption equilibrium point for the β_i-cut of $1(M_j)$

Theorem 2 warrants the uniqueness of x and x for every M (associated with one β) and u, respectively.

Theorem 3: The fuzzy u-consumption equilibrium set is equal to the fuzzy M-consumption equilibrium set, i.e., $E_u = E_M$.

Proof: There are two parts in this proof.

I. We consider two partition sets as described in Fig. 2. It is obvious that $E_{u_i} \subset E_{M_j}$, because E_{u_j} is a component of E_{M_j} with $\beta = 1$. It is known that if $A_1 \subset B_1, A_2 \subset B_2, \ldots, A_n \subset B_n$, then $A_1 \cup \ldots \cup A_n \subset B_1 \cup \ldots \cup B_n$. Thus

$$\sum_{u > u_j} E_{u_j} \subset \sum_{M > M_j} E_{M_j}$$

and $\sum_{u>0} E_{u_j} \subset \sum_{M>0} E_{M_j}$

or $E_u \subset E_M$.

II. Now we consider two other possible partition sets as described in Fig. 3 in which $1(u_j)$ is the support of $1(M)$. Obviously, the membership grade of every β consumption equilibrium point for every β-cut of $1(M)$ is equal or smaller than that of u-consumption equilibrium point corresponding to the α-cut set of $l(M)$, where $\alpha = \beta$. Hence

$$\sum_{M > M_j} E_{M_j} \subset \sum_{u > u_j} E_{u_j}$$

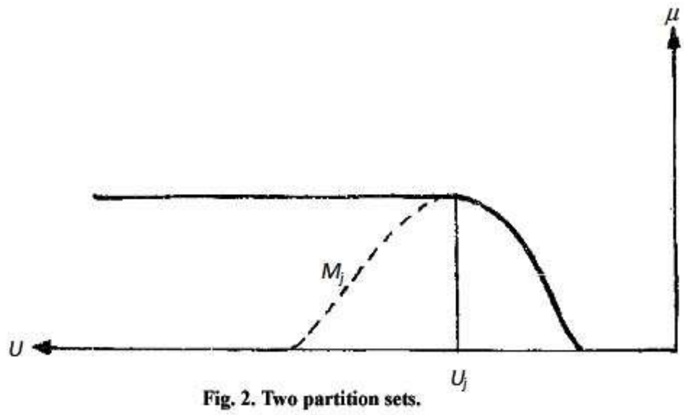

Fig. 2. Two partition sets.

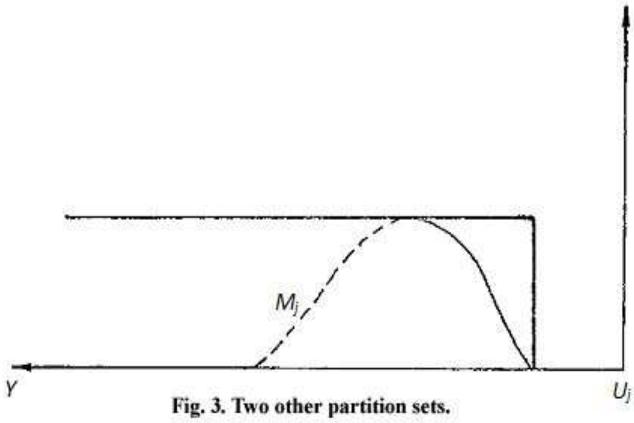

Fig. 3. Two other partition sets.

where M_j ranges over $F(U)$, u_j ranges over U we have

$E_M \subset E_u$.

thus $E_M = E_u$, since E_M and E_u include each other.

It is of interest to note the existence of multiple consumption equilibriums. We consider, for example, a fuzzy utility function defined over nonfuzzy wheat and fuzzy labor services. Such a fuzzy utility function is depicted in Fig. 4. With the exceptions of the corner points, all contours of various α-cut of weak u preference set for every u are nonintersecting owing to the convexity of utility indicators. Assuming marginal utility is diminishing, the contours of equal α-cut of various weak u preference sets are also nonintersecting. However, the contours of different levels α-cut in different u may interest each other. It follows that the points of tangency between the budget lines and the contours of the α-cut in different u cannot overlap.

Theorem 4: If all the utility indicators of a fuzzy utility function are comparable, optimum equilibrium set reduces to a singular point.

Proof: Construct a plane vertical to X and passing through the budget line. We can identify all the intersections between this plane and the fuzzy utility relation in Fig. 5(a).

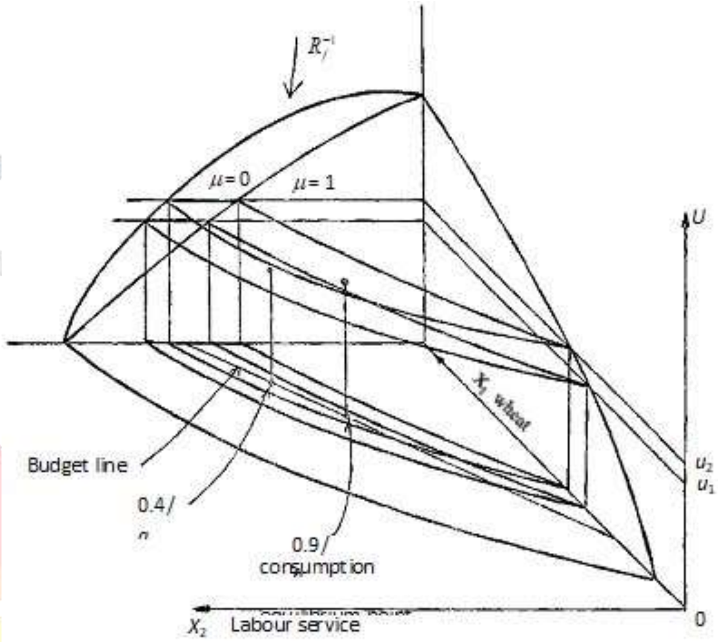

Fig. 4. The existence of multiple consumption equilibriums.

We suppose the contour of α_1-cut with α_1 = 0.3 for example, is tangent, at point a, to the straight line P, which runs in parallel with the budget line. The tangency point a is a consumption equilibrium with membership grade α_1. Because of the convexity of R_f^{-1}, the membership grades of all the other points on the budget line for the same utility level must be smaller than α_1. In this case, where all of the utility indicators are comparable, the membership grade of the point a is maximum at any of the other utility levels. The case in which the utility indicators are not comparable is depicted in Fig. 5(b).

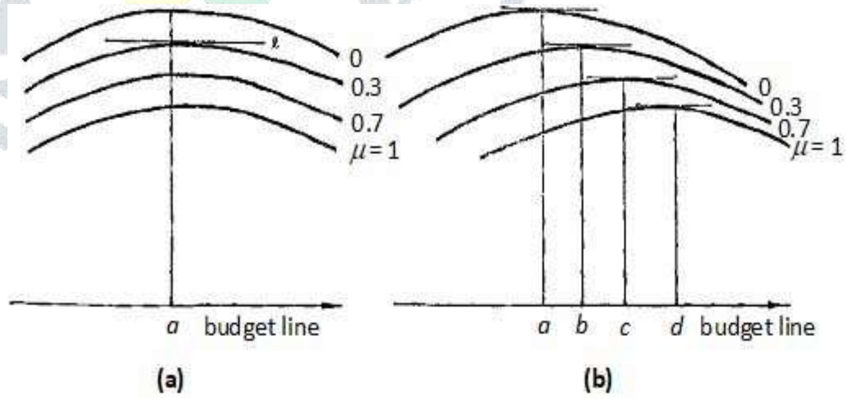

Fig. 5. Two kinds of optimum equilibrium.

Theorem 5: From the above discussions, it follows immediately that the uniqueness of the consumption equilibrium requires the fulfillment of the following three conditions:

(1) utility indicators are convex;

(2) utility indicators are comparable; and

(3) marginal utility of every iso-membership grade surface of a fuzzy utility function is diminishing.

We consider an analytically representable fuzzy utility function described by

$$U = (1 - \frac{k}{2} x_2)(1 - e^{(x_2^2 - x_1 + 1)/10})$$

where x_1 represents one kind of commodity and x_2 another kind. U represents the fuzzy utility indicators of commodity bundles (x_1, x_2). Parameter k, $0 \le k \le 2$, relates the membership grade μ, $0 \le \mu \le 1$, via the following equations

$$\begin{cases} \mu = k, & 0 \le k \le 1 \\ \mu = 2 - k, & 1 < k \le 2 \end{cases}$$

If x_1 represents a certain amount of wheat and x_2 the hours of labor service required for exchanging the x_1 wheat, then U is the degree of satisfaction of the person after this trade, Obviously, for any given x_1 and x_2, a fuzzy set $U_1 \in F(U)$ can be obtained. When $x_2 = 0$, U becomes independent of k (therefore, also μ); since we assumed that the consumer in question was precise in the estimation of utility of wheat but fuzzy in that of labor.

It was pointed out in Theorem 2 that the lagging edges of all fuzzy utility indicators failed to make any contribution to the construction of the weak preference set, hence, also the consumption equilibrium. Thus, it is sufficient to consider the range $0 \le k \le 1$. In Fig. 6 some iso-membership surfaces of fuzzy utility function within this range are depicted in company with the budget line $x_1 = 4x_2 + 2$. Fig. 7 shows the intersection between the iso-membership surface and the budget plane. The maxima of these curves determine the consumption equilibrium points. From Fig. 8, it is seen that the membership grade of a point in the consumption equilibrium set and the correspondent utility indicator run in the opposite directions. We express consumption equilibrium points only by one of their components x_2, since x_2 is given, x_1 can be obtained from the budget line.

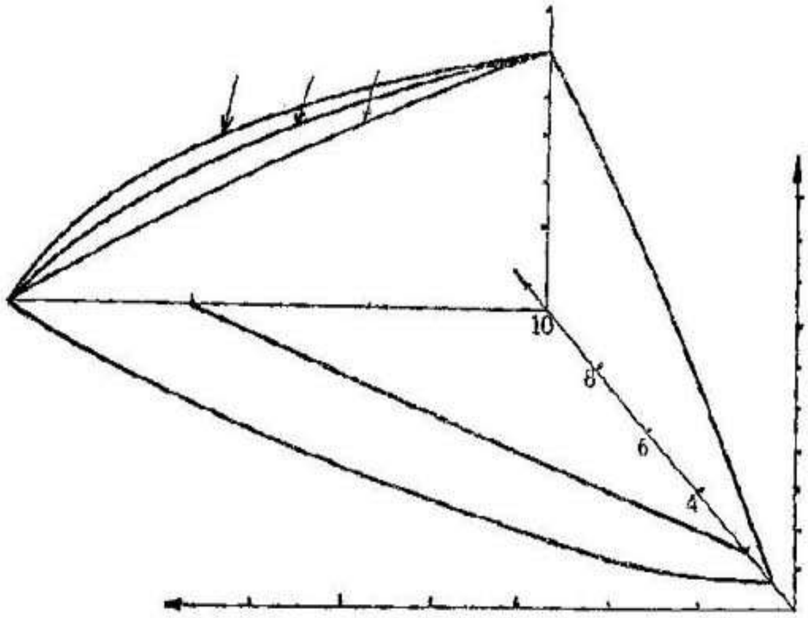

Fig. 6. Two kinds of optimum equilibrium.

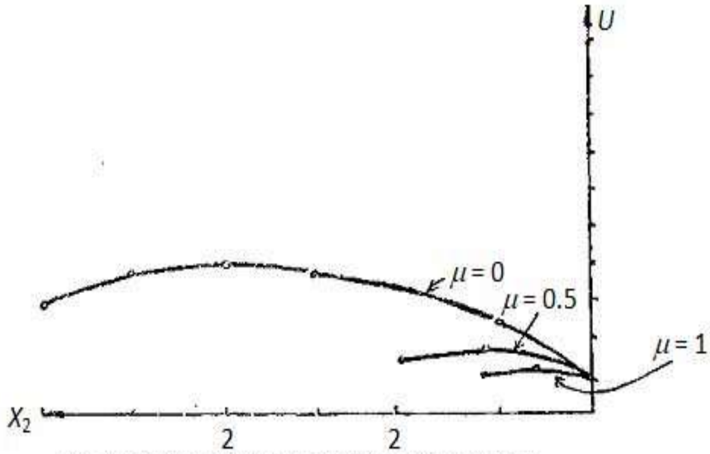

Fig. 7. Relation between U and X2 on the budget line.

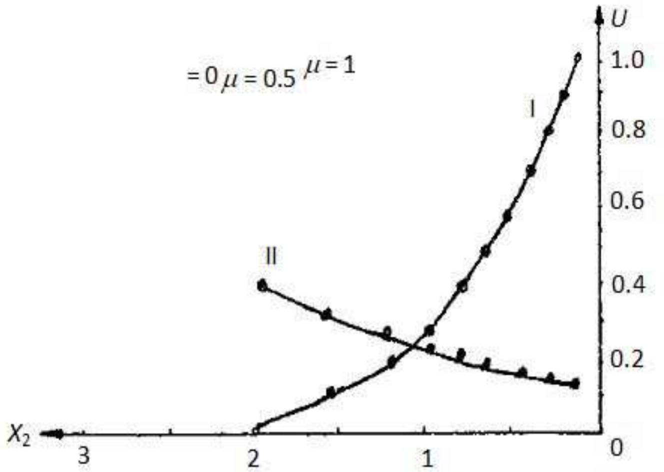

Fig. 8. I equilibrium point and its μ.
II utility on equilibrium point.

In the preceding analysis, several interesting properties of a fuzzy utility function have been established. Given diminishing marginal utility of every iso-membership grade surface of a fuzzy utility relation, the optimum consumption equilibrium is a fuzzy set on the budget line in the commodity space, provided that every fuzzy utility indicator of a fuzzy utility relation is convex and normalized. It has been pointed out that the utility level associated with a consumption equilibrium increases as the membership grade of this equilibrium decreases. Since the membership grade implies the degree of confidence, the consumption equilibrium with the highest confidence level is at the point where the membership grade equals one.

When utility function is fuzzy, multiple consumption equilibriums emerge. The resulting demand relation becomes fuzzy in that a given price is associated with various quantities of a commodity. The quantity purchased depends on both the prevailing price and the membership grade; the consumption equilibrium set reduce to a singular point when all the utility indicators of a fuzzy utility function are comparable.

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