# MATHEMATICAL MODELLING OF TWO-LAYERED BLOOD FLOW IN AN INCLINED STENOSED ARTERY

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***Abstract:*** In this paper we have studied two layered blood flow in an inclined stenosed arteryunder the influence of body acceleration taking blood as Bingham-plastic fluid in core region and as Newtonian fluid at peripheral region. Governing equation are solved using perturbation technique and the velocity profile, plug radius, wall shear stress are displayed graphically.

***Index Terms*: Inclined artery, Bingham-plastic, Body acceleration.**

**I. INTRODUCTION** :Blood flow in the human circulatory system depends upon the pumping action of the heart, which in turn produces a pressure gradient throughout the system. But in many situations of day to day life, e.g. driving a vehicle, flying in an aircraft, etc., a human body may be subjected to vibratory or acceleratory motion. It may occur for a short or a long period of time, sometimes deliberately and sometimes unintentionally.

Majhi and Nair [1994] in their model considered blood as an incompressible third grade fluid and estimated the effects of the body acceleration on the velocity the flow rate and the wall shear stress numerically through an implicit finite difference scheme for femoral and coronary arteries. Masoudi and Phuoc [2008] studied pulsatile flow of blood using a modified second grade fluid. Biswas and Chakraborty [2010] investigated two-layered pulsatile blood flow in a stenosed artery under the influence of body acceleration and slip at wall assuming blood as Bingham plastic fluid in the core region and Newtonian fluid in the peripheral region of plasma.Nanda et.al.[2017] studied a mathematical model to study the effect of multiple stenosis on flow characteristics of streaming blood through atherosclerotic artery taking blood as Bingham plastic. Misra and Adhikary [2017] studied the flow of a Bingham plastic fluid in a porous bed under the action of an external magnetic field. Bhatnagar et.al.[2018] analyzed the effect of external body acceleration and slip velocity on the non-Newtonian pulsatile flow of blood through a constricted blood vessel taking blood as Bingham-Plastic fluid. Yadav and Kumar [2012] have investigated the effects of length of stenosis, shape parameter, parameter $\gamma$ on the resistance to flow for a Bingham plastic flow of blood through a generalized artery having multiple stenoses locate at equal distances. Maiti [2014] discussed mathematical model for blood flow through an axially symmetric but radially non- symmetric stenosed artery has been considered. The effect of non-Newtonian nature of blood has been taken into account by modelling blood as a Bingham plastic fluid. Mirza et.al. [2017] have studied non-Newtonian blood flow together with magnetic particles in a stenosed artery, using a magneto-hydrodynamic approach.

In this paper we have studied the pulsatile flow of blood through inclined stenosed artery by assuming the blood as a two fluid model with the suspension of all the erythrocytes in the core region as a Non-Newtonian fluid and the plasma in the peripheral layer as a Newtonian fluid under the influence of body acceleration and slip velocity at wall .The Non-Newtonian fluid in the core region is assumed as Bingham-plastic fluid model.

**II. FORMULATION OF THE PROBLEM**

In the present work the blood flow is taken as pulsatile, one dimensional, laminar,fully developed and blood is assumed as a two fluid model in the core region as Bingham plastic fluid and in the peripheral layer as a Newtonian fluid under the influence of body acceleration and slip velocity at wall. Artery is taken as inclined artery.

**STENOSIS GEOMETRY**

The stenotic protuberance is assumed to be an axisymmetric surface generated by a cosine curve Neeraja and K. Vidya [2012].

$$R(z)=\begin{cases} R_0-\delta_p\left(1+\cos\left(\dfrac{\pi z}{2z_0}\right)\right) & -2z_0\le z\le 2z_0 \\ R_0 & otherwise \end{cases} \qquad (2.1)$$

$$R_1(z)=\begin{cases}\beta R_0-\delta_c\left(1+\cos\left(\dfrac{\pi z}{2z_0}\right)\right) & -2z_0\le z\le 2z_0\\ \beta R_0 & otherwise\end{cases} \qquad (2.2)$$

where 4 $z_0$ is the length of the stenotic region, $2\delta_p$ is max. projections of the stenosis , $2\delta_c$ = max. projections of the stenosis in the peripheral and core region respectively, $R$ and $R_1$ is radius of the stenosed artery with the peripheral region and core region respectively, and $R_0$ is the radius in non-stenotic region.

## III. GOVERNING EQUATION

The governing equation in cylindrical polar coordinates can be taken by the Navier-Stoke's equation:

$$\rho_B\frac{\partial u_B}{\partial t}=-\frac{\partial p}{\partial z}-\frac{1}{r}\frac{\partial}{\partial r}(r\tau_B)+F(t)+\rho g\sin(al)\quad 0\le r\le R_1(z) \qquad (3.1)$$

$$\rho_N\frac{\partial u_N}{\partial t}=-\frac{\partial p}{\partial z}-\frac{1}{r}\frac{\partial}{\partial r}(r\tau_N)+F(t)+\rho g\sin(al)\quad R_1(z)\le r\le R(z) \qquad (3.2)$$

where $\rho_B$ and $\rho_N$ are densities of the fluid, $\tau_B$ and $\tau_N$ are the shear stress ,$u_B$ and $u_N$ are the velocities of the fluid in the core region and peripheral region respectively and $al$ is small angle of inclination. $F$(t) represents time periodic body acceleration term.

The constitutive equation for fluids in the motion in the core region (Bingham plastic fluid) and peripheral region (Newtonian fluid) are given by:

$$\frac{\partial u_B}{\partial r}=0\ if\ \tau_B\le\tau_y,\quad 0\le r\le R_p(z) \qquad (3.3)$$

$$\tau_B=-\mu_B\left(\frac{\partial u_B}{\partial r}\right)+\tau_y,\qquad \tau_B\ge\tau_y,\qquad R_p(z)\le r\le R_1(z) \qquad (3.4)$$

$$\tau_N=-\mu_N\left(\frac{\partial u_N}{\partial r}\right),\quad R_1(z)\le r\le R(z) \qquad (3.5)$$

where $\mu_B$ and $\mu_N$ are viscosities of Bingham-plastic fluid and Newtonian fluid respectively , $\tau_y$ is the yield stress, $R_p$ is the plug radius in the core region.

## BOUNDARY CONDITIONS

$\tau_B$ is finite and $\dfrac{\partial u_B}{\partial r}$ = 0 at r = 0 (3.6)

$$u_N=u_s\ \ at\ r=R(z) \qquad (3.7)$$

$$\tau_B=\tau_N\ ,\ u_B=u_N\ \ at\ \ r=R_1(z) \qquad (3.8)$$

## IV. RESULT AND DISCUSSIONS:

Applying perturbation technique in this paper we get the expression for velocity profile , plug radius ,wall shear stress and displayed graphically.

Figure 1 to 6 shows axial velocity distribution through the mild stenosed inclined artery taking blood as Bingham-plastic fluid in core region w.r.t. r for different values of body acceleration parameter $A$, pressure gradient parameter $e$, slip velocity $u_s$, stenosis height $\delta$, axial distance z and yield stress $\theta$ respectively. Figure 1 illustrates velocity u w.r.t. radial distance r for the different values of body acceleration parameter $A$. It shows that velocity increases with increasing value of $A$. Figure 2 demonstrates velocity distribution w.r.t. radial distance r for the different values of pressure gradient parameter $e$. It depicts that the velocity increases with the increasing value of $e$. It is noticed in computation that plug radius decreases with increasing values of $e$.Figure 3 depicts that the velocity increases with increasing value of slip velocity $u_s$. Figure 4 represents velocity distribution w.r.t. r at different stenosis heights $\delta$ ,where $\delta=\delta_p=\delta_c$ It depicts that velocity decreases with increasing $\delta$ and there is significant changes in plug velocity. Figure 5 shows velocity distribution w.r.t r at different positions in stenotic region. It reveals that lowest velocity occurs at the throat of the stenosis, then velocity increases at the different position on upstream of the stenosis and highest velocity occurs at the end of the constriction profile.Figure 6 demonstrates velocity distribution w.r.t. r for different values of yield stress $\theta$.It depicts that velocity decreases with increasing values of $\theta$.It is noticed in computation that peripheral layer's velocity does not get affected by values of $\theta$ .It is also noticed that plug radius increases with increasing value of yield stress. Figure 7 shows that the plug velocity increases with increasing value of *A*, which is in conformity with Fig 1. Figure 8 reveals that plug velocity increases with increasing value of *al* (inclination angle) and again it is noticed that effect of *al* on plug velocity at starting and end of the constriction profile is more than at the throat of the stenosis.

## V. CONCLUSION:

Main object of the present study is to find out the velocity profile in different cases. This study is useful to determine low velocity region and shows the behavior of blood flow in different conditions which is very useful to medical practitioner and research

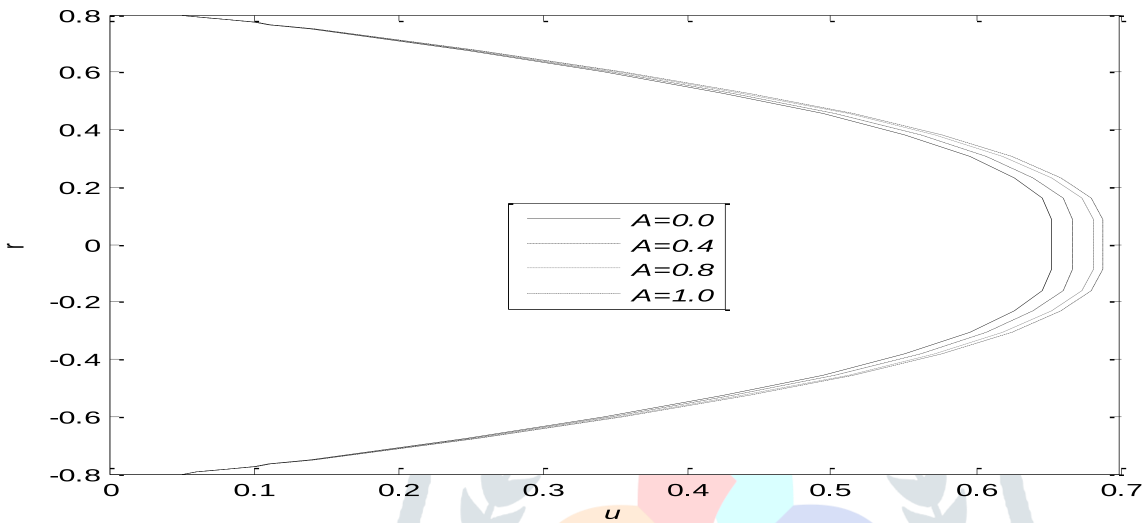


**Figure 1 : Variation of axial velocity profile w.r.t. radial distance r for different values of body acceleration parameter *A***

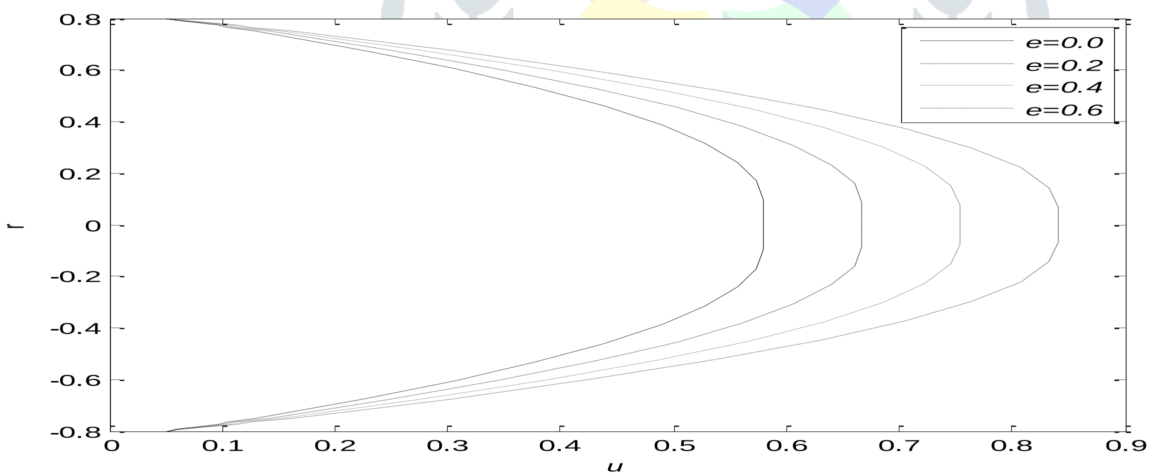


**Figure 2 : Variation of axial velocity profile w.r.t. radial distance r for different values of pressure gradient parameter *e***

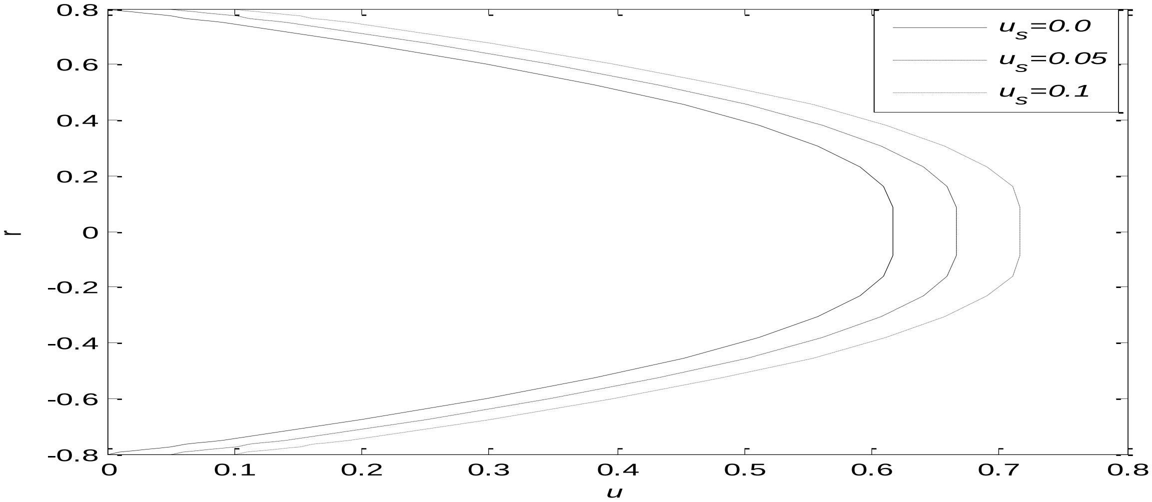


**Figure 3 : Variation of axial velocity profile w.r.t. radial distance r for different values of slip velocity $u_s$**

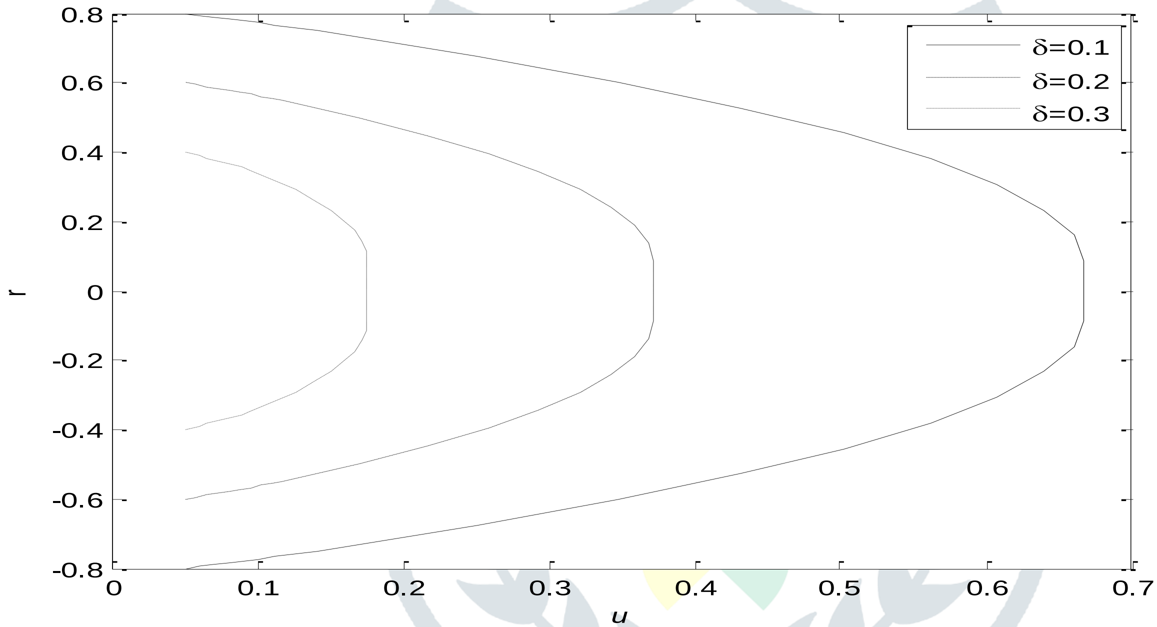


**Figure 4 : Variation of axial velocity profile w.r.t. radial distance r for different stenosis height $\delta$**

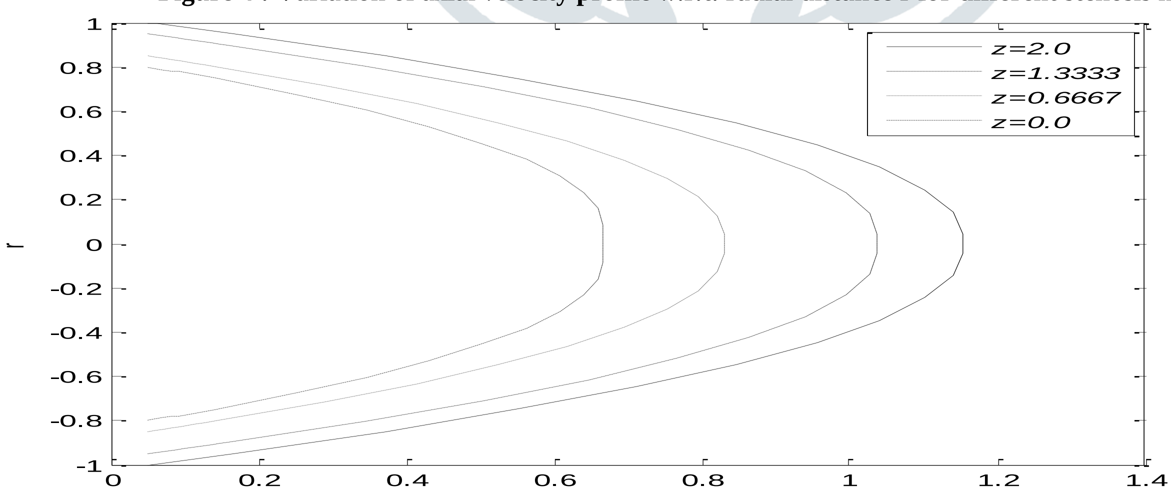


**Figure 5 : Variation of axial velocity profile w.r.t. radial distance r for different values of axial distance z at stenotic region**

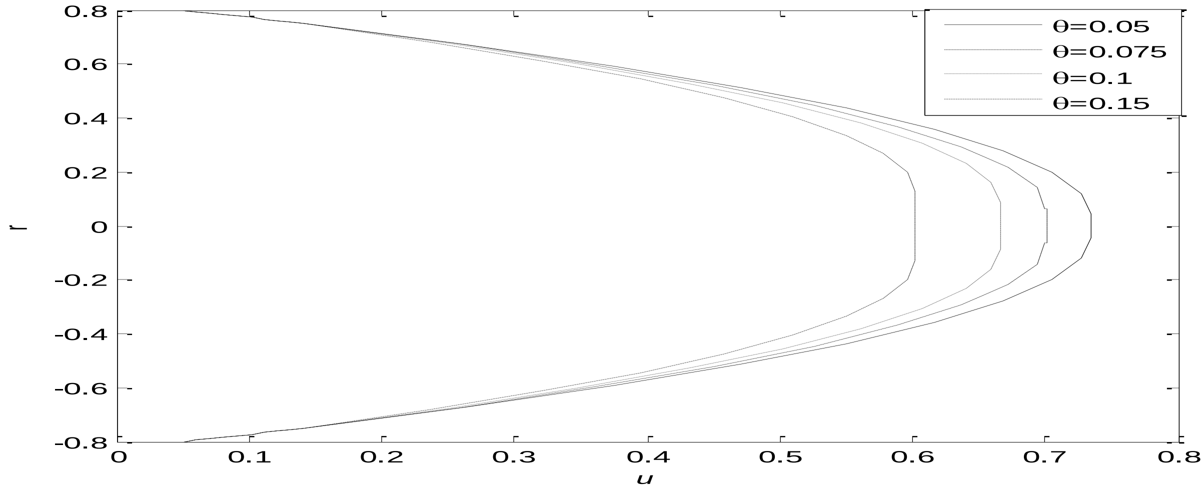


**Figure 6 : Variation of axial velocity profile w.r.t. radial distance r for different values of yield stress $\theta$**

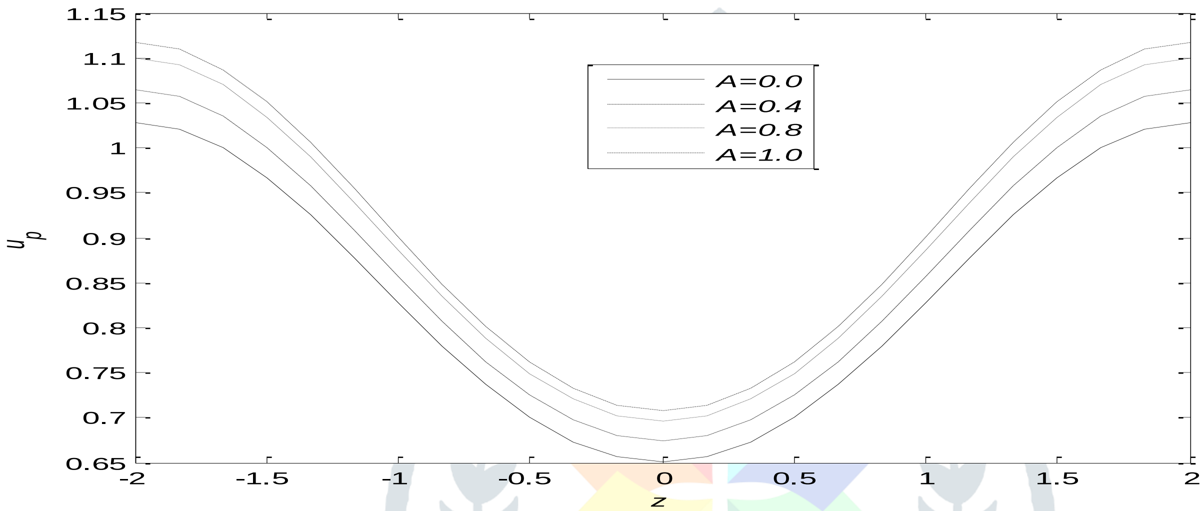


**Figure 7: Variation of plug velocity profile w.r.t. axial distance z for different values of body acceleration parameter *A***

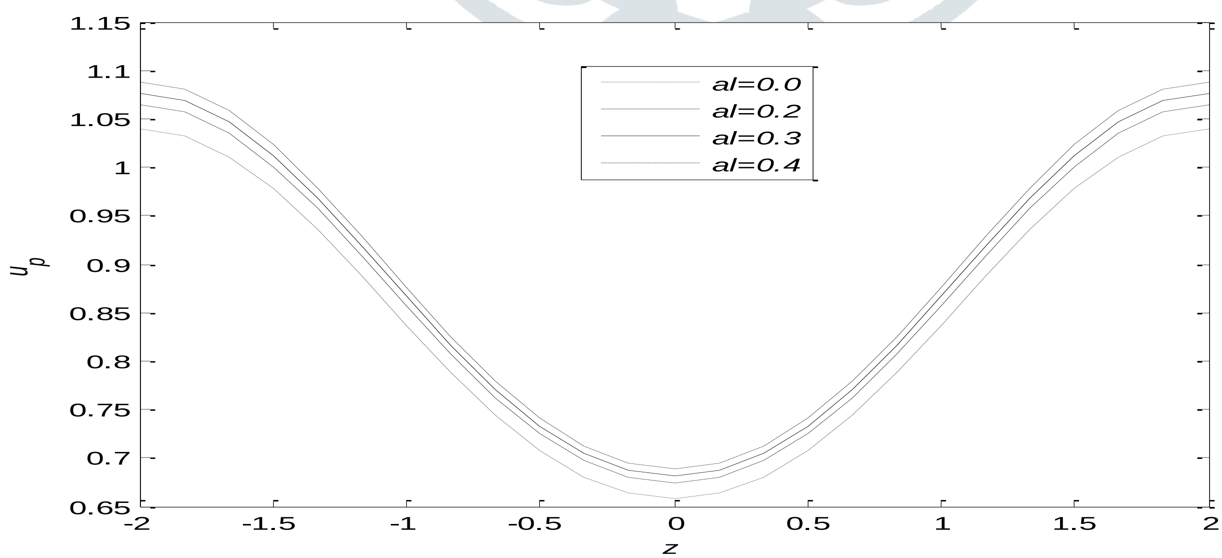


**Figure 8 : Variation of plug velocity profile w.r.t. axial distance z for different values of inclination angle *al***

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