Left$(\alpha, 1)$ Derivations on Semirings

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Abstract : *The idea of the definition Left derivation taken from the paper [Dr D Bharathi et all,] and in this paper we introduce two sided Left α derivations and Left $(\alpha,1)$ derivation on a semiring with examples. In this paper we proved for s be an additively cancellative and commutative semiring and let I be an ideal of s which contains zero. Let d be a two sided left α derivation on s such that $\alpha(I) = I$ and if d acts as a homomorphism on I then $d(I) = 0$.*

IndexTerms - Derivations, Semi ring, Prime ring, Characteristic of the ring, α derivation and (α,1) derivation.

I. INTRODUCTION

DEFINITION:

Let α be an endomorphism on S. An additive map $d : S \to X$ is called a

1. $(\alpha, 1)$ derivation if $d(xy) = \alpha(x)d(y) + d(x)y$
2. $(1, \alpha)$ derivation if $d(xy) = xd(y) + d(x)\alpha(y)$

DEFINITION:

An additive map $d : S \to X$ is called a two sided α derivation if d is an $(\alpha, 1)$ derivation as well as $(1, \alpha)$ derivation.

DEFINITION:

Let α be an endomorphism on S. An additive map $d : S \to X$ is called a

1. $(\alpha, 1)$ left derivation if $d(xy) = \alpha(x)d(y) + yd(x)\ \forall x, y \in S$.
2. $(1, \alpha)$ left derivation if $d(xy) = yd(x) + \alpha(x)d(y)\forall x, y \in S$.

DEFINITION:

An additive map $d : S \to X$ is called a two sided α Left derivation if d is an $(\alpha, 1)$ Left derivation as well as $(1, \alpha)$ Left derivation.

Example 1: Let S be a commutative semiring.

Let $M_2(S) = \left\{\begin{pmatrix} a & 0 \\ b & c \end{pmatrix} / a, b, c \in S\right\}$

Define $d: M_2(S) \to M_2(S)$ *and* $\alpha(S): M_2(S) \to M_2(S)$ *by*

$$d\left[\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}\right] = \begin{pmatrix} 0 & 0 \\ b & 0 \end{pmatrix}$$

and

$$\alpha\left[\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}\right] = \begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix}$$

respectively.

Then d is called two sided $(1, \alpha)$ left derivation.

Example 2 : Let $\alpha(S): M_2(S) \to M_2(S)$ *defined by*

$$\alpha\left[\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}\right] = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$$

Then d is an $(\alpha, 1)$ left derivation but not a $(1, \alpha)$ left derivation.

Example 3: Let $\alpha(S): M_2(S) \to M_2(S)$ *defined by*

$$\alpha\left[\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}\right] = \begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix}$$

Then d is an $(1, \alpha)$ left derivation but not a $(\alpha, 1)$ left derivation.

2.Main Results:

Lemma 1: Let S be a prime semiring and I be a nonzero ideal of S. Led d be a nonzero $(\alpha, 1)$ left derivation on S. If $d(x + y - x - y) = 0\ \forall x, y \in I$, then $\alpha(x + y - x - y)d(z) = 0\ \forall x, y \in I$.

Proof: Assume that $d(x + y - x - y) = 0\ \forall x, y \in I$.

Let $x = xz$ *and* $y = yz$,

we have

$$\Rightarrow d(xz + yz - xz - yz) = 0$$

$\Rightarrow d\big((x + y - x - y)z\big) = 0$

$\Rightarrow \alpha(x + y - x - y)d(z) + z\, d(x + y - x - y) = 0$

$$\Rightarrow \alpha(x + y - x - y)d(z) = 0.$$

Lemma 2: Let S be a prime semiring and I be a nonzero ideal of S. Let αbe a non zero $(\alpha, 1)$ left derivation on$S$. If $x \in S$ and $xd(I) = 0$ then $x = 0$.

Proof: since $xd(I) = 0$, we have

$xd(ua) = 0\ \forall a \in I, u \in S$

$x\big(\alpha(u)d(a) + ad(u)\big) = 0$

$x\alpha(u)d(a) + xad(u) = 0\ \forall a \in I, u \in S$

$xad(u) = 0$

Replacing u by uv, we have

$xad(uv) = 0$
$xa(\alpha(u)d(v) + vd(u)) = 0$
$xa\alpha(u)d(v) + xavd(u) = 0$
$xaSd(u) = 0$
$xISd(u) = 0$

Since S is prime, $d(u) = 0 \; or \; xI = 0$.
Since $d \neq 0, xI = 0$.
Since $I \neq 0, x = 0$.

Theorem 1: Let S be an additively cancellative semiring and I a multiplicatively subsemigroup of S. Let d be an $(\alpha, 1)$ Left derivation of S and $\alpha(I) = I$.

1. If d acts acts a homomorphism on I then
$xd(y)d(y) = xyd(y) = x\alpha(y)d(y)$

$\forall x, y \in I$

2. If d acts acts a antihomomorphism on I then
$d(x)yd(x) = xyd(x) = y\alpha(x)d(x)$

$\forall x, y \in I$

Proof: (i) Since d is a $(\alpha, 1)$ Left derivation of S and it is a homomorphism we have

$$d(yx) = \alpha(y)d(x) + xd(y) \quad (1)$$

Substitute x=xy in (1) we have

$d(yxy) = \alpha(y)d(xy) + xyd(y)$
$= \alpha(y)d(x)d(y) + xyd(y)$ (2)

also

$d(yxy) = d(yx)d(y)$
$= [\alpha(y)d(x) + xd(y)]d(y)$
$= \alpha(y)d(x)d(y) + xd(y)d(y)$ (3)

From (2) and (3) we have

$xd(y)d(y) = xyd(y)$

Substitute y=xy in (1) we have

$d(xyx) = \alpha(xy)d(x) + xd(yx)$
$= \alpha(x)\alpha(y)d(x) + xd(yx)$ (4)

But

$d(xyx) = d(x)d(yx)$
$= d(x)[\alpha(y)d(x) + xd(y)]$
$= d(x)\alpha(y)d(x) + d(x)xd(y)$
$= d(x)\alpha(y)d(x) + xd(y)d(x)$
$= d(x)\alpha(y)d(x) + xd(yx)$ (5)

From (4) and (5) we have

$$\alpha(x)\alpha(y)d(x) = d(x)\alpha(y)d(x)$$

Replace x by y and y by x we have

$$\alpha(y)\alpha(x)d(y) = d(y)\alpha(x)d(y)$$

Since $\alpha(I) = I, we \; have$

$$x\alpha(y)d(y) = xd(y)d(y)$$

therefore

$$xd(y)d(y) = xyd(y) = x\alpha(y)d(y) \quad \forall x, y \in I$$

(ii) Since d is a $(\alpha, 1)$ Left derivation of S and it is a homomorphism we have

$$d(xy) = \alpha(x)d(y) + yd(x) \quad (6)$$

Substitute y=xy we have

$d(xxy) = \alpha(x)d(xy) + xyd(x)$
$= \alpha(x)d(x)d(y) + xyd(x)$ (7)

But

$d(xxy) = d(x)d(xy)$
$= d(x)[\alpha(x)d(y) + yd(x)]$
$= d(x)\alpha(x)d(y) + d(x)yd(x)$

$$d(xxy) = \alpha(x)d(x)d(y) + d(x)yd(x) \quad (8)$$

From (7) and (8) we have

$xyd(x) = d(x)yd(x)$

Substitute $x = xy$ in (6), we have

$d(xyy) = \alpha(xy)d(y) + yd(xy)$
$= \alpha(x)\alpha(y)d(y) + yd(xy)$ (9)

But

$d(xyy) = d(xy)d(y)$
$= [\alpha(x)d(y) + yd(x)]d(y)$
$= \alpha(x)d(y)d(y) + yd(x)d(y)$
$= \alpha(x)d(y)d(y) + yd(xy)$ (10)

From (9) and (10) we have

$$\alpha(x)\alpha(y)d(y) = \alpha(x)d(y)d(y)$$

From $\alpha(I) = I$ we have

$x\alpha(y)d(y) = d(y)xd(y)$

Replace y by x and x by y we have

$$y\alpha(x)d(x) = d(x)yd(x)$$

Therefore

$$d(x)yd(x) = xyd(x) = y\alpha(x)d(x) \quad \forall x, y \in I$$

Theorem 2: Let s be an additively cancellative and commutative semiring and let I be an ideal of s which contains zero. Let d be a two sided left α derivation on s such that $\alpha(I) = I$ and if d acts as a homomorphism on I then $d(I) = 0$.

Proof: By the theorem 1,

$$xd(y)d(y) = x\alpha(y)d(y)$$

Replace x by y and y by x we have

$$d(x)yd(x) = \alpha(x)yd(x)$$

Multiply with $d(z)$ we have

$$d(z)d(x)yd(x) = d(z)\alpha(x)yd(x)$$

$$\Rightarrow d(zx)yd(x) = d(z)\alpha(x)yd(x) \quad (11)$$

Since d is $(\alpha, 1)$ left derivation, then

$$[\alpha(z)d(x) + xd(z)]yd(x) = d(z)\alpha(x)yd(x)$$

Which gives

$\alpha(z)d(x)yd(x) = 0$

Since $\alpha(I) = I$, therefore

$$zd(x)yd(x) = 0 \quad (12)$$

Taking ny instead of y in the above equation, we have

$$zd(x)nyd(x) = 0 \quad \forall x, y, z \in I, n \in S.$$

$$zd(x)Syd(x) = 0$$

By primeness,

$$yd(x) = 0 \text{ and } \alpha(I) = I$$

We have

$$\alpha(y)d(x) = 0 \; \forall x, y \in I. \quad (13)$$

Substitute xn for x and multiply in the right hand side by $d(y)$

$\alpha(y)d(xn)d(y) = 0$

$\Rightarrow \alpha(y)[\alpha(n)d(x) + nd(x)]d(y) = 0$

$\Rightarrow [\alpha(y)\alpha(n)d(x) + \alpha(y)nd(x)]d(y) = 0$

$\Rightarrow \alpha(y)\alpha(n)d(x)d(y) + \alpha(y)nd(x)d(y) = 0 \quad (14)$

Which implies

$\alpha(y)nd(x)d(y) = 0$

$\Rightarrow \alpha(y)nd(xy) = 0$

$\Rightarrow \alpha(y)n[\alpha(x)d(y) + yd(x)] = 0$

$\Rightarrow \alpha(y)n\alpha(x)d(y) + \alpha(y)nyd(x) = 0$

$\Rightarrow \alpha(y)n\alpha(x)d(y) = 0$

$\Rightarrow \alpha(y)Sxd(y) = 0 \quad (15)$

By prime ness

$xd(y) = 0 \quad (16)$

From (13) and (16), we have

$\alpha(y)d(x) + xd(y) = 0$

$\Rightarrow d(yx) = 0$

Now replace y by ny, we have

$d(nyx) = 0$

$\Rightarrow d(ny)d(x) = 0$

$\Rightarrow [a(n)d(y) + yd(n)]d(x) = 0$

$\Rightarrow a(n)d(y)d(x) + yd(n)d(x) = 0$

$\Rightarrow a(n)d(yx) + yd(n)d(x) = 0$

$\Rightarrow yd(n)d(x) = 0 \Rightarrow d(x) = 0 \quad \forall x \in I.$

Corollary 1: Let S be a semiprime ring and I be a semigroup ideal of S containing zero. Let d be a two sided left $(\alpha, 1)$ derivative on S such that $\alpha(I) = I$ and if d acts as homomorphism on I then $d = 0$.

Proof : From theorem 2, we have

$$d(x) = 0 \quad \forall x \in I.$$

Replace x by nx

$d(nx) = 0$

$\Rightarrow \alpha(n)d(x) + xd(n) = 0$

$\Rightarrow xd(n) = 0$

Again replace x by xm

$$xmd(n) = 0, \; \forall m \in S \text{ and } x \in I.$$

$\Rightarrow xSd(n) = 0$

$\Rightarrow ISd(n) = 0, \forall n \in S$

By primeness

$I = 0 \text{ or } d(n) = 0$

Since $I \neq 0$, therefore

$d = 0.$

II. ACKNOWLEDGMENT

Author express his sincere thanks to Prof. D. Bharathi, Prof in Mathematics, Srivenkateswara University, Tirupati, A.P., India, her valuable guidance to complete this paper and also express sincere thanks on reviewer's their valuable suggestions.

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