Reliability Analysis of Two Non-Identical Components System with Common Cause Failure

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Abstract

In this paper we investigated reliability analysis of two non- identical components system with common cause failure (CCF). The component may fail simultaneously due to any common cause. The lifetimes as well as repair time of the components are assumed to be exponentially distributed. The Markov model is used for the number of failed component in the system. A set of differential difference equations has been constructed in terms of state dependent failure rates and repair rates. By taking the Laplace transform of the equation, the matrix method is used to obtain the probabilities. The sensitivity analysis is providing to explore the effect of different parameters on system reliability.

Keywords: System reliability, Two components, Common cause failure, MTTF.

1. Introduction:

The reliability is the most important factor of the system or equipment associated with the dependent and use of the components. To understand the behavior of redundant repairable system, the reliability models are needed which describe how the components are failed and how the components are repaired. The repair ability is the probability that a failure of the system can be repaired under the specified conditions. The most important performance measure for repairable system is system reliability. The value depends on the system as well as the components reliability which decrease as the components age increase. The importance of allowing repair of failed components in the system should be obvious when considering systems with repairable components because if repair is possible for a failed component without affecting the overall operable system then it is desirable to the chance are returning this component to either operation or an operating state before it lack of operation common cause failure of the system. The systems in which repair could be include may simple series and parallel system. Series system with repair offers no increase in reliability since as soon as the component fails the system fails.

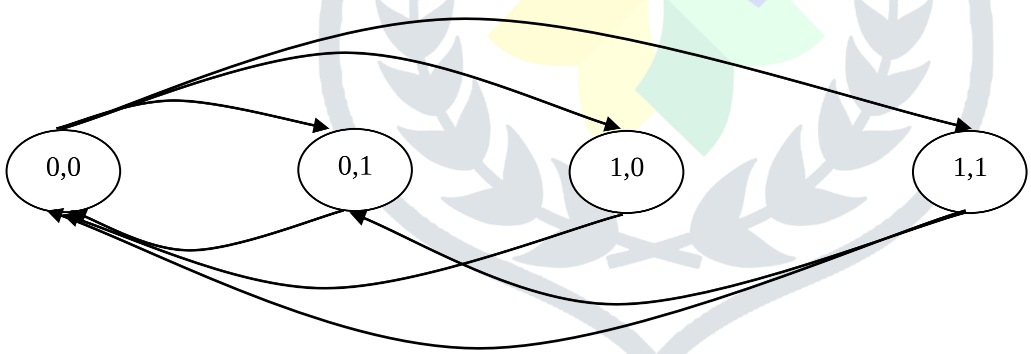
Several researchers have developed the reliability models in different frameworks under common cause failures. Atwood (1986) analysed studied a reliability model with binomial failure rate and common cause failure. Hughes (1987) considered a new approach to examine the effect of common cause failure on reliability indices. A Markovian failure approach to investigate the system reliability and availability measure in presence of common cause failure was used by Chari (1988) and Chari et al. (1991). Verma and Chari (1991, 1994) obtained the availability and frequency of failures of non-identical two component system in the presence of change common cause failures. Chari and Shastri (1994) studied the system reliability analysis in the presence of lethal and non-lethal common cause shocks failures. Amari et al. (1999) discussed the optimal reliability of systems subject to imperfect fault-coverage.

Coit (2001) considered the cold-standby redundancy optimization for non-repairable systems. Kvam and Miller (2002) considered the common cause failure prediction using data mapping. The Bayesian study of a two component non-identical system with common cause failures was introduced by Yadavalli et al. (2005). Makespan minimization for two parallel machines with availability constraint and common cause failures was studied by Liao et al. (2005). Dhillon and Shah (2007) analyzed availability of a generalized maintainable three-state device parallel system with human error and common-cause failures. Guo and Yang (2007) analyzed a simple reliability block diagram method for safety integrity verification. Guo and Yang (2008) discussed the automatic creation of Markov models for reliability assessment of safety systems. Chaturvedi, Pati & Tomer (2014) discussed Robust Bayesian analysis of Weibull failure model. Dey, Alzaatreh, Zhang & Kumar (2017) analyzed a new extension of generalized exponential distribution with application to Ozone data. Han, Park, & Thoma (2018) discussed Why do we need to employ Bayesian statistics and how can we employ it in studies of moral education : with practical guidelines to use JASP for educators and researchers.

The purpose of this paper is to develop a comprehensive approach to compute the reliability and performance indices for non-identical two units system under individual LCCS (Lethal Common Cause Failure) and NCCS (Non-Lethal Common Cause Failure) failures. A discrete state, Markov chain is used to constitute a set of differential difference equations for transient probabilities governing the model. With the aid of an inverse Laplace transform, we derive a complete analytic solution of the simultaneous differential equation. For series and parallel configuration, reliability function and MTTF are determined. The series and parallel both configurations of two non-identical components system in which the influence of common cause failure of the system with the frequency of operating and failed state is considered.

2. The Model:

In this section, we consider the two non-identical components system with common cause failure. The only possible configurations of such systems are series and parallel. The repair facility provided will help the series system in the sense that it may increase the reliability of the system due to the failure of individual component. Let us consider the repairable system consists of two non-identical components each with failure rates λ_i, repair rates μ_i (i = 0, 1, 2, 3) and common cause failure rate λ_c. Each component has two mutually exclusive states an operable and failed state. The states of the model are generated based on the components being in one of these two states. It is assumed that both components are originally operating if any component has failed then it is immediately sent for repair to repair facility; if both components failed then we say that the system has failed.



State transition diagram of two unit system

The system of two non- identical components must be divided in one of the four following states:

i. State (0, 0): both components are in operating state.

ii. State (0, 1): Second component has failed and first component is operating state.

iii. State (1, 0): First component has failed and second component is operating state.

iv. State (1, 1): Both components have failed.

It is assumed that the failure rates and repair rates of two non-identical components of different states are Poisson distributed under common cause failure. The intensities of the transition between these states are negligible. The commutation mechanism is perfect.

3. Notations:

The following notations have been used as follows:

λ_i : Failure rate (i = 0, 1, 2, 3)

μ_i : Repair rate (i = 0, 1, 2, 3)

λ_c : Common cause failure rate

μ_c : Common cause repair rate

$\tilde{L}(s)$: Laplace parameters

$P_o(t)$: Probability of operating components at time t.

$P_i(t)$: Probability of i^{th} (i = 1, 2) components is in failed state at time t while the one is in operating state.

$P_3(t)$: Probability of both components has failed due to CCF at time t.

$R_s(t)$: System reliability of the series configuration.

$R_p(t)$: System reliability of the parallel configuration.

4. Mathematical Equations and Analysis:

The differential difference equations for the system state probabilities are constructing as follows:

$$\frac{d}{dt}P_{0,0}(t) = -(\lambda_0 + \lambda_1 + \lambda_c)P_0(t) + \mu_0 P_1(t) + \mu_1 P_2(t) + \mu_c P_3(t) \quad \ldots(4.1)$$

$$\frac{d}{dt}P_{0,1}(t) = \lambda_0 P_0(t) - \mu_0 P_1(t) + \mu_3 P_3(t) \quad \ldots(4.2)$$

$$\frac{d}{dt}P_{1,0}(t) = \lambda_1 P_0(t) - (\lambda_2 + \mu_1)P_2(t) + \mu_2 P_3(t) \quad \ldots(4.3)$$

$$\frac{d}{dt}P_{1,1}(t) = \lambda_c P_0(t) + \lambda_2 P_2(t) - (\mu_2 + \mu_3 + \mu_c)P_3(t) \quad \ldots(4.4)$$

Using Laplace transformation, the set of equations (4.1) - (4.4) with initial conditions, $P_0(0) = 1$, $P_i(0) = 0$, $i \neq 0$ can be solved. The Laplace transforms of (4.1) - (4.4) yield

$$(s + \lambda_0 + \lambda_1 + \lambda_c)\tilde{L}_0(s) - \mu_0 \tilde{L}_1(s) - \mu_2 \tilde{L}_2(s) - \mu_c \tilde{L}_3(s) = 1 \quad \ldots(4.5)$$

$$-\lambda_0 \tilde{L}_0(s) + (s + \mu_0)\tilde{L}_1(s) - \mu_3 \tilde{L}_3(s) = 0 \quad \ldots(4.6)$$

$$-\lambda_1 \tilde{L}_0(s) + (s + \mu_1 + \lambda_2)\tilde{L}_2(s) - \mu_2 \tilde{L}_3(s) = 0 \quad \ldots(4.7)$$

$$-\lambda_c \tilde{L}_0(s) - \lambda_2 \tilde{L}_2(s) + (s + \mu_2 + \mu_3 + \mu_c)\tilde{L}_3(s) = 0 \quad \ldots(4.8)$$

The equations (4.5) - (4.8) written in matrix form as:

$$A(s)\, P(s) = \mathrm{I} \quad \ldots(4.9)$$

where

$$A(s) = \begin{bmatrix} (s + \lambda_0 + \lambda_1 + \lambda_c) & -\mu_0 & -\mu_1 & -\mu_c \\ -\lambda_0 & (s + \mu_0) & 0 & -\mu_3 \\ -\lambda_1 & 0 & (s + \mu_1) & -\mu_2 \\ -\lambda_c & 0 & -\lambda_2 & (s + \mu_2 + \mu_3 + \mu_c) \end{bmatrix}$$

$$\tilde{\boldsymbol{P}}(s)=[\tilde{P}_0(s),\tilde{P}_1(s),\tilde{P}_2(s),\tilde{P}_3(s)] \quad \ldots(4.10)$$

$$\boldsymbol{I} = [1, 0, 0, 0]^T \quad \ldots(4.11)$$

Eq. (4.9) has a unique solution. By using Cramer's rule, we obtain

$$\tilde{P}_{n-1}(s)=\frac{\det A_n(s)}{\det A(s)},\ \ n=1,\ 2,\ 3,\ 4 \quad \ldots(4.12)$$

Where $A_n(s)$ is the matrix obtained from $\boldsymbol{A}(s)$ by replacing the n^{th} (n = 1, 2, 3, 4) column of $A(s)$ by the vector I. By inspection of the matrix $\boldsymbol{A}(s)$, it is not difficult to see that determinant of the matrix $\boldsymbol{A}(s)$ must be a polynomial in s of degree 4 with leading coefficient 1, and so we can write

$$det\ \boldsymbol{A}(s) = s\prod_{i=1}^{3}(s-s_i) \quad \ldots(4.13)$$

Where each s_i is a root of the polynomial. It can be easily verified that these roots must be distinct. By algebraic manipulations, we obtain

$$s_1=(\mu_0+\mu_1+\mu_2+\mu_3+\mu_c+\lambda_0+\lambda_1+\lambda_2+\lambda_c) \quad \ldots(4.14)$$

$$s_2=\lambda_0(\mu_1+\mu_2+\mu_3+\mu_c)+\lambda_1(\mu_0+\mu_2+\mu_3+\mu_c)$$
$$+\lambda_2(\mu_0+\mu_1+\mu_3+\mu_c)+\lambda_c(\mu_0+\mu_1+\mu_2+\mu_3)$$
$$+\mu_0(\mu_1+\mu_2+\mu_3+\mu_c)+\mu_1(\mu_2+\mu_3+\mu_c) \quad \ldots(4.15)$$

$$s_3=\lambda_0(\mu_2\mu_1+\mu_3\mu_1+\mu_1\mu_c)+\lambda_1(\mu_1\mu_2+\mu_1\mu_3+\mu_1\mu_c)$$
$$+\lambda_2(\mu_0\mu_1+\mu_0\mu_3+\mu_1\mu_c)+\lambda_c(\mu_0\mu_1+\mu_0\mu_2+\mu_1\mu_3)$$
$$+\mu_0(\mu_2\mu_1+\mu_1\mu_3+\mu_1\mu_c) \quad \ldots(4.16)$$

Now the state probabilities of the system are as given:

$$P_0(t)=[\{(s_1^3+m_0s_1^2+n_0s_1+r_0)/s_1(s_1-s_2)(s_1-s_3)\}]\exp(s_1t)$$

$$+[\{(s_2^3+m_0s_2^2+n_0s_2+r_0)/s_2(s_2-s_1)(s_2-s_3)\}]\exp(s_2t)$$

$$+[\{(s_3^3+m_0s_3^2+n_0s_3+r_0)/s_3(s_3-s_2)(s_3-s_1)\}]\exp(s_3t)-r_0/s_1s_2s_3 \quad \ldots(4.17)$$

$$P_1(t)=[\{(m_1s_1^2+n_1s_1+r_1)/s_1(s_1-s_2)(s_1-s_3)\}]\exp(s_1t)$$

$$+[\{(m_1s_2^2+n_1s_2+r_1)/s_2(s_2-s_1)(s_2-s_3)\}]\exp(s_2t)$$

$$+[\{(m_1s_3^2+n_1s_3+r_1)/s_3(s_3-s_2)(s_3-s_1)\}]\exp(s_3t)-r_1/s_1s_2s_3 \quad \ldots(4.18)$$

$$P_2(t)=[\{(m_2s_1^2+n_2s_1+r_2)/s_1(s_1-s_2)(s_1-s_3)\}]\exp(s_1t)$$

$$+[\{(m_2s_2^2+n_2s_2+r_2)/s_2(s_2-s_1)(s_2-s_3)\}]\exp(s_2t)$$

$$+[\{(m_2s_3^2+n_2s_3+r_2)/s_3(s_3-s_2)(s_3-s_1)\}]\exp(s_3t)-r_2/s_1s_2s_3 \quad \ldots(4.19)$$

$$P_3(t)=[\{(m_3s_1^2+n_3s_1+r_3)/s_1(s_1-s_2)(s_1-s_3)\}]\exp(s_1t)$$

$$+[\{(m_3s_2^2+n_3s_2+r_3)/s_2(s_2-s_1)(s_2-s_3)\}]\exp(s_2t)$$

$$+[\{(m_3s_3^2+n_3s_3+r_3)/s_3(s_3-s_2)(s_3-s_1)\}]\exp(s_3t)-r_3/s_1s_2s_3 \quad \ldots(4.20)$$

where

$$m_0 = (\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_c + \lambda_2);$$

$$m_1 = (\lambda_0); \quad m_2 = (\lambda_1); \quad m_3 = (\lambda_c)$$

$$n_0 = \mu_0 (\mu_1 + \mu_2 + \mu_3 + \mu_c + \lambda_2) + \mu_1(\mu_2 + \mu_3 + \mu_c) + \lambda_2(\mu_3 + \mu_c)$$

$$n_1 = \mu_1 + \mu_2 + \mu_3 + \mu_c + \lambda_2 + \lambda_c$$

$$n_2 = \lambda_1 (\mu_0 + \mu_2 + \mu_3 + \mu_c)$$

$$n_3 = \lambda_c(\mu_0 + \mu_1 + \lambda_2) + \lambda_1\lambda_2$$

$$r_0 = \mu_0\mu_1(\mu_2 + \mu_3 + \mu_c) + \mu_0\lambda_2(\mu_3 + \mu c)$$

$$r_1 = \mu_1(\mu 2 + \mu 3 + \mu_c + \lambda 2 + \lambda_c) + \mu_3(\lambda 1\lambda 2 + \lambda c\mu 1 + \lambda 2\lambda c) + \lambda_2(\mu 3 + \mu_c)$$

$$r_2 = [\mu_0\lambda_1(\mu 2 + \mu 3 + \mu_c) + \mu 2\lambda_c]$$

$$r_3 = \mu_0\lambda_c(\mu_1 + \lambda_c)$$

5. The Reliability Function:

The reliability functions $R_s(t)$ and $R_P(t)$ for both series and parallel configurations are obtained as follows:

System Reliability of Series Configuration:

In this case, state 1 itself is an absorbing state and therefore, no transition takes place from the neighboring states. So the reliability is given by

$$R_s(t) = P_0(t)$$

$$=[\{(s_1^3+m_0 s_1^2+n_0 s_1+r_0)/s_1(s_1-s_2)(s_1-s_3)\}]\exp(s_1 t)$$

$$+[\{(s_2^3+m_0 s_2^2+n_0 s_2+r_0)/s_2(s_2-s_1)(s_2-s_3)\}]\exp(s_2 t)$$

$$+[\{(s_3^3+m_0 s_3^2+n_0 s_3+r_0)/s_3(s_3-s_2)(s_3-s_1)\}]\exp(s_3 t)-r_0/s_1 s_2 s_3$$

…(5.1)

System Reliability of Parallel Configuration

The reliability of the parallel system can be obtained as:

$$R_P(t)=P_0(t)+P_1(t)+P_2(t)$$

$$=[\{(s_1^3+ms_1^2+ns_1+r)/s_1(s_1-s_2)(s_1-s_3)\}]\exp(s_1 t)$$

$$+[\{(s_2^3+ms_2^2+ns_2+r)/s_2(s_2-s_1)(s_2-s_3)\}]\exp(s_2 t)$$

$$+[\{(s_3^3+ms_3^2+ns_3+r)/s_3(s_3-s_2)(s_3-s_1)\}]\exp(s_3 t)-r/s_1 s_2 s_3$$

…(5.2)

where $m = m_0 + m_1 + m_2$; $n = n_0 + n_1 + n_2$; $r = r_0 + r_1 + r_2$.

Mean Time To Failure (MTTF):

The mean time to failure of the system is as follows:

$$MTTF=\int_0^\infty R_p(t)\,dt \qquad \ldots(5.3)$$

6. Numerical Results:

The numerical results for system performance indices are calculated with the help of software MATLAB. Table – (8.1) shows the probabilities for system states at different times by taking fixed value of various parameters. The graphical presentations for reliability (series and parallel) by varying different parameters are done in figures (9.1) – (9.6). Figures – (9.1) & (9.2) depict the variation of reliability (series and parallel) for value of λ_0. In these figures, it is noted that initially reliability decreases gradually up to time t = 4, and then after it becomes almost constant. By increasing failure rates λ_0 the reliability decreases as we expect.

In figures – (9.3) & (9.4), we examine the effect of different repair rate μ_0 respectively on the reliability for series and parallel system configuration. We find that on increasing the values of μ_0, the system reliability increases remarkably up to time t = 2, the effect is not much significance as t grows. By this observation, we conclude that by increasing repair rates, the reliability can be improved to a certain extent; also as time increases, the reliability tends to a constant value.

Figure – (9.5) & (9.6), illustrated the effect of λ_c on system reliability and it is noted that reliability decreases as λ_c increases.

In figure – (9.7) & (9.8), we have seen effect of λ_c on mean time to failure with the variation of λ_0 and μ_0. We observe that mean time to failure decreases with the increasing values of λ_0 and μ_0.

7. Conclusion:

In the present investigation, we have derived explicit results for system reliability measures when both units of the system are either in series or parallel configuration. In case of common cause failure, the probabilities of different states are evaluated in terms of transient probabilities. The reliability entered a new area with the advent of subsonic and supersonic air craft, electronics, missiles, nuclear energy applications and computers as well as domestic applications.

8. Table:

t	$P_0(t)$	$P_1(t)$	$P_2(t)$	$P_3(t)$
0	1.00000	0	0	0
1	0.43044	0.28200	0.17700	0.06300
2	0.28065	0.11971	0.09500	0.07800
3	0.20719	0.08459	0.06600	0.05100
4	0.16888	0.06602	0.03390	0.02900
5	0.14882	0.05629	0.01880	0.02080
6	0.13831	0.05119	0.01460	0.01920
7	0.13281	0.04852	0.01250	0.01770
8	0.12992	0.04712	0.01250	0.01600
9	0.12841	0.04639	0.01300	0.01490
10	0.12762	0.04600	0.01360	0.01640

Table. – (8.1): State probabilistic of two components systems

9. Figures:

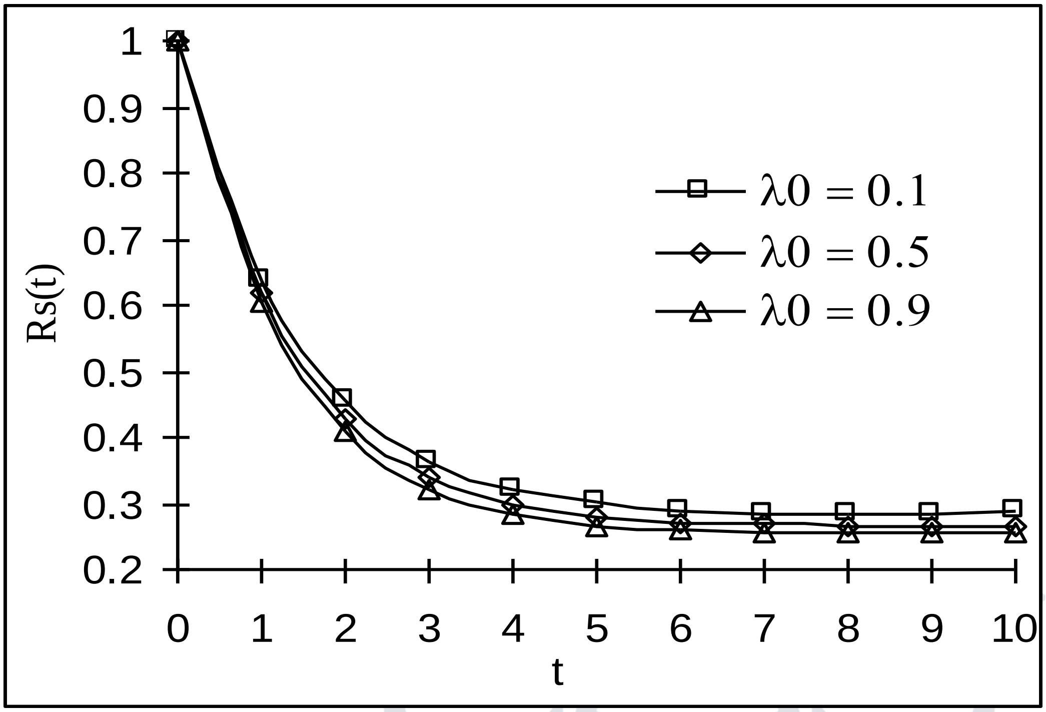


Fig. – (9.1): Effect of λ_0 on $R_s(t)$ by varying t.

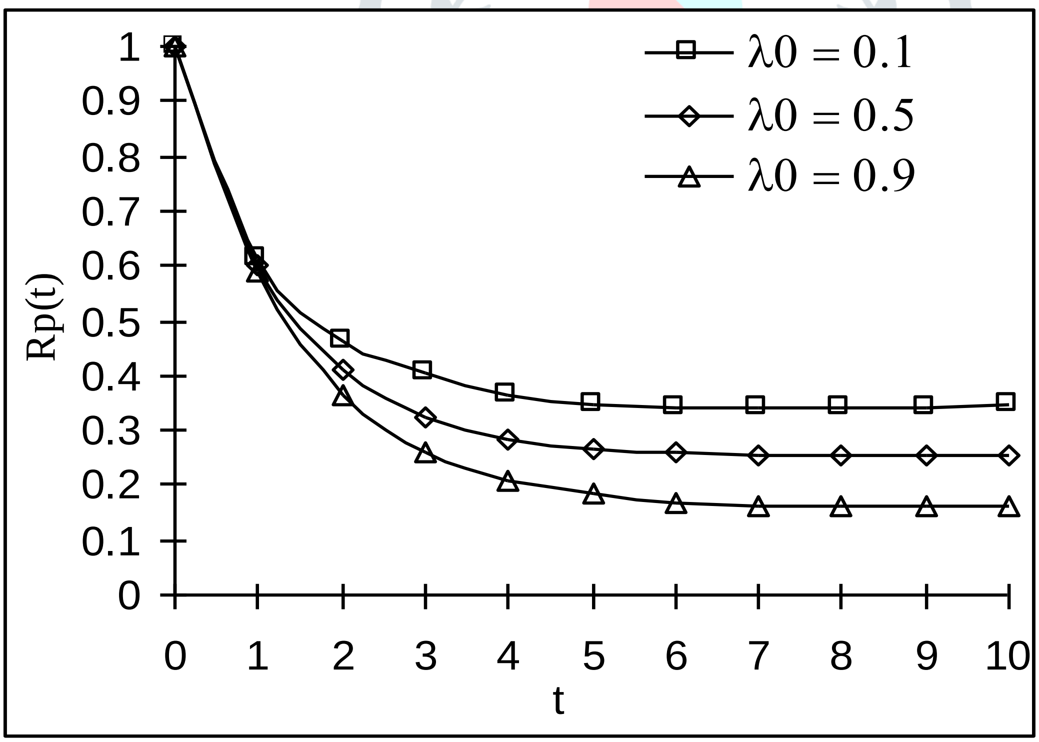


Fig. – (9.2): Effect of λ_0 on $R_p(t)$ by varying t.

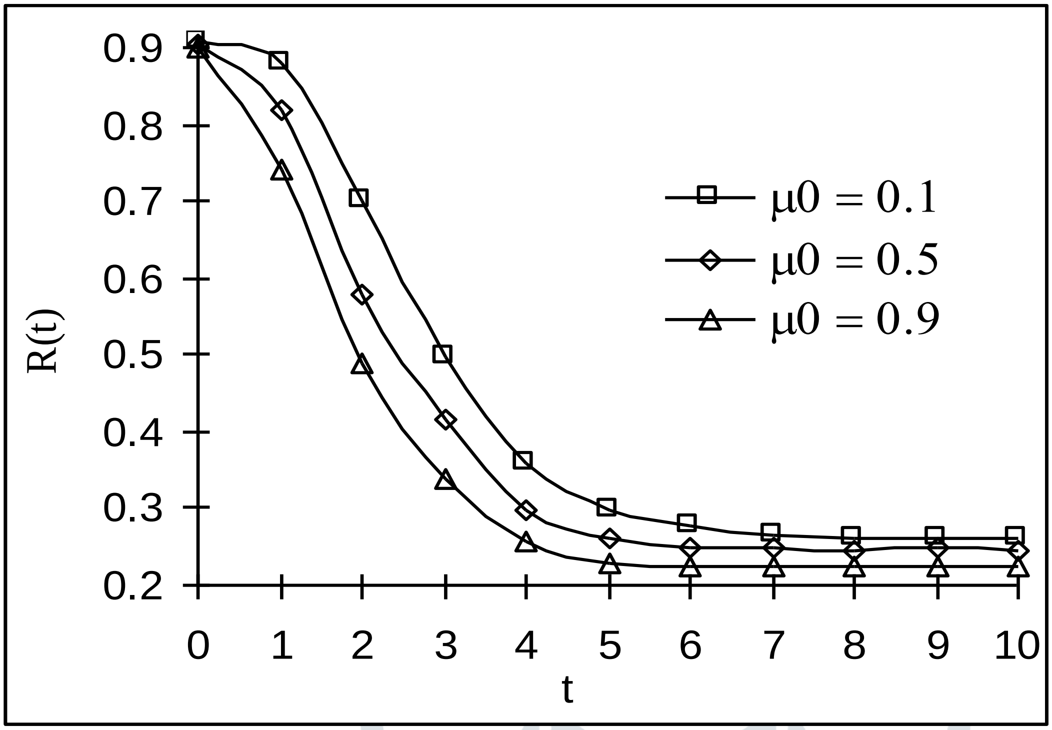


Fig. – (9.3): Effect of μ_0 on $R_s(t)$ by varying t.

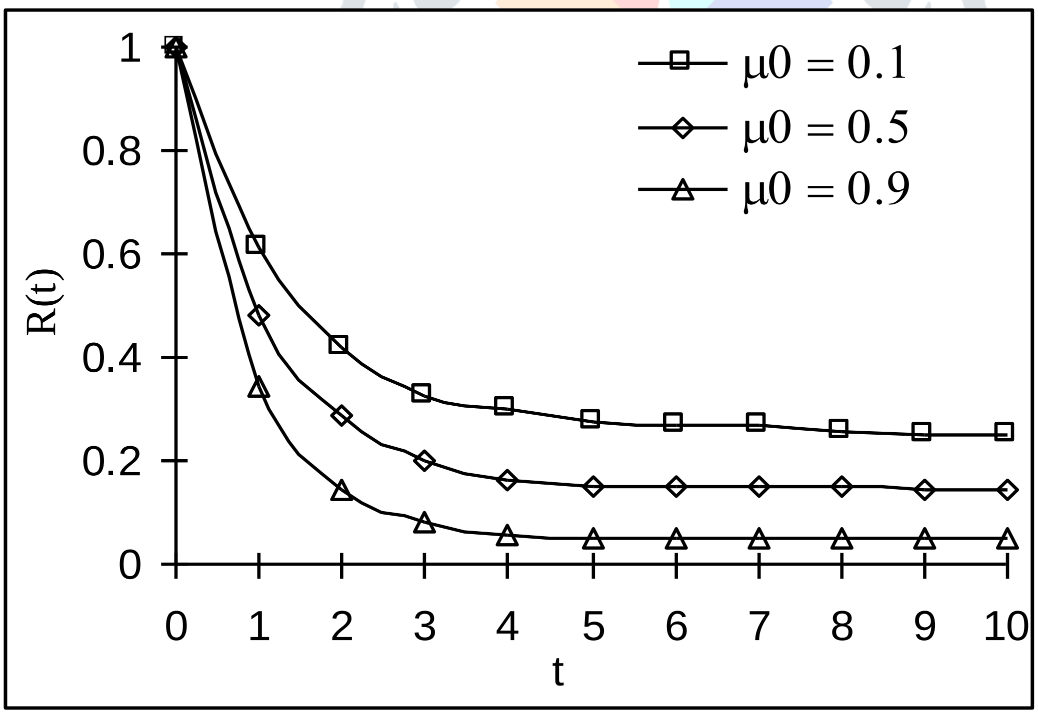


Fig. – (9.4) Effect of μ_0 on $R_p(t)$ by varying t.

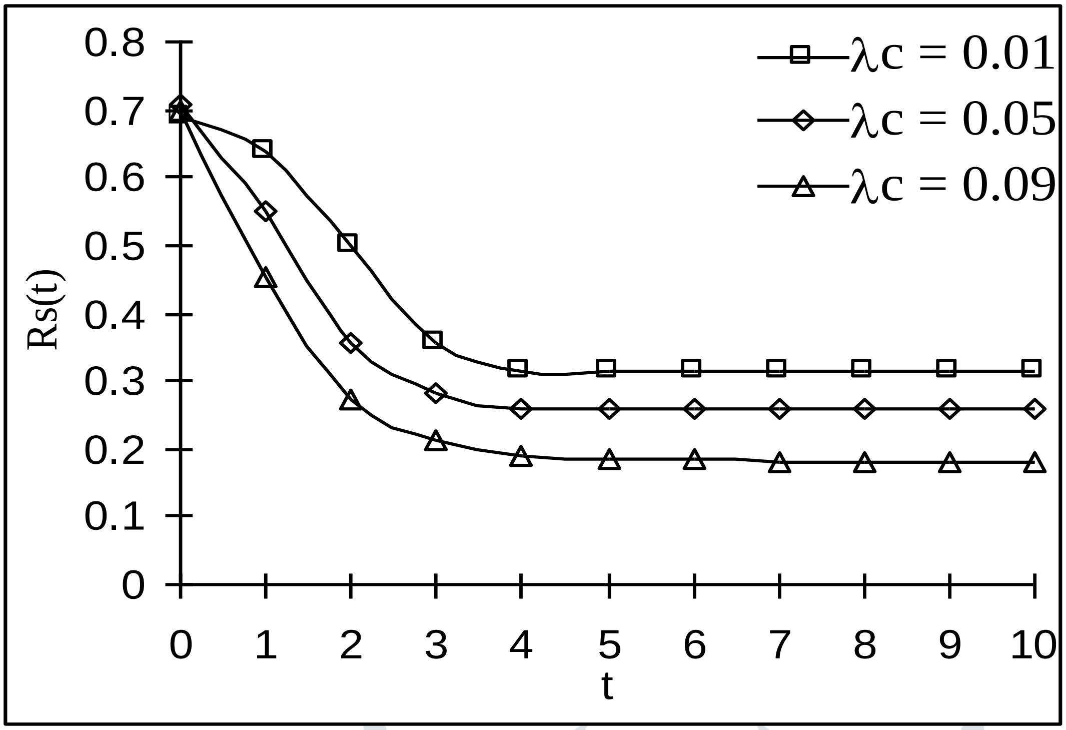


Fig. – (9.5): Effect of λ_c on $R_s(t)$ by varying t.

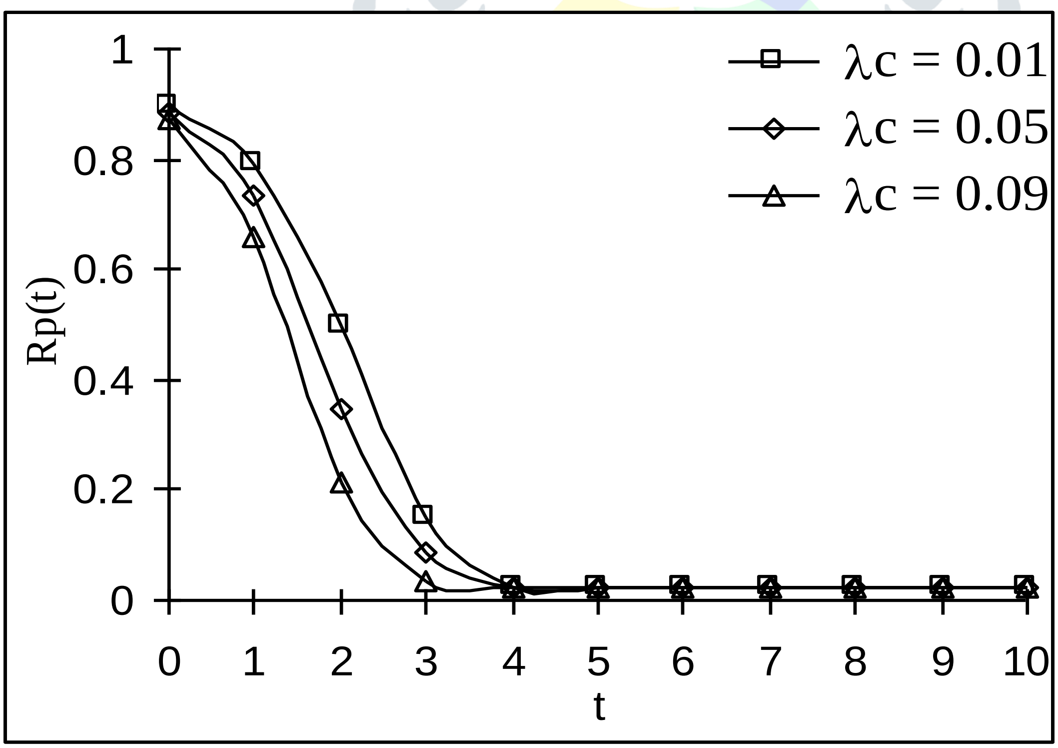


Fig. – (9.6): Effect of λ_c on $R_p(t)$ by varying t.

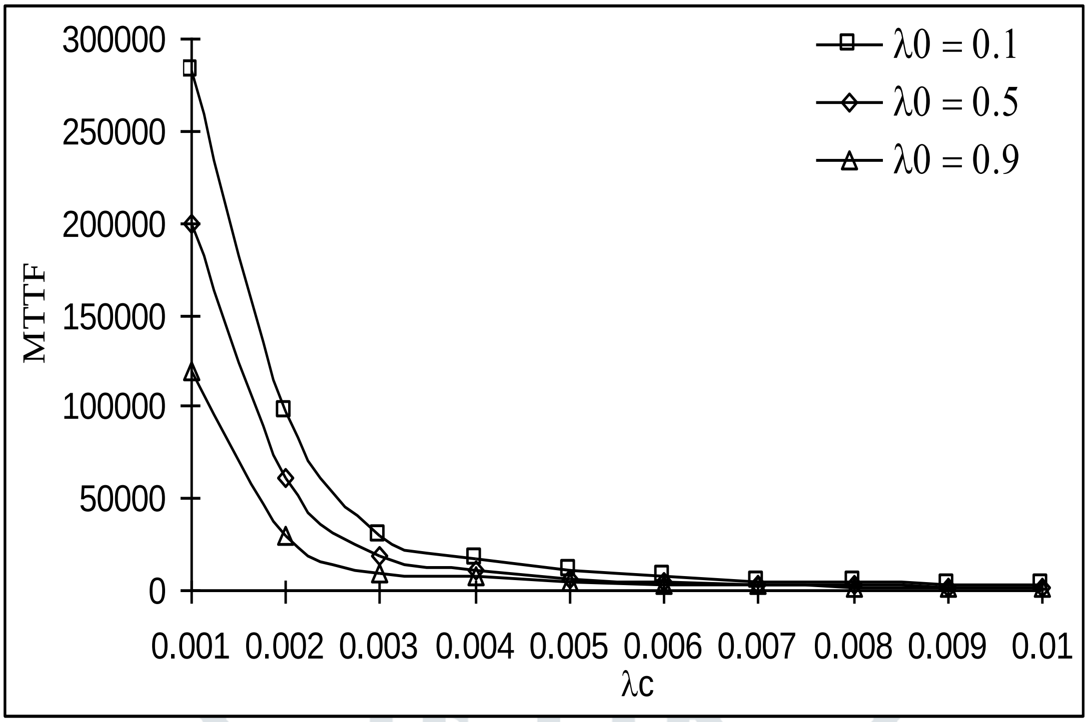


Fig. – (9.7): Effect of λ_0 on MTTF by varying λ_c

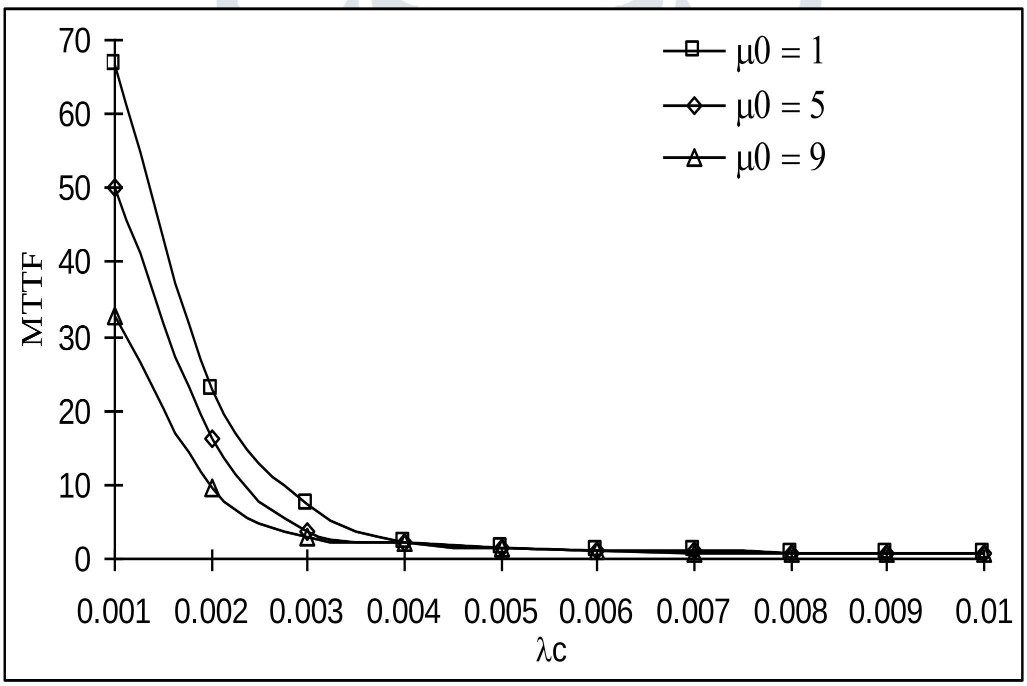


Fig. – (9.8): Effect of μ_0 on MTTF by varying λ_c

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