SOME STUDY ON COMPLETE REGULAR HAMILTONIAN GRAPHS

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ABSTRACT:

This paper presents some study and results about regular graphs, symmetric graphs, complete regular Hamiltonian Graphs with definitions and supportive illustrations. In this paper also presents some proofs about Complete Regular Hamiltonian Graph which have a Second Hamiltonian Cycle.

KEYWORDS:

Complete graph, Hamiltonian cycle, Hamiltonian graph, regular Graph, symmetric graphs

MSC 2020. 05C45; 05C07; 05C30

INTRODUCTION:

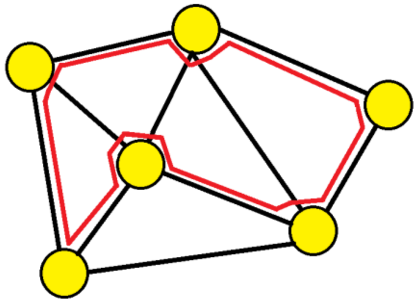
A Hamiltonian graph is a connected graph that contains a Hamiltonian cycle or circuit. Hamiltonian cycle is a path that visits each and every vertex exactly once and goes back to starting vertex. Hamiltonian graphs are used for finding optimal paths, Computer Graphics, and many more fields.

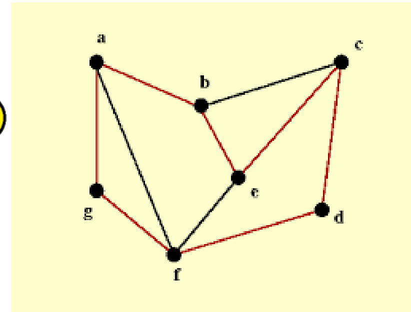
The origin of graph theory started with the problem of Koinsberbridge, in 1735. This problem lead to the concept of Eulerian Graph. Euler studied the problem of Koinsberg bridge and constructed a structure to solve the problem called Eulerian graph. In 1840, A.F Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems.

The concept of tree, a connected graph without cycles was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. In 1852, Thomas Gutherie found the famous four colour problem. Then in 1856, Thomas. P. Kirkman and William R. Hamilton studied cycles on polyhydra and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. In 1856, Hamiltonian introduced the Hamiltonian Graph where a Graph which is covered all the vertices without repetition and end with starting vertex.

1.1 Definition (Hamiltonian Cycles):

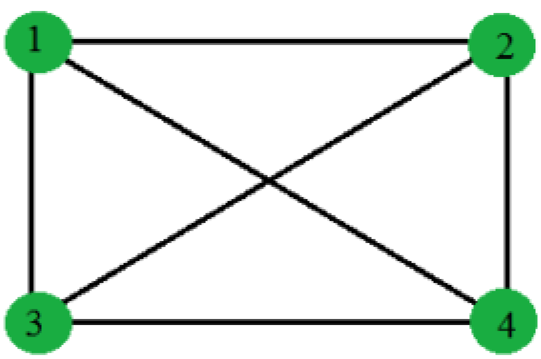
A cycle $\boldsymbol{C}$ of a graph $\boldsymbol{G}$ is Hamiltonian if $\boldsymbol{V}(\boldsymbol{C}) = \boldsymbol{V}(\boldsymbol{G})$. A graph is Hamiltonian if it has a Hamiltonian cycle.





1.2 Definition (Regular Graphs):

A graph $\boldsymbol{G}$ is said to be regular, if all its vertices have the same degree. In a graph, if the degree of each vertex is $'\boldsymbol{k}'$, then the graph is called a $'\boldsymbol{k-regular\ graph}$. The size of a r-regular graph is its number of edges. The order of a $\boldsymbol{r-regular\ graph}$ is its number of vertices. The degree of each vertex of an r-regular graph is $\boldsymbol{r}$. Hence the total of all the degrees of an $\boldsymbol{r-regular\ graph\ of\ order\ n\ is\ nr}$.



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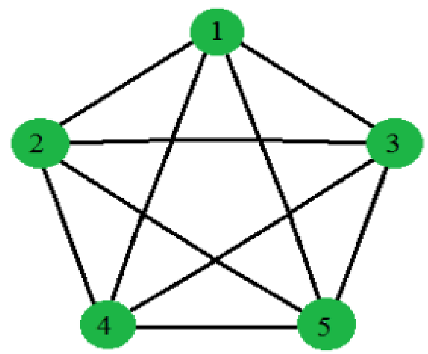

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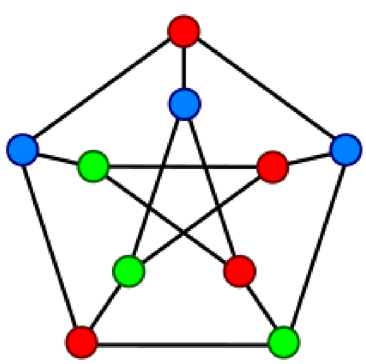
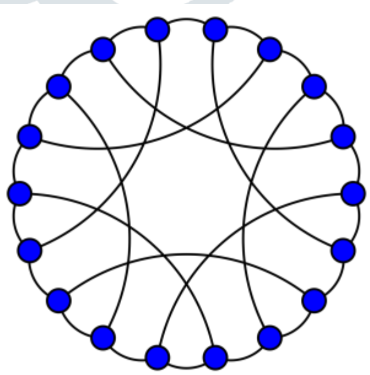
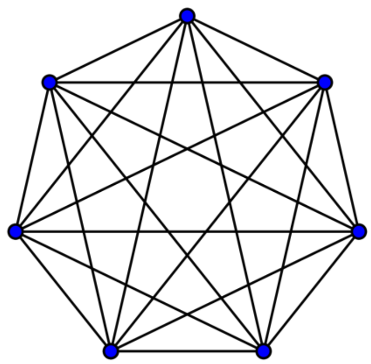
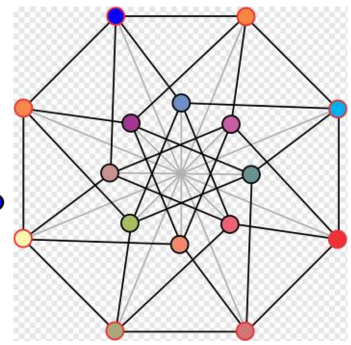
1.3 Definition (Symmetric graphs):

A regular graph that is edge-transitive but not vertex-transitive is called a semi symmetric graph. Neither the graph complement nor the line graph of a symmetric graph is necessarily symmetric.

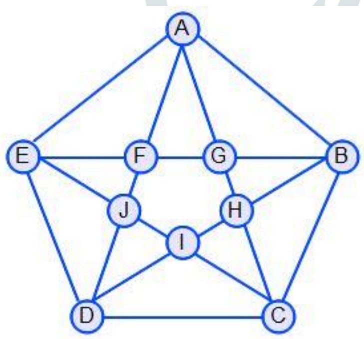


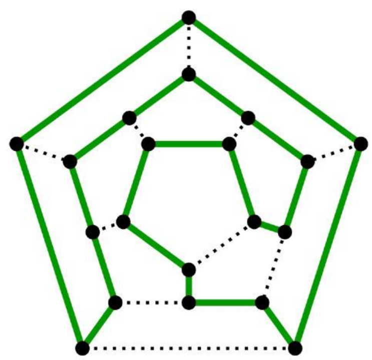


1.4 Definition (Compete Graph): A simple graph in which there exists an edge between every pair of vertices is called a complete graph.

1.5 Definition (Hamilton circuit): Hamilton circuit is a path that visits every vertex in the graph exactly once and return to the starting vertex. Determining whether such paths or circuits exist is an NP-complete problem. Hamilton Circuit diagram be





1.5 Definition: A graph usually denoted $G(V,E)$ or $G = (V,E)$ consists of set of vertices $\boldsymbol{V}$ together with a set of edges $\boldsymbol{E}$.the number of vertices in a graph is usually denoted $\boldsymbol{n}$ while the number of edges is usually denoted $\boldsymbol{m}$.

1.6 Definition: Vertices are also known as nodes, points and in social networks as actors, agents or players.

1.7 Definition: Edges are also known as lines and in social networks as ties or links. An edge $\boldsymbol{e} = (\boldsymbol{u},\boldsymbol{v})$ is defined by the unordered pair of vertices that serve as its end points.

2.1 Theorem. Let $\boldsymbol{G}$ be a Complete Regular Hamiltonian Graph of degree r. If $\boldsymbol{r} \geq \mathbf{300}$, then $\boldsymbol{G}$ contains a second Hamiltonian cycle.

Proof:

It is almost certain that 300 can be reduced considerably at least to 100 using arguments similar to those used in Thomason's proof.

We know every complete Hamiltonian 4-regular graph has at least one more Hamiltonian cycle is true and then every complete Hamiltonian regular graph except the cycle has at least two Hamiltonian cycles.

This follows by noting that we only have to prove the result for graphs of even degree because we know that every edge in a cubic graph lies on an even number of Hamiltonian cycles gives us the odd degree case.

So suppose that $\boldsymbol{G}$ is complete regular of even degree at least six.

Then the graph obtained from $\boldsymbol{G}$ by removing the edges of a Hamiltonian cycle $\boldsymbol{H}$ is complete regular of even degree.

A longstanding result we know that complete even regular graphs have a spanning graph, $\boldsymbol{S}$, which is the union of cycles.

Now $\boldsymbol{S}\ together\ with\ \boldsymbol{H}\ forms$ a 4-regular connected graph.

We know that every Hamiltonian 4-regular graph has at least two Hamiltonian cycles.

Hence does $\boldsymbol{G}$.

The above discussion suggests that it is not possible to find complete regular graphs which have only one Hamiltonian cycle. Certainly if they exist they will have even degrees less than 300.

2.2 Theorem. There exists a constant $\boldsymbol{c} > 0$ such that there are innately many 4-regular, 4-connected graphs, each containing exactly $\boldsymbol{c}$ Hamiltonian cycles.

Proof:

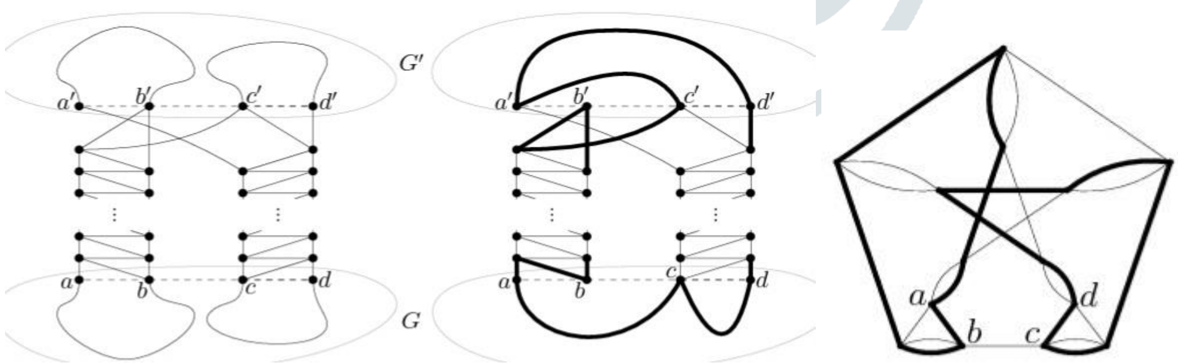
Suppose that $\boldsymbol{G}$ is a 4-regular 4-edge-connected graph containing a path abcd such that

(i) $\boldsymbol{G}$ has no Hamiltonian cycle;

(ii) $\boldsymbol{G}$ has a 2-factor consisting of two cycles $\boldsymbol{C}$ and $\boldsymbol{Cl}$ such that **C** contains ab and $\boldsymbol{Cl}$ contains cd;

(iii) $\boldsymbol{G - bc}$ has no Hamiltonian path joining two of $\boldsymbol{a}$; $\boldsymbol{b}$; $\boldsymbol{c}$; $\boldsymbol{d}$. If $\boldsymbol{v}$ is any vertex in $\{\boldsymbol{a,b,c,d}\}$, then $\boldsymbol{G - v - bc}$ has no Hamiltonian path joining two of $\boldsymbol{a}$; $\boldsymbol{b}$; $\boldsymbol{c}$; $\boldsymbol{d}$.

Let us construct the graph $\boldsymbol{HG}$ as follows.



Let $\boldsymbol{Gl}$ be a copy of **G** and $\boldsymbol{l \geq 1}$ a natural number. Consider disjoint union of G, Gl denote for a $vertex\ \boldsymbol{v}\ in\ \boldsymbol{G}$ it's copy in Gl by $\boldsymbol{vl}$. Now, delete the edges **ab**; **bc**; **cd**; $\boldsymbol{a'b'}$; $\boldsymbol{b'c'}$; $\boldsymbol{c'd'}$; add four pair wise disjoint paths Pa; Pb; Pc; Pd joining a; $\boldsymbol{c'}$ and b; $\boldsymbol{b'}$ and c; $\boldsymbol{a'}$and d; $\boldsymbol{d'}$, respectively, where P_a and P_d have length $l+1$, while P_b and P_c have $length\ `.Every\ Hamiltonian\ cycle\ in\ HG\ will\ contain\ the\ paths\ Pa;\ Pb;\ Pc;\ Pd.Note\ that$
$the\ number\ of\ Hamiltonian\ cycles\ of\ HG\ is\ independent\ of\ \ l.The\ bold\ edges\ form$
$the\ 2-factor\ satisfying\ (ii).It\ is\ straightforward\ to\ infer\ from\ the\ non-Hamiltonicity$
$of\ Petersen's\ graph\ that\ G\ satisfies\ (i)\ and\ (iii).$

CONCLUSION:

It is understand of a complete regular Hamiltonian graph through various definitions illustrative examples, some theorems.

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