

Proposed Methodology of Compton Scattering

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Abstract

Compton scattering is a technique for determining the momentum distribution of electrons in condensed matter. When monochromatic photons are Compton scattered in a fixed direction, the observed energy spectrum of the scattered photons is Doppler-broadened due to the motion of the target electrons. The objective of this review is to present the Compton scattering theory to researchers generally unfamiliar with this phenomenon and to lead the researchers to understanding of the fundamental principles of the Compton Scattering Theory and of the way in which they are employed in logical deductions and analyses. In this review, the theoretical and experimental considerations and energy limitations of the Compton scattering method are discussed. The method for extracting information about ground-state electron momentum densities through an analysis of the Compton line shape is presented. Today the Compton scattering is acknowledged as a crucial technique for investigating the electronic structure of materials; it provides a sensitive test for the accuracy of the resulting electron wave functions obtained from different theoretical models.

Keywords: Compton scattering, high peak energy, coefficient.

Introduction: Compton scattering of a beam with a relativistic beam has been used to generate intense, highly polarized and nearly monoenergetic x-ray or gamma-ray beams at many facilities. The ability to predict the spatial, spectral, and temporal characteristics of a Compton gamma-ray beam is crucial for the optimization of the operation of a Compton light as well as for the applications utilizing the Compton beam. In this paper, we present two approaches, one based upon analytical calculations and therefore the other based upon Monte Carlo simulations, to review the Compton scattering process for various electron and laser-beam parameters also as different gamma-beam collimation conditions. The various Compton sources and Compton scattering in current use are reviewed. Since 1970 Compton profile measurements became more frequent and therefore the experimental results for several Z-elements reported within the literature are quoted to an accuracy of higher than 1% for the total profiles. This has been demonstrated in view of the Compton scattering experiments successfully performed over a wide range of incident photon energies (10 - 662 KeV) used in various Compton spectrometer systems distributed around the world. The ability to predict the spectral, spatial, and temporal characteristics of a Compton gamma-ray beam is crucial for the optimization of the gamma-ray beam production as well as for research applications utilizing the beam. While the idea of particle-particle (or electron-photon) Compton scattering, which is like the scattering between a monoenergetic beam and a monoenergetic beam with zero transverse sizes, is well documented in literature^[4], there remains a need to fully understand the characteristics of the gamma-ray beam produced by Compton scattering of a laser beam and an electron beam with specific spatial and energy distributions, i.e., the beam scattering. Study of beam-beam Compton scattering has been recently reported in [7,8]. However, the algorithms used in these works are based upon the Thomson scattering cross section, i.e., an elastic scattering of electromagnetic radiation by a charged particle without the recoil effect. For scattering of a high-energy beam and a beam, the recoil of the electron must be taken under consideration. The Compton scattering cross section has been used to study characteristics of Compton gamma-ray beams by Duke scientists in the 1990s^[9,10]. However, the effects of incoming beam parameters and the effects of gammabeam collimation were not fully taken into account. Compton scattering is one of the most important processes on assessment interaction of radiation with material in which the ray will be attenuated quantitatively as Compton attenuation coefficient. This coefficient is a practical and applicable parameter for improving the images in nuclear medicine as well as decreasing the absorbed

radiation dose along with the photoelectric absorption one. Also, each organ in the body is at a specific situation where acts the functional and physiological engagements in order to obtain health and high growth of cells. Every change in this spatial situation leads to appear abnormalities, lesions and loss of performance as well. Therefore, estimation of the organs' depth plays a key role in medicine due to providing helpful information for evaluating their performance, early diagnosis, as well as the better treatment. While those are deformed, the depth may be changed, and this can be probably utilized to detect the type and degree of disease. Clearly, the calculation of the depth depends on the peripheral tissues, the attenuation coefficient and the organ size that were considered in the existing methods.^[1-7] Nosil et al. employed two radioisotopes that their method was independent of the size, but assuming the attenuation coefficient of water to be constant.^[3] On the other hand, Starck and Carlsson proposed a method in which the modulation transform function and the mean linear attenuation coefficient used, in contrast with depending size.^[5] These methods were related to either the size or the total attenuation coefficient, as it has been done comparison of the methods on this matter, but all of them related to the other variable distinct parameters. In this study, a new method proposed free from all aforementioned parameters in which convolution of the scattering and primary photons functions (CSPF) along with the triple energy-window (TEW) and extended triple energy-window (ETEW) methods is used for estimating depth of organs along with estimating Compton scattering attenuation coefficient and the photoelectric absorption coefficient using the energy spectra.

Methods: The radiopharmaceutical, containing the nuclear materials, is used in order to evaluation of metabolic performance and physiological parameters of the tissues.^[11,12] Due to the nature of nuclear radiation, several interactions between radiation and tissue occur according to different energies of the radiopharmaceutical. Photoelectric and Compton scattering are usually dominant phenomena in diagnostic nuclear medicine.^[13] Although, the scattered gamma ray reduces image contrast,^[14] but we have demonstrated that the depth can be found by this and some mathematical concepts, and then the Compton attenuation and the photoelectric absorption coefficients will be estimated. Scatter estimation using triple energy-window The TEW method,^[8] has better performance than other methods in nuclear medicine,^[9,10] and is based on the energy spectrum. The subtraction and trapezoidal laws are

used in this method which the subtraction is carried out using two sets of data: One set is acquired with a main window centered at photopeak energy, and the other is acquired with two subwindows on both sides of the main window. The scattered photons (C_{scat}) included in the main window are estimated from the counts acquired with the subwindows and then they are subtracted from the count acquired with the main window. The count of primary photons (C_{prim}) is given by

$$C_{prim} = C_{total} - C_{scat} \quad (1)$$

The C_{scat} is estimated from the count data C_{left} and C_{right} acquired with the two subwindows that are located at both sides of the main window. Assuming that the width of the main window as W_m and that of the subwindow as W_s , the C_{scat} can be estimated from a trapezoidal region having a left height of C_{left}/W_s , a right height of C_{right}/W_s , and a base of W_m as follows:

$$C_{scat} \cong \left(\frac{C_{left}}{W_s} + \frac{C_{right}}{W_s} \right) W_m / 2 \quad (2)$$

The C_{prim} can be calculated using Eqs. (1, 2). The choice of the energy-window width (EWW) for the values of W_m and W_s is critical in these methods because the accuracy is related severely to the EWW value and the detector system. The photopeak point has spatial shift more to the left side on the spectrum when the C_{scat} is increased. This shift may also be considered as a preliminary estimation of the C_{scat} which this is beyond the scope of this study, and it is not considered at accounting process here. This matter may be an error source in CSPF method.^[16]

Scatter estimation using extended triple energy-window proposed this method in order to improve the quality of the nuclear medicine images.^[17] The ETEW method estimates scatter counts with the trapezoidal approximation as follows,

$$C_s = \left(\frac{C_{left}}{W_{left}} - \frac{C_{right}}{W_{right}} \right) (W_1 + W_2) \frac{W_m}{2W} + \frac{C_{right}}{W_{right}} W_m \quad (3)$$

Where W is difference between the centers of the right and left subwindows, W_1 is difference between the center of the right subwindow and lower bound of the main window, and W_2 is difference between the center of the right subwindow and upper bound of the main window. Convolved scatter and primary functions method The aim was to present a new method for calculating the organ depth independent of the linear attenuation coefficient. Estimation of the depth is useful for measuring the amount of radioactive tracker taken up by an organ in the body. This method is based on the mathematical relations

as convolution of two exponential functions that both the related parameters of these functions and its result mapping on the spectrum curve which they were determined by the Monte Carlo method and Matlab software. Obviously, the convolution operator is the effectiveness of a function on the other function with progressing some known variables. In the interaction of radiation with matter, the energy value is decreased gradually, and it seems that these energies will act together totally on the system so that one may consider the CSPF appeared in the energy spectrum. However, the spectrum is a final result from these interactions. In CSPF method, two pseudo-analogical expressions are introduced on the spectra. The first, the spectrum function (C (E)) is the convolution of two exponential functions, and the second, the scattered (Cs (E)) and primary (Cp (E)) photon functions are as exponential functions in which the origin of coordinates is metaphorical. One may obtain the function of C (E) as follows,

$$C(E) = C_S(E) * C_P(E) = \varphi e^{-\alpha E} * \omega e^{-\beta E} = \frac{\varphi\omega}{\beta-\alpha} (e^{-\alpha E} - e^{-\beta E}) \quad (4)$$

Where α , β and ω are the constant parameters, which are determined by the some distinct points on the spectrum curve using Matlab software. The C_{scat} and C_{prim} are calculated by the TEW and ETEW methods at various depths. In this method, these values are the integrand of the $C_S(E)$ and $C_P(E)$ functions over the DECS and DECP windows as follows,

$$C_{SCAT} = \int C_S(E) dE = \int_{140-0.5\Delta E_{CS}}^{140+0.5\Delta E_{CS}} \varphi e^{-\alpha E} dE \quad (5)$$

$$C_{SCAT} = \int C_P(E) dE = \int_{140-0.5\Delta E_{CP}}^{140+0.5\Delta E_{CP}} \omega e^{-\beta E} dE \quad (6)$$

The energies are in terms of keV and the technetium-99m (Tc-99m) source has 140-keV gamma ray. The ΔE_{CS} and ΔE_{CP} values will be calculated from the energy spectra at different depths by solving the Eqs. (5, 6) using Matlab software. The distance from the detector, namely depth (d), is related to the energy-windows of Δ

$$\Delta E_{CS} \cong \Delta E_{CP} = \sigma e^{-\tau x d} \quad (7)$$

The aim is the determination of the σ and τ unknown parameters. Therefore, one may estimate the depth value using Eq. (7) in which the energy-window calculated by both the ETEW and CSPF methods on the energy spectrum obtained in nuclear imaging. This flowchart shows the step by step for the calculation of these unknown parameters. Finally, with characterizing Eq. (7), one the first obtains the spectrum and procedures for determining the ΔE_{CS} or ΔE_{CP} values, and then the depth will be determined.

Determination Of The Compton Attenuation Coefficient:

The second, the relationship between the counts ratio and the Compton attenuation coefficient has been evaluated. The radiopharmaceuticals concentrated in the special organs may be considered as the volumetric sources along with a special pharmaceutical signal to noise ratio (S/N) in which the rays employed to determine the anatomical and functional parameters. For instance, a Tc-99m point source is positioned beyond an attenuator that decreases both the number and energy of the rays at the different situations. Some of the rays can pass through the material without scattering to be recorded under the photopeak region. The ray on the straight path does not react with the matter that will pass from the hole of collimator and will reach into the detector. With regard to the resolution and related parameters on the detection, the rays will be detected and recorded under the photopeak region on the energy spectrum. In contrast, the ray passing through the attenuator with the small angle (θ) relative to normal line interacts at the distance of x from the source at the dx thickness included in the attenuator due to its Compton attenuation coefficient that will be scattered so that Compton phenomenon will act as priority one. The scattered photon will travel through the distance of L and will be recorded in the Compton region on the spectrum. The primary counts (C_p) has been calculated by,

$$C_{prim} = k I_0 e^{-\mu d} \quad (8)$$

Where k, I_0 , and μ are the buildup factor along with the other parameters of the detector, the primary intensity of the source, and total linear attenuation coefficient, respectively. To estimate theoretically the number of the scattered photons recorded in the Compton region of the spectrum, a thickness of dx away from a distance of x from the source is considered. The number of descending photons to this thickness is as follows,

$$k' I_0 e^{-\mu x} \quad (9)$$

That the number of the scattered photons with respect to the Compton scattering attenuation coefficient (μ_{sc}) and the thickness of dx is shown in fig 1.

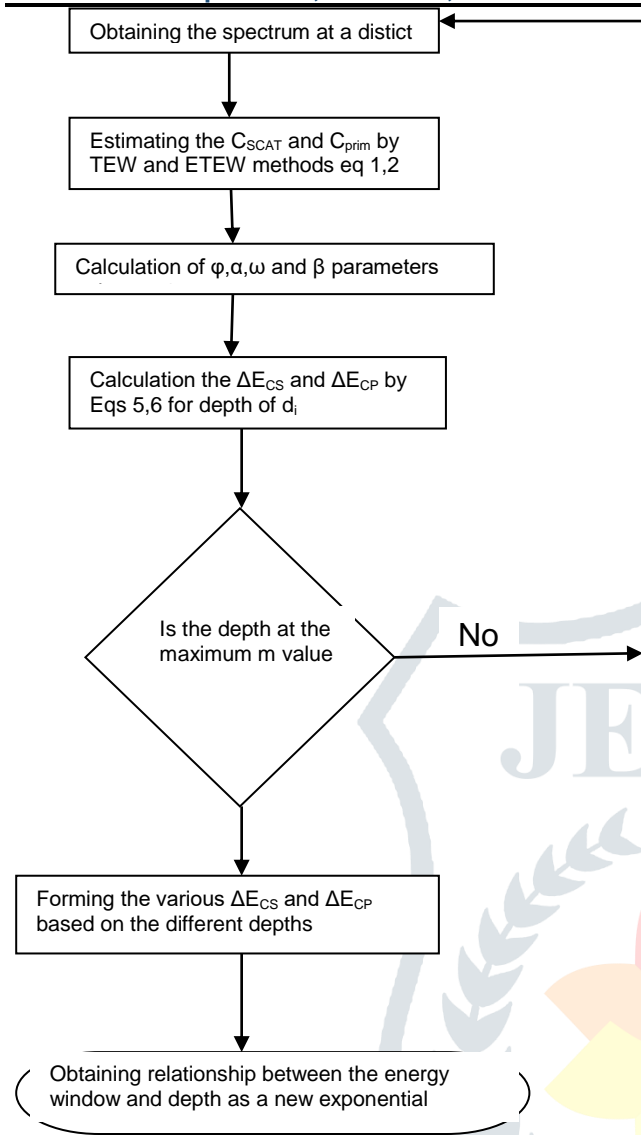


Fig 1: Flow chart of implementation of the proposed method

Simulation method and computation of the parameters To study the effects of interactions, Monte Carlo N-Particle version 4C (MCNP4C) code was used here. The input files would specify geometry of source objects, collimators and detector planes for a distinct aim, which input files and geometry specifications are often complex and can be very cumbersome

Results: The response of the detector is simulated with the F8 tally and E8 card. As known, the choice of energy-window at the energy spectra is important for the SNR and image contrast. We have demonstrated that the EWW is proportional to the depth with respect to the primary and scattered photon counts. While the energy spectra obtained both as experimentally by the detector systems and as theoretically either by the simulation using the Monte Carlo method or the calculation by the existing formulas are accessible, one may extract the more information on tracer, detector system, and the

geometrical specifications of organ. The TEW and ETEW methods have been used for estimating the scattered and primary counts accurately. Though the spectra of the scattered photons vary with object size, source distribution, and source energy, estimation of the scattered photons as a trapezoid is good approximation. The other problem is the energy value used in the field of imaging that leads to appearing the Compton scattering and photoelectric absorption processes. Emitted gamma radiation interacts with the body based on these processes, producing a significant attenuation in the primary beam at energies. These mechanisms are well known, and those were a basis in our method. The simulated results indicate that the EWW value is decreased with increasing depth due to the more attenuation and higher cross-sections. It is found that the relationship between the EWW and depth is as exponential function. This method may be used for estimating energy resolution of detector system. Also, it is estimated a distinct distance that Compton scattering regions at depths lower than this distance (4 cm) are similar to with each other. This distinct depth is probably useful to better compensation of scattering for organs close to the skin. The scattered photons of the photopeak window are mainly contributed by the first-order Compton scatter.[20] The Compton scattering which may be identified by the cross-sections that will vary with the energy of gamma ray has a key role in this study, although in the field of imaging is unsuitable and must be compensated in order to having a better diagnosis. The C_{scat} value is important both to improve SNR and to estimate depth because the increase in SNR and the reduction of noise followed by the rejection of scattering that it can be clearly observed as well as to provide better quality in the reconstructed images. Corrections for scattering are necessary in order to obtain the higher quantitation accuracy, which at all categories, the depth parameter is not considered. It can be used to compensate some effects due to scattering that is undesirable for forming a qualified image. Also, it seems that the noise is an important factor to accuracy estimation of depth as well as the rigid and flexible motions. To decrease these effects, it must be prepared some methods before obtaining the spectra. Some theoretical formulas could be used to rapidly assess the impact of different scatter correction strategies on image quality.

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