Power 3 Mean Labeling of Line Graphs

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Abstract:

In this paper we contribute some new results on Power 3 mean labeling of graphs. We investigate on some standard graphs that accept Power 3 mean labeling and proved that the Line graphs of these Power 3 mean graphs are also Power 3 mean graphs. We proved that the Line graphs of Path, Cycle, Comb, $P_n \odot K_1$, $P_n \odot K_{1,2}$ are Power 3 mean graphs.

Key words: Graph, Power 3 mean graph, Line Graph, Path, Cycle, Comb, $P_n \odot K_1$, $P_n \odot K_{1,2}$.

AMS Subject Classification: 05C78

1.Introduction:

All graphs in this paprer are finite, simple, and undirected graph $G = (V, E)$ with p vertices and q edges. For all detailed survey of Graph labeling, we refer to J.A.Gallian [1]. For all other standard terminology and notations we follow Harary [2]. The concept of mean labelling has been introduced by S.Somasundaram and R.Ponraj [3] in 2004. S.Somasundaram and S.S.Sandhya introduced Harmonic mean labeling [4] in 2012. S.S.Sandhya, S.Sreeji introduced Power 3 mean labeling.

In this paper we investigate the Line graphs of some standard Power 3 mean graphs Path, Cycle, Comb, $P_n \odot K_{1,2}$. We will provide a brief summary of definitions and other information which are necessary for our present investigation.

A Path P_n is a walk in which all the vertices are distinct. A **Cycle C_n** is a Closed Path. The graph obtained by joining a single pendant edge to each vertex of a Path is called a **Comb**. **A Complete Bipartite** graph $K_{m,n}$ is a bipartite graph with bipartition (V_1, V_2) such that every vertex of V_1 is joined to all the vertices of V_2, Where $|V_1| = m$ and $|V_2| = n$. $P_n \odot K_{1,2}$ is a graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,2}$.

Definition 1.1:

A graph G with p vertices and q edges is called a power 3 mean graph, if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1,2, \dots \dots, q+1$ in such a way that in each edge $e = uv$ is labelled with $f(e = uv) = \left\lceil \left(\frac{x^3+y^3}{2}\right)^{\frac{1}{3}} \right\rceil$ or $\left\lfloor \left(\frac{x^3+y^3}{2}\right)^{\frac{1}{3}} \right\rfloor$. Then , the edge labels are distinct. In this case f is called Power 3 Mean labelling of G.

Remark: 1.2

If G is a Power 3 mean graph, then '1' must be a label of one of the vertices of G, since an edge should get label '1'.

Remark: 1.3

If u gets label '1', then any edge incident with u must get label 1 (or) 2 (or) 3. Hence this vertex must have a degree ≤ 3.

Definition 1.4:

Let $G = V, E$ be a non - trivial graph. Now each edge in E can be considered as a set of two elements of V. So E is a non - empty collection of distinct non empty subsets of V, such that their union is V. So there is a intersection graph $\Omega(E)$. Their graph $\Omega(E)$ is called the line graph of G and is dented by $L(G)$.

We observe that the vertices of $L(G)$ are the edges of G. Further two vertices of $L(G)$ are adjacent iff their corresponding edges are adjacent in G. Thus the vertices a, b in $L(G)$ are adjacent iff $a = uv$ and $b = vw$ are in G.

Theorem 1.5: Any Path P_n is a Power 3 mean graph.

Theorem 1.6: Any Cycle $C_n, n \geq 3$ is a Power 3 mean graph.

Theorem 1.7: Any Comb $P_n \odot K_1$ is a Power 3 mean graph.

Theorem 1.8: $P_n \odot K_{1,2}$ is a Power 3 mean graph.

Remark : 1.9 Line graph of path P_n is a Power 3 mean graph.

Remark : 1.10 Line graph of Cycle C_n is a Power 3 mean graph

Remark : 1.11 In Path and Cycle, Graph G and Line graph $L(G)$ are isomorphic to each other.

2. Main Results:

Theorem 2.1:

The line graph of $K_{1,3}$ is a Power 3 mean graph

Proof:

The graph $K_{1,3}$ is displayed below.

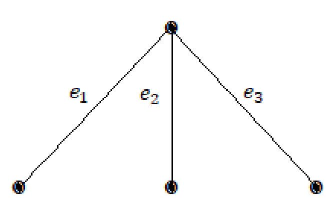

Figure : 1

The Line graph of $K_{1,3}$ is displayed below.

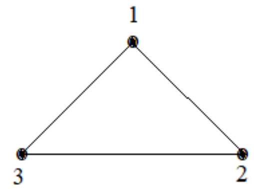

Figure : 2

Theorem 2.2:

Line graph of Comb $P_n \odot K_1$ is a Power 3 mean graph.

Proof:

Let G be a graph obtained from a Path $P_n = u_1 u_2 \ldots \ldots u_n$ by joining the vertex u_i to v_i; $1 \leq i \leq n$.

Graph G of Comb $P_5 \odot K_1$ is displayed below.

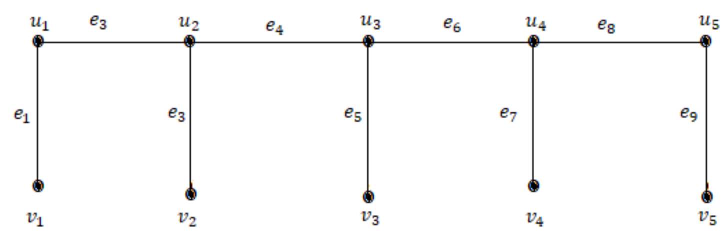

Figure : 3

Let e_i be the vertices of $L(G)$. The Line graph $L(G)$ Comb $P_5 \odot K_1$ is shown in figure : 9

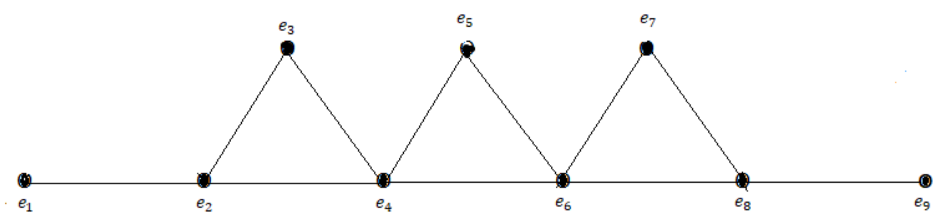

Figure : 4

In general, the Line graph $L(G)$ of Comb $P_n \odot K_1$ is shown in figure : 10

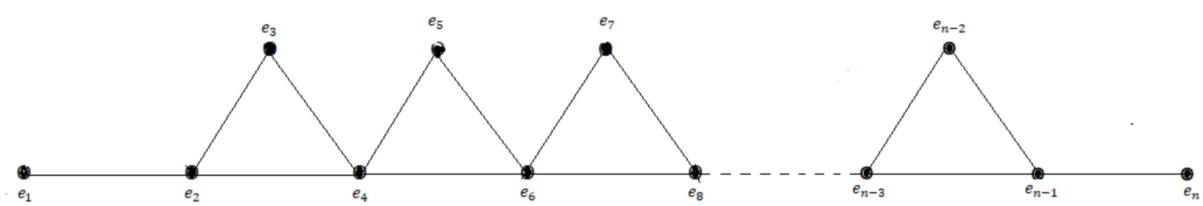

Figure : 5

Let u_i, v_i be the vertices of Line graph $L(G)$.

Define a function $f(G) \rightarrow \{1,2, \dots \dots, q+1\}$ by,

$f(u_1) = 1\ ;\ f(u_2) = 2\ ;\ f(u_i) = 3i - 2\ ;\ 3 \leq i \leq n-1$

$f(u_n) = f(u_{n-1}) + 2\ ;\ f(v_i) = 3i\ ;\ 1 \leq i \leq n$

Edges are labeled with, $f(u_1u_2) = 1\ ;\ f(u_iu_{i+1}) = 3i - 3\ ;\ 2 \leq i \leq n-2$

$f(u_{n-1}u_n) = f(u_{n-1})\ ;\ f(v_iu_{i+1}) = 3i - 1\ ;\ 1 \leq i \leq n-3\ ;\ f(v_iu_{i+2}) = 3i + 1$

Hence $L(G)$ of Comb $P_n \odot K_1$ is a Power 3 mean graph.

Example : 2.3 Power 3 mean labeling of Line graph of Comb $P_5 \odot K_1$ is shown below.

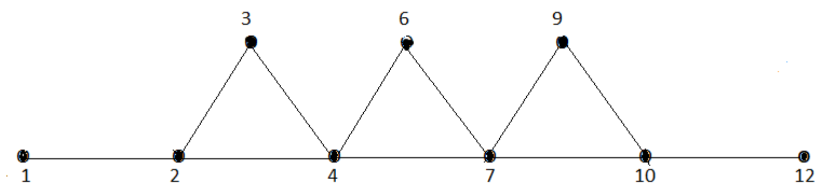

Figure : 6

Theorem 2.4:

Line graph of $P_n \odot K_{1,2}$ is a Power 3 mean graph.

Proof:

Graph G of $P_4 \odot K_{1,2}$ is displayed below.

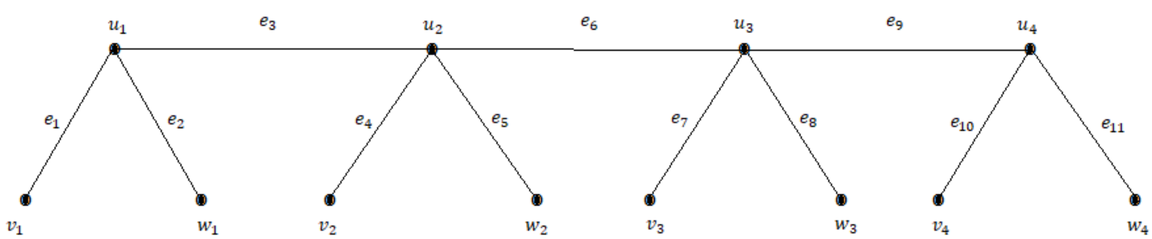

Figure : 7

The Line graph $L(G)$ of $P_4 \odot K_{1,2}$ is shown in figure : 13

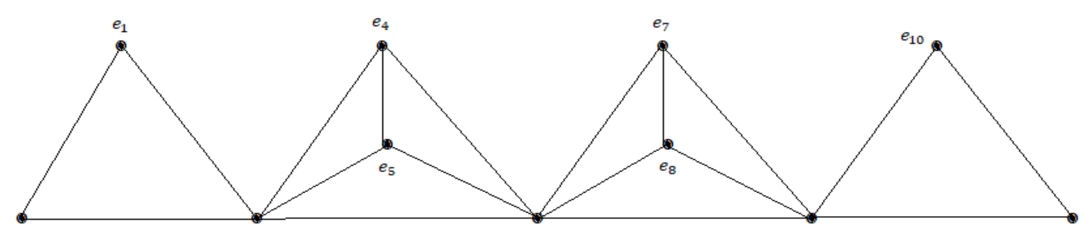

Figure : 8

In general, The Line graph of $P_4 \odot K_{1,2}$ is shown in figure : 14

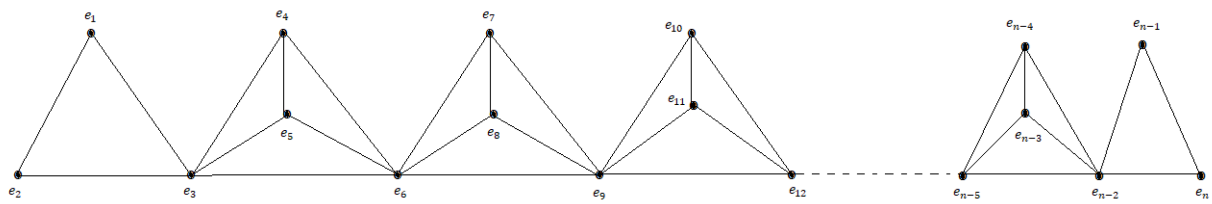

Figure : 9

Let $L(G)$ be the line graph of $P_n \odot K_{1,2}$ and u_i, v_i, w_i be the vertices of $L(G)$.

Define a function $f: V(G) \rightarrow \{1,2, \ldots \ldots, q+1\}$ by

$f(u_1) = 1$; $f(u_i) = 6i - 9$; $2 \leq i \leq n-1$;

$f(u_n) = f(u_{n-1}) + 4$

$f(v_1) = 2$;

$f(v_i) = 6i - 7$; $2 \leq i \leq n-1$;

$f(w_i) = 6i + 1$; $1 \leq i \leq n-3$

Edges are labeled with, $f(u_1 u_2) = 2$;

$f(u_i u_{i+1}) = 6i - 5$; $2 \leq i \leq n-2$

$f(u_{n-1} u_n) = f(u_{n-1}) + 2$;

$f(u_1 v_1) = 1$;

$f(u_i v_i) = 6i - 8$;

$f(v_1 u_2) = 3$;

$f(v_2 u_3) = 8$

$f(v_i u_{i+1}) = 6i - 4$; $3 \leq i \leq n-2$;

$f(v_n u_{n+1}) = f(v_n) + 1$;

$f(w_i u_{i+1}) = 6i - 1$; $1 \leq i \leq n-3$

$f(w_i u_{i+1}) = 6i$; $1 \leq i \leq n-3$;

$f(w_i u_{i+2}) = 6i + 3$; $1 \leq i \leq n-3$

Thus, f admits Power 3 mean labeling of G. Hence $L(G)$ of $P_n \odot K_{1,2}$ is a Power 3 mean graph.

Example : 2.5 Power 3 mean labeling of Line graph $P_4 \odot K_{1,2}$ is shown below.

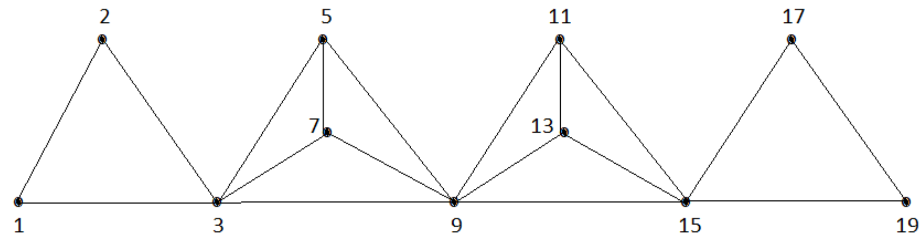

Figure : 10

Theorem 2.9:

Line graph of $K_3 \odot K_1$ is a Power 3 mean graph.

Proof:

The graph of $K_3 \odot K_1$ is shown below.

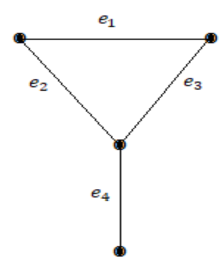

Figure : 11

The Line graph of $L(K_3 \odot K_1)$ is shown below.

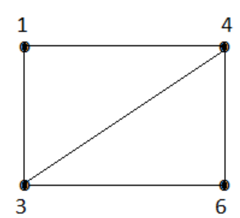

Figure : 12

In the above figure, the vertices and edges are get distinct labels.

Hence Line graph of $K_3 \odot K_1$ is a Power 3 mean graph.

3.Conclusion:

The study of Power 3 mean labelling of Line graphs is important due to its diversified applications. Line graphs of all Power 3 Mean Graphs are not Power 3 Mean Graphs. It is very interesting to investigate graphs which admits Power 3 Mean Labeling. In this Paper, We proved that Line Graph of Path, Cycle, Comb, Star, $P_n \odot K_{1,2}$ are Power 3 Mean Graphs. The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

4. Acknowledgement:

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