Graph Energies of Anthraquinone: Energy, Randic Energy, Colour Energy, Laplacian Energy, Sign less Laplacian Energy, Normalized Laplacian Energy.

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Abstract:

In this paper I compute Energy, Randic energy, Colour energy, Laplacian energy, Sign less Laplacian energy, Normalized Laplacian energy of Anthraquinone.

Key words:

Anthraquinone, Eigen values, Characteristic equation, Energy, Randic energy, Colour energy, Laplacian energy, Sign less Laplacian energy, Normalized Laplacian energy.

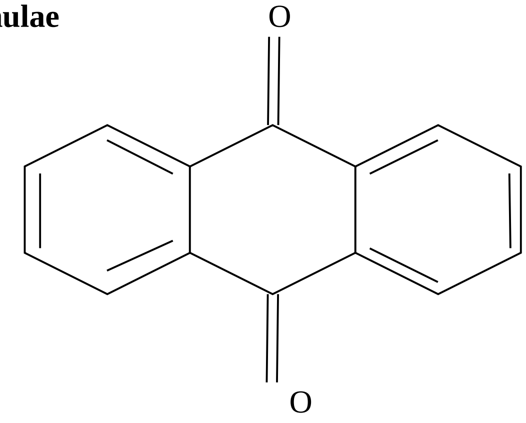
1. Introduction

Anthraquinones are active components of many plant blends which are used as medicines and exhibit laxative, diuretic, estrogenic and immunomodulatory effects. Anthraquinones are structurally related to anthracene and possess the 9,10-dioxoanthracene. Anthraquinones typically occur in their glycosidic forms. These compounds impart colour to plants and have been widely utilized as natural dyes. Anthraquinone is an important and widely used raw material for the manufacture of vat dyes, which are a class of water-insoluble dyes that can easily be reduced to a water soluble and usually colourless leuco form that readily impregnates fibres and textiles. In addition, they are also used as laxatives and possess antifungal and antiviral activities. It is also used as a seed dressing or in seed treatments. Other major uses are as a pesticide, as a bird repellent (especially for geese), and as an additive in chemical alkaline pulp processes in the paper and pulp industry.

So far, 79 naturally occurring anthraquinones have been identified which include emodin, physcion, cascarin, catenarin and rhein. A large body of literature has demonstrated that the naturally occurring anthraquinones possess a broad spectrum of bioactivities, such as cathartic, anticancer, anti-inflammatory, antimicrobial, diuretic, Vaso relaxing and phytoestrogen activities, suggesting their possible clinical application in many diseases.

In this paper I compute Energy, Randic energy, Colour energy, Laplacian energy, Sign less Laplacian energy, Normalized Laplacian energy of Anthraquinone.

2. Structural and Molecular formulae

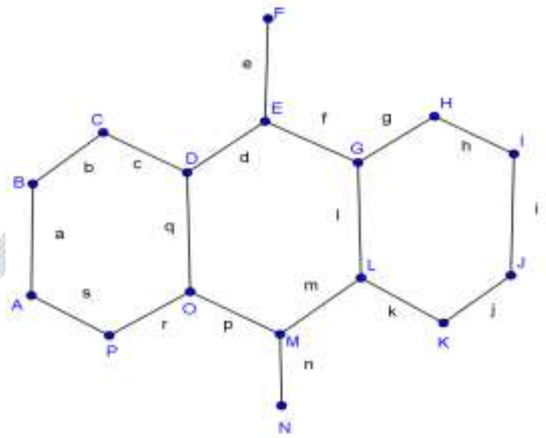


Molecular formulae: $C_{14}H_8O_2$

3. Energy of a Graph

The energy of a graph is one of the emerging concepts within graph theory. This concept serves as a frontier between chemistry and mathematics and is defined in1978 by I. Gutman [1] and originating from theoretical chemistry. In this paper we consider all graphs are simple without loops and multiple edges, finite and undirected. For standard terminology and notations related to graph theory, we follow Balakrishnan and Ranganathan [2]. The energy of a graph is zero if and only if it is trivial. The energy of a graph is one of the emerging concepts within graph theory. This concept serves as a frontier between chemistry and mathematics [3].

Let us consider the graph of Anthraquinone (i.e., Graph G) as shown in the following fig.



Graph G: Graph of Anthraquinone

Here the vertices A, B, C, D, ...M, N, O, P are treated as the vertices $v_1, v_2, ...v_{13}, v_{14}, v_{15}, v_{16}$.

In general, G be a graph possessing n vertices and m edges. Let $v_1, v_2, ...v_n$ be the vertices of G. Then the adjacency matrix $A(G)$ of the graph G is the square matrix of order n whose (i, j) entry is defined as

$$a_{ij} = \begin{cases} 1 & if \quad i \neq j \quad and\ v_i\ and\ v_j\ are\ adjacent \\ 0 & if \quad i \neq j \quad and\ v_i\ and v_j\ are\ not\ adjacent \\ 0 & if \quad i = j \end{cases}$$

The eigenvalues $\lambda_1, \lambda_2, \lambda_3, ...\lambda_n$ of the graph G are the eigen values of its adjacency matrix. Since $A(G)$ is real symmetric, the eigen values of G are real numbers whose sum equal to zero.

The energy of a graph G is the sum of absolute values of the eigen values of a graph G and denoted it by $E(G)$. Hence

$$E(G) = \sum_{i=1}^{n} |\lambda_i|$$

$E(G)$ will be referred as the ordinary energy of the graph G.

Energy of the Anthraquinone:

Adjacency matrix of Anthraquinone is shown in matrix 1.

The Characteristic equation is

$$x^{16} - 18x^{14} + 127x^{12} - 456x^{10} + 903x^8 - 998x^6 + 593x^4 - 168x^2 + 16 = 0$$

The Eigen values of above characteristic equation are

$\lambda_1 = -2.49889, \lambda_2 = 2.49889, \lambda_3 = -2, \lambda_4 = 2, \lambda_5 = -1.6624, \lambda_6 = 1.6624, \lambda_7 = -1.49592, \lambda_8 = 1.49592\ \lambda_9 = -1, \lambda_{10} = 1, \lambda_{11} = -1, \lambda_{12} = 1,$
$\lambda_{13} = -0.757366, \lambda_{14} = 0.757366, \lambda_{15} = -0.424945, \lambda_{16} = 0.424945.$

The Energy of Anthraquinone is

$$\varepsilon(C_{14}H_8O_2)= |-2.49889|+|2.49889|+|-2|+|2|+|-1.6624|+|1.6624|+|-1.49592|+|1.49592|+|-1|+|1|+|-1|$$
$$+|1|+|-0.757366|+|0.757366|+|-0.424945|+|0.424945|$$
$$= 21.679042$$

4. Randic energy of a graph:

In 1975 Milan Randic [4] invented a molecular structure descriptor defined as

$$R = R(G) = \sum_{v_i v_j \in E(G)} \frac{1}{\sqrt{d_i d_j}}$$

Where the summation goes over all pairs of adjacent vertices of the underlying(molecular) graph. This graph invariant is nowadays known under the name Radic index.

Let G be graph of order n with vertex set $V=\{v_1, v_2, \ldots\ldots, v_n\}$ and edge set E. Randic matrix of G is a $n \times n$ symmetric matrix defined by $R(G)$ whose (i,j)th entry is defined as

$$r_{ij} = \begin{cases} \frac{1}{\sqrt{d_i d_j}} & \text{if } v_i \text{ and } v_j \text{ are adjacent} \\ 0 & \text{if } v_i \text{ and } v_j \text{ are not adjacent} \\ 0 & \text{if } v_i = v_j \end{cases}$$

Denote the eigenvalues of the Randic matrix $R = R(G)$ by $\lambda_1, \lambda_2, \ldots\ldots \lambda_n$ and label them in non-increasing order. The multi set $S_{PR} = S_{PR}(G) = \{\lambda_1, \lambda_2, \ldots\ldots\ldots\ldots \lambda_n\}$ will be called the R-spectrum of the graph G. In addition, $\phi_R(G,\lambda) = \det(\lambda I_n - R)$ will be referred to as the R-characteristic polynomial of G. Here I_n is denoted the unit matrix of order n. The Randic energy of G is defined as $RE = RE(G) = \sum_{i=1}^{n} |\lambda_i|$.

Randic Energy of Anthraquinone:

Randic matrix of Anthraquinone is as shown in matrix 2.

The Characteristic equation is

$$x^{16} - \frac{7}{2}x^{14} + \frac{2137}{432}x^{12} - \frac{85151}{23328}x^{10} + \frac{284041}{186624}x^8 - \frac{33965}{93312}x^4 - \frac{77}{23328}x^2 + \frac{1}{11664} = 0$$

The Eigen values are

$\lambda_1 = -1, \lambda_2 = -0.906264, \lambda_3 = -0.740627, \lambda_4 = -0.72613, \lambda_5 = -0.466855, \lambda_6 = -0.406376, \lambda_7 = -0.393924, \lambda_8 = -0.254206$
$\lambda_9 = 0.254206, \lambda_{10} = 0.393924, \lambda_{11} = 0.406376, \lambda_{12} = 0.466855, \lambda_{13} = 0.72613, \lambda_{14} = 0.740627, \lambda_{15} = 0.906264, \lambda_{16} = 1$.

The Randic Energy of Anthraquinone is

$$RE(C_{14}H_8O_2) = |-1| + |-0.906264| + |-0.740627| + |-0.72613| + |-0.466855| + |-0.406376| + |-0.393924|$$
$$+|-0.254206| + |0.254206| + |0.393924| + |0.406376| + |0.466855| + |0.72613| + |0.740627|$$
$$+|0.906264| + |1| = 9.788764$$

5. Colour energy of a graph:

In 2013, Chandrashekar Adiga, E. Sampath Kumar, M.A. Sriraj and A. S. Shrikanth [5] have studied the energy of the coloured graph and complement of coloured graph. A colouring of graph G is colouring of its vertices such that no two adjacent vertices receive the same colour.

Let G be a simple graph of order n with vertex set $V=\{v_1,v_2,.......,v_n\}$ and edge set E. The colour matrix of G is the $n\times n$ matrix defined by $A_c(G)=(a_{ij})$,

where

$$a_{ij}=\begin{cases}1 & \text{if } v_i \text{ and } v_j \text{ are adjacent with } c(v_i)\neq c(v_j)\\ -1 & \text{if } v_i \text{ and } v_j \text{ are non-adjacent with } c(v_i)=c(v_j)\\ 0 & \text{otherwise}\end{cases}$$

The characteristic polynomial of $A_c(G)$ is denoted by $f_n(G,\lambda)=\det(\lambda I-A_c(G))$. If the colour used is minimum then the adjacency matrix is denoted by $A_\chi(G)$. The eigen values of the graph G are the eigen values of $A_c(G)$. Since $A_c(G)$ is real and symmetric, its eigenvalues are real numbers and we label them in non-increasing order $\lambda_1\geq\lambda_2\geq\lambda_3\geq.......\geq\lambda_n$. The colour energy of G is defined as $E_c(G)=\sum_{i=1}^{n}|\lambda_i|$. If the colour used is minimum then the energy is called chromatic energy and it is denoted by $E_\chi(G)$.

If a graph G is coloured with minimum number of colours χ, then $E_\chi(G)$ is the colour energy of G, $A_\chi(G)$ is the colour matrix, $P_\chi(G,\lambda)$ is the characteristic polynomial and $Spec_\chi(G)$ is the spectrum of the graph G.

Colour Energy of Anthraquinone:

Colour matrix of Anthraquinone is as shown in matrix 3.

The Characteristic equation is

$$x^{16}-74x^{14}+276x^{13}+681x^{12}-6148x^{11}+12980x^{10}-4560x^{9}-19160x^{8}+21544x^{7}$$
$$+4140x^{6}-13536x^{5}+1888x^{4}+2304x^{3}-576x^{2}=0$$

Eigen values are

$\lambda_1=-9.29423$, $\lambda_2=-4.82288$, $\lambda_3=-1$, $\lambda_4=-0.654449$, $\lambda_5=-0.577302$, $\lambda_6=0$, $\lambda_7=0$, $\lambda_8=0.311346$ $\lambda_9=0.487678$, $\lambda_{10}=1.56485$, $\lambda_{11}=1.72442$, $\lambda_{12}=2$, $\lambda_{13}=2$, $\lambda_{14}=2.60113$, $\lambda_{15}=2.65943$, $\lambda_{16}=3$.

The Colour Energy of Anthraquinone is

$$E_c(C_{14}H_8O_2)=|-9.29423|+|-4.82288|+|-1|+|-0.654449|+|-0.577302|+|0|+|0|+|0.311346|+|0.487678|$$
$$+|1.56485|+|1.72442|+|2|+|2|+|2.60113|+|2.65943|+|3|$$
$$=32.697715$$

6. Laplacian Energy:

Laplacian matrix $L=L(G)$ of (n,m) graph is defined via its matrix elements as

$$l_{ij}=\begin{cases}-1 & \text{if } i\neq j, v_i \,\&\, v_j \text{ are adjacent}\\ 0 & \text{if } i\neq j, v_i \,\&\, v_j \text{ are not adjacent}\\ d_i & \text{if } i=j\end{cases}$$

Where d_i is the degree of the i^{th} vertex of G.

To find the eigen values, we can solve for λ, $\det(\lambda I-L(G))=0$. The eigen values of $L(G)$ are denoted by $\lambda_1,\lambda_2,........\lambda_n$

Laplacian energy [6] can be represented as $LE = LE(G) = \sum_{i=1}^{n} \left| \lambda_i - \frac{2m}{n} \right|$

Laplacian Energy of Anthraquinone:

Laplacian matrix of Anthraquinone is as shown in matrix 4.

The Characteristic equation is

$$x^{16} - 36x^{15} + 586x^{14} - 5708x^{13} + 37109x^{12} - 170048x^{11} + 565450x^{10} - 1383964x^{9} + 2502638x^{8}$$
$$-3326648x^{7} + 3204952x^{6} - 2182288x^{5} + 1007925x^{4} - 294968x^{3} + 48264x^{2} - 3264x = 0$$

Eigen values are

$\lambda_1 = 0$, $\lambda_2 = 0.198062$, $\lambda_3 = 0.447358$, $\lambda_4 = 0.491519$, $\lambda_5 = 1.10477$, $\lambda_6 = 1.12061$, $\lambda_7 = 1.32036$, $\lambda_8 = 1.55496$ $\lambda_9 = 2.54624$, $\lambda_{10} = 2.82578$, $\lambda_{11} = 3.24698$, $\lambda_{12} = 3.3473$, $\lambda_{13} = 3.5916$, $\lambda_{14} = 4.36234$, $\lambda_{15} = 4.53209$, $\lambda_{16} = 5.31003$.

The Laplacian Energy of Anthraquinone is

$$LE(G) = \sum_{i=1}^{n} \left| \lambda_i - \frac{2m}{n} \right| = 23.524721$$

7. Sign less Laplacian Energy:

The Sign less Laplacian matrix $L^+ = L^+(G)$ of (n,m) graph is defined via its matrix elements as

$$l_{ij}^{+} = \begin{cases} +1 & if\ i \neq j, v_i \,\&\, v_j \text{ are adjacent} \\ 0 & if\ i \neq j, v_i \,\&\, v_j \text{ are not adjacent} \\ d_i & if\ i = j \end{cases}$$

Where d_i is the degree of the i^{th} vertex of G.

To find the eigen values, we can solve for λ, $\det(\lambda I - L^+(G)) = 0$. The eigen values of $L^+(G)$ are denoted by $\lambda_1, \lambda_2, \ldots\ldots\lambda_n$

Sign less Laplacian energy [9] can be represented as

$$LE^+ = LE^+(G) = \sum_{i=1}^{n} \left| \lambda_i - \frac{2m}{n} \right|$$

Sign less Laplacian Energy of Anthraquinone:

Sign less Laplacian matrix of Anthraquinone is as shown in matrix 5.

The Characteristic equation is

$$x^{16} - 36x^{15} + 586x^{14} - 5708x^{13} + 37109x^{12} - 170048x^{11} + 565450x^{10} - 1383964x^{9} + 2502638x^{8}$$
$$-3326648x^{7} + 3204952x^{6} - 2182288x^{5} + 1007925x^{4} - 294968x^{3} + 48264x^{2} - 3264x = 0$$

Eigen values are

$\lambda_1 = 0$, $\lambda_2 = 0.198062$, $\lambda_3 = 0.447358$, $\lambda_4 = 0.491519$, $\lambda_5 = 1.10477$, $\lambda_6 = 1.12061$, $\lambda_7 = 1.32036$, $\lambda_8 = 1.55496$ $\lambda_9 = 2.54624$, $\lambda_{10} = 2.82578$, $\lambda_{11} = 3.24698$, $\lambda_{12} = 3.3473$, $\lambda_{13} = 3.5916$, $\lambda_{14} = 4.36234$, $\lambda_{15} = 4.53209$, $\lambda_{16} = 5.31003$.

The Sign less Laplacian Energy of Anthraquinone is

$$LE^+(G) = \sum_{i=1}^{n} \left| \lambda_i^+ - \frac{2m}{n} \right| = 23.524721$$

The oriented incidence energy of G, $OIE(G)=E(Q)=\sum_{i=1}^{n}\sqrt{\lambda_i}$ = 21.64588

8. The Normalized Laplacian energy:

The normalized Laplacian matrix of a graph *G*, denoted by $L_N(G)$, is defined to be the matrix with entries

$$L(x,y)=\begin{cases}1 & if \quad x=y \quad and \quad d_y\neq 0,\\ -\dfrac{1}{\sqrt{d_x d_y}} & if \quad x\ and\ y\ are\ adjacent\ in\ G,\\ 0 & otherwise\end{cases}$$

The normalized Laplacian matrix $L_N(G)$ of G is defined as

To find the eigen values, we can solve for λ, $\det(\lambda I-L_N{}^{+}(G))=0$. The eigen values of $L^{+}(G)$ are denoted by $\lambda_1,\lambda_2,........\lambda_n$

The normalized Laplacian energy [7] of G is defined as $L_N E(G)=\sum_{i=1}^{n}\left|\lambda_i(G)-1\right|$

The Normalized Laplacian energy of Anthraquinone:

Normalized Laplacian matrix of Anthraquinone is as shown in matrix 6.

The characteristic equation is

$$x^{16}-16x^{15}+\frac{2101}{18}x^{14}-\frac{4627}{9}x^{13}+\frac{659249}{432}x^{12}-\frac{116161}{36}x^{11}+\frac{38949331}{7776}x^{10}-\frac{22542887}{3888}x^{9}+$$
$$\frac{1565092967}{311040}x^{8}-\frac{126901847}{38880}x^{7}+\frac{121643851}{77760}x^{6}-\frac{4215475}{7776}x^{5}+\frac{8151991}{62208}x^{4}-\frac{1619963}{77760}x^{3}$$
$$+\frac{306049}{155520}x^{2}-\frac{7067}{77760}x+\frac{17}{12960}=0$$

Eigen values are

$\lambda_1=0.024609$, $\lambda_2=0.0937355$, $\lambda_3=0.268454$, $\lambda_4=0.384341$, $\lambda_5=0.533145$, $\lambda_6=0.604535$, $\lambda_7=0.606076$, $\lambda_8=0.752295$ $\lambda_9=1.2477$, $\lambda_{10}=1.39394$, $\lambda_{11}=1.39545$, $\lambda_{12}=1.46686$, $\lambda_{13}=1.61566$, $\lambda_{14}=1.73155$, $\lambda_{15}=1.90626$, $\lambda_{16}=1.97539$

The normalized Laplacian energy of G is defined as $L_N E(G)=\sum_{i=1}^{n}\left|\lambda_i(G)-1\right|$ = 9.46573

9. Sum connectivity energy:

The sum connectivity matrix $SC(G)=(S_{i\,j})_{n\times n}$ is defined as

$$S_{i\,j}=\begin{cases}\dfrac{1}{\sqrt{d_i+d_j}} & if \quad v_i\ and\ v_j\ are\ adjacent\\ 0 & otherwise\end{cases}$$

The characteristic polynomial of $SC(G)$ is denoted by the sum connectivity matrix of G. The characteristic polynomial of $SC(G)$ is denoted by $\phi_{SC}(G,\lambda)=\det(\lambda I-SC(G))$. Since the sum connectivity matrix is real and symmetric, its eigenvalues are real numbers and we label them in non-increasing order $\lambda_1>\lambda_2>\lambda_3>........>\lambda_n$.

The sum connectivity energy [8] is given by $SC\,E(G)=\sum_{i=1}^{n}|\lambda_i|$

Sum connectivity energy of Anthraquinone:

The sum connectivity matrix is as shown in matrix 7.

The characteristic equation is

$$x^{16}-3.93333x^{14}+6.16583x^{12}-4.98751x^{10}+2.25609x^{8}-0.578358x^{6}+0.0811831x^{4}$$
$$-0.00557815x^{2}+0.000134584=0$$

The eigen values are

$\lambda_1=-1.11079$, $\lambda_2=-0.964388$, $\lambda_3=-0.759836$, $\lambda_4=-0.733511$, $\lambda_5=-0.501462$, $\lambda_6=-0.446735$, $\lambda_7=-0.392705$, $\lambda_8=-0.220867$
$\lambda_9=0.220867$, $\lambda_{10}=0.392705$, $\lambda_{11}=0.446735$, $\lambda_{12}=0.501462$, $\lambda_{13}=0.733511$, $\lambda_{14}=0.759836$, $\lambda_{15}=0.964388$, $\lambda_{16}=1.11079$

The sum connectivity energy is given by $SC\,E(G)=\sum_{i=1}^{n}|\lambda_i|$=10.260588

$$A(C_{14}H_8O_2)=\left[\begin{array}{ccccccccccccccccc}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16}\\
v_1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
v_2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
v_3 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
v_4 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
v_5 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
v_6 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
v_7 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
v_8 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
v_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
v_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
v_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0\\
v_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0\\
v_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0\\
v_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
v_{15} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1\\
v_{16} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]$$

Matrix 1: Adjacency Matrix of Anthraquinone

$$
R\left(C_{14}H_8O_2\right)=\begin{array}{cc}
 & \begin{array}{cccccccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \end{array} \\
\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \\ v_{16} \end{array} &
\begin{bmatrix}
0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\
\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 \\
0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & \frac{1}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{\sqrt{3}} & \frac{1}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{\sqrt{6}} \\
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0
\end{bmatrix}
\end{array}
$$

Matrix 2: Randic Matrix of Anthraquinone

$$A_c\left(C_{14}H_8O_2\right)=\begin{array}{cc} & \begin{array}{cccccccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \\ v_{16} \end{array} & \begin{bmatrix} 0 & 1 & -1 & 0 & -1 & 0 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & -1 & 1 \\ 1 & 0 & 1 & -1 & 0 & -1 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 1 & -1 & 0 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 1 & -1 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & 1 & -1 \\ -1 & 0 & -1 & 1 & 0 & 1 & 1 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & -1 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & -1 & 1 & -1 & 0 & 1 & -1 & 0 & -1 & 1 & -1 & 0 & 0 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 & -1 & 0 & -1 & 0 & -1 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 & -1 & 1 & 0 & 1 & -1 & 0 & -1 & 0 & 0 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 & 0 & -1 & 1 & 0 & 1 & -1 & 0 & -1 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 & -1 & 0 & -1 & 1 & 0 & 1 & -1 & 0 & 0 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 & 1 & -1 & 0 & -1 & 1 & 0 & 1 & -1 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 & -1 & 0 & -1 & 0 & -1 & 1 & 0 & 1 & 1 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 & 0 & -1 & 0 & -1 & 0 & -1 & 1 & 0 & -1 & 0 \\ -1 & 0 & -1 & 1 & -1 & 0 & 0 & -1 & 0 & -1 & 0 & -1 & 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 & 0 & -1 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Matrix 3: Colour Matrix of Anthraquinone

$$L\left(C_{14}H_8O_2\right)=\begin{array}{cc} & \begin{array}{cccccccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \\ v_{16} \end{array} & \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 3 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 3 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 3 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 3 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 3 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix} \end{array}$$

Matrix 4: Laplacian Matrix of Anthraquinone

$$L^{+}\left(C_{14}H_{8}O_{2}\right) \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \\ v_{16} \end{array} \begin{array}{c} \begin{array}{cccccccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \end{array} \\ \left[\begin{array}{cccccccccccccccc}
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 3 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2
\end{array}\right] \end{array}$$

Matrix 5: Sign less Laplacian Matrix of Anthraquinone

$$
L_N\left(C_{14}H_8O_2\right) \quad
\begin{array}{cc}
 & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \end{matrix} \\
\begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \\ v_{16} \end{matrix} &
\begin{bmatrix}
1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
-\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 1 & -\frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{\sqrt{6}} & 1 & -\frac{1}{\sqrt{15}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{3} & 0 \\
0 & 0 & 0 & -\frac{1}{\sqrt{15}} & 1 & -\frac{1}{\sqrt{5}} & -\frac{1}{\sqrt{15}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{\sqrt{5}} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{\sqrt{15}} & 0 & 1 & -\frac{1}{\sqrt{6}} & 0 & 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{6}} & 1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 1 & -\frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{\sqrt{6}} & 1 & -\frac{1}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{3} & 1 & -\frac{1}{\sqrt{3}} & -\frac{1}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{3}} & 1 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{3} & 0 & 1 & -\frac{1}{\sqrt{6}} \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{6}} & 1
\end{bmatrix}
\end{array}
$$

Matrix 6: Normalized Laplacian Matrix of Anthraquinone

$$SC(C_{14}H_8O_2)=\begin{array}{c|cccccccccccccccc}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\
\hline
v_1 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\
v_2 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_3 & 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{5}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_4 & 0 & 0 & \frac{1}{\sqrt{5}} & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 \\
v_5 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & \frac{1}{2} & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_6 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_7 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 \\
v_8 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
v_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
v_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{5}} & 0 & 0 & 0 & 0 \\
v_{12} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & \frac{1}{\sqrt{5}} & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 \\
v_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & \frac{1}{2} & \frac{1}{\sqrt{6}} & 0 \\
v_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\
v_{15} & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & \frac{1}{\sqrt{5}} \\
v_{16} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{5}} & 0
\end{array}$$

Matrix 7: Sum Connectivity Matrix of Anthraquinone

10. Conclusion:

In this article, I compute Randic energy, Colour energy, Laplacian energy, Sign less Laplacian energy, Normalized Laplacian energy of Anthraquinone.

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