An Approach for Multilevel Programming problems

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Abstract : In Multilevel Programs the optimization is carried out in a hierarchical manner each division controlling its decision variables. In this article a solution method to solve a fuzzy multilevel system with linear constraints over a convex polyhedral set is developed. The advantage of this approach is that it finds the solution to the problem without the use of membership function making the technique simpler. The objective functions conflict with each other, a solution maximizing each of the objectives may not exist.Efficiency is considered as a solution concept. The objectives at each level are optimized and an algorithm is developed to solve Multilevel Programming Problem (MPP) which makes the algorithm non-complex and less time consuming .The algorithm finds solution y for a given value of variables controlled by the decision maker at previous level. The algorithm is demonstrated numerically with three decision makers at lower level.

Keywords: Multilevel programming, Decentralized Systems, Quadratic Programming, Ideal solution .

I. INTRODUCTION

In many situations there arise many planning problems that can be represented by a multilevel programming problems. In Multilevel Programs , a planner at a certain level of hierarchy may have his objective function and decision space determined by other levels under partial cooperation. The execution of decisions is sequential. The decision maker at each level maximizes its own benefits, but is affected by the decisions of decision makers at other levels through externalities. The upper level decision maker first sets his goals and then asks each subordinate level about their solutions which are computed in isolation. Multilevel Programming Problems are characterized by planner at a certain hierarchical level determining his/her own objective and constraint space by successive levels partially under cooperation. The decision maker at first level optimizes his/her decision first and for a given choice of the variables under its control, the lower level decision makers optimize their objectives.
Further, each of the decision makers can influence the other level decisions to achieve his own objectives.

Multilevel Programming has been drawing considerable attention from scientific community in recent years. Many problems need to be modelled as multilevel programs as a result of which new efficient methods have been evolved. During the past two decades of last century, several approaches for solving MLPPs have been deeply studied by Bard and Falk [2], Cao D. [4], Candler and Townsley [3].

Most developments concentrate on Linear Programs (LPP) as a class of MLPP. Anandalingam [1] focused on bilevel non-linear programs. Three-level programs were introduced along with their solution methods viz extreme-point search algorithm .

Many authors [4,5,6,7] have studied multilevel and bilevel programming problems. Ayhan and Kilic[2] proposed a two stage method method using fuzzy AHP to generate solutions to the problem considered. The main theme is that it does not need to construct membership functions as is done in most of fuzzy programming. It is made possible by the introduction of additional constraints in the solution space using reduced ranges of the variables.

2. MATHEMATICAL FORMULATION OF PROBLEM

Let us consider the t-level multilevel system in which (L1) controlling the decision variables $\bar{x}_i = (x_1^1, x_1^2, ..., x_1^{n_1})$ with the lower following divisions controlling the decision variables $\bar{x}_j = (x_j^1, x_j^2, ..., x_j^{n_j})$, $1 \leq j \leq r$

The multilevel system, under consideration possesses a feasible set $X' \subset E^{n_1+n_2+...+n_t}$ for X_j , $1 \leq j \leq t$ as the solution space.

Let the L_i be the i- th decision maker ($1 \leq i \leq \tau$)

A maximization t-level Multilevel Decentralized Programming Problem (MPP) can be formulated as

(MPP) max Z_1 (X)

max Z_2 (X)

max (Z_{i1}(X) ,Z_{i2}(X)……,Z_{ie}(X))

:

: max Z_t (X)

subject to

$$A_{i1}\bar{X}_1 + A_{i2}\bar{X}_2 + ... + A_{it}\bar{X}_t \quad (\leq, =, \geq) \; b_i , \; 1 \leq i \leq r$$

$$X_1 \geq 0, X_2 \geq 0 ... X_t \geq 0$$

where Z $(1 \leq i \leq t)$ denotes the i-th level objective function, Z_1 being quadratic (quasiconcave), and $Z_i(2 \leq i \leq t)$ being linear fractional; the i-th level decision maker can have more than one decision maker ;

$\bar{X}_1 = (x_1^1, x_1^2, ...x_1^{n_1})^t$ are decision variables controlled by first level, $(x_2^1, x_2^2, ... x_2^{n_2})^t$ are decision variables controlled by (2), ...

$\underline{x}_2 = (x_2^1, x_2^2, \ldots x_2^{n_2})$

$X_t = (x_t^1, x_t^2, \ldots x_t^{n_t})$ are decision variables controlled by (t). where

$\underline{n} = \underline{n}_1 + \underline{n}_2 + \ldots + \underline{n}_r$

Let $X = X_1 \cup X_2 \cup \ldots \cup X_t$

While the process of decision making, each decision maker finds its best/ideal solution to arrive at a solution.

4.ALGORITHM

The algorithm summarized as follows:

Step 1:Find the optimal solution to each of the t-levels for the (MPP) having *m*-constraints and n- variables

Let $\overline{X_r}^0$, $r = 1, 2, \ldots t$ be their respective optimal solutions, where

$\overline{X}_r^0 = \{x_r^{10}, x_r^{20}, \ldots, x_r^{n_t 0}\}$

Step 2: Determine the ideal range making use of the r-optimal solutions determined in the initial step 1 and set it as the range of that variable

i.e. $x_r^1 \in (x^{1L}, x^{10})$;

$x_r^2 \in (x^{2L}, x^{20})$;

..

.

$x_r^{n_t} \in (x^{n_t L}, x^{n_t 0})$, $r = 1, 2, \ldots R$

Step 3: Obtain reduced ranges of each of the decision variables to achieve a se solution followed by successive lower level decision makers.

Step 4: Take t = 1

Let $\overline{x_r}^0 = x^0{}_1$ optimal solution of (1)

Step 5: One and only one of the following conditions hold

(a) $x^1{}_t{}^o \in (x^{1L}, x^{10})$;

(b) $x^{10}{}_t = x^{IL}$

(c) $x^{10}{}_t = x^{10}$

Step 6: For variables $x_r^{20}, x_r^{30} \ldots x_n^{n_r}$ under the control of r-th decision maker (Lr),follow step 5.

Step 7: Take the ranges for variable not controlled by (Lr) as constraints so as to shrink the size of feasible region.

Step 8 : For the lower levels, form 2*n* additional constraints along with the m original constraints.

STEP 9; Sett →t + 1. Read X_r^{1C} in place of X_r^{10} . Follow Step 5.

5. CONCLUSIONS

The algorithm proposed converges in a finite number of steps. The proposed approach is supposed to have a contribution in the study of the MLPPs in future. The author hopes that the solution approach proposed in this article will open up new possibilities of research for dealing with multilevel problems having multiple decision makers in the decision making arena.

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