INVERSE DOMINATION IN INTERVAL GRAPHS

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Abstract : A dominating set for a graph $G=(V,E)$ is a subset D of V such that every vertex not in D is adjacent to at least one member of D. The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set of G . In this paper we write an algorithm to find inverse dominating set of an interval family.

***Index Terms* – Inverse dominating set, Inverse dominating number, Interval graph.**

I. INTRODUCTION

The concept of Inverse dominating set(IDS) was introduced by Kulli and Domke[1]. A subset D' of vertex set V is said to be an inverse dominating set of G with respect to a minimum dominating set (MDS) D of a graph G if the induced sub graph $<V-D>$ contains a dominating set D' of G. The cardinality of a smallest inverse dominating set of G is called an inverse domination number $\gamma'(G)$ of G. From the definition of an inverse dominating number $\gamma'(G)$, for any graph without isolated vertex, we have $\gamma(G)+\gamma'(G)\leq n$, where n is number of vertices. Also, G.S. Domke [2] characterized the graphs which satisfy $\gamma(G)+\gamma'(G)=n$. Many other inverse domination parameters in domination theory were studied, for example[3,4,5,6].

Let J = {j_1, j_2, ……, j_n } be the interval family. Each interval j in J is represented by [a_j, b_j] for j = 1,2,…..,n. Here a_j and b_j are called left end point and right end points of the intervals in J_j. Here we are assuming that all end points of the intervals in J which are distinct between 1 and 2n. Here intervals are labelled in the increasing order of their right end points. If there is a 1-to-1 correspondence between vertex set V and Interval family J such that two vertices of G are joined by an edge in edge set E if and only if their corresponding intervals in J intersect then the graph $G=(V,E)$ is called an interval graph.

Let MDS = Minimum dominating set,
IDS IDS = Inverse dominating set,
$Nhd[i]$ = The set of all intersecting intervals of an interval i,
$Nhd^{+}[i]$ = The set of all right end side intersecting intervals of an interval i,
$NIS(i)$ = First non-intersecting interval of an interval i and
$Max(A)$ = The largest number in a set A.

An Algorithm to find IDS with respect to MDS.
Input : Interval family J.
Out put : An inverse dominating set with respect to a minimum dominating set.

Step 1 : $MDS=\{\ \}$
Step 2 : $IDS=\{\ \}$
Step 3 : $x=1$
Step 4 : $b=0$
Step 5 : Find $Nhd^{+}[x]$
Step 6 : $S=\{y\,/\,y$ belongs to $Nhd^{+}[x]$ and y is an interval which intersets all other intervals which are in $Nhd^{+}[x]\}$
Step 7 : If there exists an interval 'a' in S such that $|Nhd(a)|$ is maximum among all the intervals of S then
  Step 7.1 : $MDS=MDS\cup\{a\}$
  Else
  Step 7.2 : $a=Max(S)$
  Step 7.3 : $MDS=MDS\cup\{a\}$
Step 8 : Find $S_1=S-\{a$ and the set of intervals which belongs to S and which are greater than $a\}$
Step 9 : If S_1 is null set then

Step 9.1 : If $NIS(b)$ exists then

Step 9.1.1 : $x = NIS(b)$

Step 9.1.2 : $S_2 = \{y / y$ belongs to $Nhd^+[x]$ and y is an interval which intersects all other intervals which are in $Nhd^+[x]$ and y does not belongs to $MDS\}$

Step 9.1.3 : If there exists an interval b in S_2 such that $|Nhd(b)|$ is maximum among all the intervals of S_2

Step 9.1.3.1 : $b = Max(S_2)$

Step 9.1.3.2 : $IDS = IDS \cup \{b\}$ goto step 12

Step 10 : If S_1 is not null set then

Step 10.1 : If b is not equal to 0 then

Step 10.1.1 : $S_{new} = S \cup NIS(b)$

Step 10.1.2 : If $|S_{new}| > |S_1|$ then

Step 10.1.2.1 : $b =$ First intersecting interval on right end side to $NIS(b)$

Else

Step 10.1.2.2 : $b = Max(S_1)$

Step 11: $IDS = IDS \cup \{b\}$ goto step 12

Step 12: If $NIS(a)$ exists then

Step 12.1: $x = NIS(a)$ and goto step 5

Else

Step 13: If $NIS(b)$ exists then

Step 13.1: $x = NIS(b)$

Step 13.2: $S_2 = \{y / y$ belongs to $Nhd^+[x]$ and y is an interval which intersects all other intervals which are in $Nhd^+[x]\}$

Step 13.3: If there exists an interval b in S_2 such that $|Nhd(b)|$ is maximum among all the intervals of S_2 then

Step 13.3.1: $b = Max(S_2)$

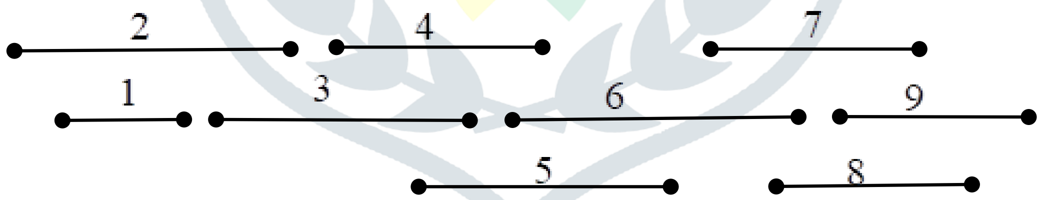
Step 13.3.2: $IDS = IDS \cup \{b\}$ goto step 5

Else go to Step 14

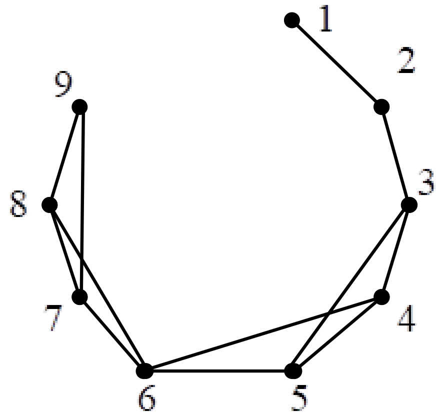
Step 14: End

5 6 7 8 9

Example: Given interval family is



Interval Family



Interval Graph

$Step\ 1: MDS = \{\}$

$Step\ 2: IDS = \{\}$

$Step\ 3: x = 1$

$Step\ 4: b = 0$

$Step\ 5: Nhd^{+}[1] = \{1,2\}$

$Step\ 6: S = \{1,2\}$

$Step\ 7: a = 2$

$MDS = \{2\}$

$Step\ 8: S_1 = \{1\}$

$Step\ 10: b = 1$

$Step\ 11: IDS = \{1\}$

$Step\ 12: x = 4$

$Step\ 5: Nhd^{+}[4] = \{4,5,6\}$

$Step\ 6: S = \{4,5,6\}$

$Step\ 7: a = 6$

$MDS = \{2,6\}$

$Step\ 8: S_1 = \{4,5\}$

$Step\ 10: b = 4$

$Step\ 11: IDS = \{1,4\}$

$Step\ 12: x = 9$

$Step\ 5: Nhd^{+}[9] = \{9\}$

$Step\ 6: S = \{9\}$

$Step\ 7: a = 9$

$MDS = \{2,6,9\}$

$Step\ 8: S_1 = \{\}$

$Step\ 9: x = 7$

$S_2 = \{7,8\}$

$b = 8$

$Step\ 11: IDS = \{1,4,8\}$

$NIS(9) = null$ and $NIS(7) = null$ then

Step 14 : End

Therefore $IDS = \{1,4,8\}$ is an inverse dominating set with respect to a minimum dominating set $MDS = \{2,6,9\}$

From the above example, $\gamma(G) = 3$

$\gamma'(G) = 3$ and n = 9

If graph has no isolated vertex $\gamma(G)+\gamma^{'}(G)\leq n$

$$3+3\leq 9$$

$$6\leq 9$$

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