Satellites: Linearised and Normalised Differential equations of Relative motion of the system

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Abstract

This paper deals with the Linearised and Normalised differential equations of relative motion of the system. We emphasize on comparative study of the Normalised and linearised vector equation of relative motion of the particle of mass m_1 with respect to the center of mass of the system.

Key words: System; Linearised; Normalise; Airdragness, Relative motion.

Introduction:

The effect of air resistance, magnetic force and oblateness of the earth on the motion of the satellites connected by a light, flexible and extensible cable in the central gravitation field of earth. The Lagrange's equations of motion of the system under the influence of air resistance, magnetic force and oblateness of' the earth. The normalized and linearised differential equations of the mass of the particle m_1 of the system have been obtained on the assumption that the length of connecting cable is very small compared to the distance of the satellites from the colure of the earth. Suppose two satellites of the system as the particles of mass m_1 and m_2 having their radius vectors $\vec{r}_1$ and $\vec{r}_2$ respectively.

Let l_0 the length of the string connecting the two particles of mass m_1 and m_2 suppose 'l'be the length of the string at any time.

The constraint of the system

$$\left|\vec{r}_1-\vec{r}_2\right|^2 \le l_0^{\,2} \qquad \text{.........}(1)$$

Mathematical Approach:

By using Lagrange's equation of motion of first kind the equation of motion of the two particles of mass m_1 and m_2 connected by an extensible string of natural length l_0. The influence of air resistance, magnetic force and oblateness of the earth in the form

$$m_1\ddot{\vec{r}}_1+\frac{m_1\mu\vec{r}_1}{r_1^{3}}+\frac{3m_1\mu k_2}{r_1^{5}}\vec{r}_1+\lambda\left[\frac{\left|\vec{r}_1-\vec{r}_2\right|-l_0}{l_0}\right]\frac{\left(\vec{r}_1-\vec{r}_2\right)}{\left|\vec{r}_1-\vec{r}_2\right|}=\vec{F}_{a_1}+Q_1\left(\vec{r}_1\times\vec{H}\right)$$

and

$$m_2 \ddot{\vec{r}}_2 + \frac{m_2 \mu \vec{r}_2}{r_2^3} + \frac{3 m_2 \mu k_2 \vec{r}_2}{r_2^5} - \lambda \left[\frac{\left|\vec{r}_1 - \vec{r}_2\right| - l_0}{l_0} \right] \frac{\left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|} = \vec{F}_{a_2} + Q_2\left(\vec{r}_2 \times \vec{H}\right)$$

Where $\vec{F}_{ai}$ (i = 1, 2, …) is the aerodynamic force

$$k_2 = \bar{\epsilon} R_e^2 / 3$$

$$R = \frac{R_e - R_P}{R_e} = \text{Earth's of oblatness}$$

We come to know that

$$\left.\begin{aligned} \vec{\rho}_1 &= \frac{m_2}{m_1 + m_2}\left(\vec{r}_1 - \vec{r}_2\right) \\ \vec{\rho}_2 &= \frac{m_1}{m_1 + m_2}\left(\vec{r}_2 - \vec{r}_1\right) \end{aligned}\right\} \qquad \text{……… (2)}$$

Now

$$m_1 \vec{\rho}_1 + m_2 \vec{\rho}_2 = 0$$

By neglecting the 2nd and higher order perturbation terms and the equation of the centre of mass of the system as

$$M\ddot{\vec{r}} + \frac{\mu M \vec{R}}{R^3} = 0 \qquad \text{……… (3)}$$

The centre of mass of the system can be assumed to move along a keplerian elliptical orbit with the higher degree of accuracy up to and order infinitesimal such as

$$\frac{\rho_1}{R} \text{ and } \frac{\rho_2}{R}.$$

In this way, we say that the centre of mass of the system of the two satellites connected by an extensible string in the central gravitation field of two satellites connected by an extensible string in the central gravi tational field of attraction moves along a given keplerian elliptical orbit.

$$\bar{\epsilon} = \alpha_R - \frac{m}{2}$$

where $m = \frac{\Omega^2 R_e}{g_e}$

Ω = Angular velocity of the earth's rotation

R_e = Equatorial radius of the earth

R_p = polar radius of the earth

g_e = Force of gravity at the equation of the earth

$$Q_i = \frac{charge \quad q_i \quad onthe \quad i^{th} \quad particle}{Velocity \quad of \quad light \quad C}; \text{ (i = 1, 2)}$$

$\vec{F}$ = Intensity of the earth's magnetic field for equatorial satellites

$$= \frac{-\vec{\nabla}\left(\vec{m}.\vec{r}\right)}{r^3}$$

$\vec{m}$ = Magnetic moment of the earth.

Suppose it is assumed that air drag varies as the square of the velocity of the moving particles.

$$\vec{F}_{a_i} = -\rho_a C_i \left|\dot{\vec{r}}_0\right| \dot{\vec{r}}_i$$

$\rho_a =$ density of the air

$C_i =$ Ballistic Coeff^H

Now,

$$m_1\ddot{\vec{r}}_1 + \frac{m_1\mu\vec{r}_1}{r_1^3} + \frac{3\mu m_1 k_2 \vec{r}_1}{r_1^5} + \lambda\left[\frac{\left|\vec{r}_1 - \vec{r}_2\right| - l_0}{l_0}\right]\frac{(r_1 - r_2)}{\left|\vec{r}_1 - \vec{r}_2\right|} = -\rho_a C_1\left|\dot{\vec{r}}_1\right|\dot{\vec{r}}_1 + Q_1\left(\dot{\vec{r}}_1 \times H\right)$$

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$$m_2\ddot{\vec{r}}_2 + \frac{m_2\mu\vec{r}_2}{r_2^3} + \frac{3m_3\mu k_2 \vec{r}_2}{r_2^5} - \lambda\left[\frac{\left|\vec{r}_1 - \vec{r}_2\right| - l_0}{l_0}\right]\frac{\left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|} = -\rho_0 C_2\left|\dot{\vec{r}}_2\right|\dot{\vec{r}}_2 + Q_2\left(\vec{r}_2 \times \vec{H}\right) \quad \text{......... (4)}$$

We emphasize that the Linearised and Normalised differential Equation of Relative motion of the system.

Now,

$$\ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 + \mu\left(\frac{\vec{r}_1}{r_1^3} - \frac{\vec{r}_2}{r_2^3}\right) + \lambda\frac{(m_1 + m_2)}{m_1 m_2}\left[\frac{\left|\vec{r}_1 - \vec{r}_2\right| - l_0}{l_0}\right]$$

$$\frac{(\vec{r}_1 - \vec{r}_2)}{\left|\vec{r}_1 - \vec{r}_2\right|} + 3\mu k_2\left(\frac{\vec{r}_1}{r_1^5} - \frac{\vec{r}_2}{r_2^5}\right) + \rho_a\left[C_1\left|\dot{\vec{r}}_1\right|\dot{\vec{r}}_1 - C_2\left|\ddot{\vec{r}}_2\right|\dot{\vec{r}}_2\right] = \frac{Q_1}{m_1}\left(\dot{\vec{r}}_1 \times \vec{H}\right) - \frac{Q_2}{m_2}\left(\dot{\vec{r}}_2 \times \vec{H}\right) \quad \text{......... (5)}$$

$$u\left(\frac{\vec{r}_1}{r_1^3} - \frac{\vec{r}_2}{r_2^3}\right) = \frac{\mu}{R^3}(\vec{r}_1 - \vec{r}_2) - \frac{3\mu\vec{R}}{R^5}\left[\vec{R}.(\vec{r}_1 - \vec{r}_2)\right]$$

$$3\mu k_2\left(\frac{\vec{r}_1}{r_1^5} - \frac{\vec{r}_2}{r_2^5}\right) = \frac{3\mu k_2}{R^5}(\vec{r}_1 - \vec{r}_2) - \frac{15\mu k_2}{R^7}\left[\vec{R}(\vec{r}_1 - \vec{r}_2)\right]\vec{R} \quad \text{......... (6)}$$

and $$\rho_a\left[C_1\left|\dot{\vec{r}}_1\right|\dot{\vec{r}}_1 - C_2\left|\dot{\vec{r}}_2\right|\dot{\vec{r}}_2\right] = \rho_a \dot{R}\dot{\vec{R}}(C_1 - C_2) + \rho_a R$$

$$\left[\frac{\dot{\vec{R}}(\dot{\vec{R}}.\vec{\rho}_1}{R^2} + \dot{\vec{\rho}}\right]\frac{(C_1 m_2 + C_2 m_1)}{m_2} \quad \text{......... (7)}$$

$$\vec{H} = -\frac{\vec{\nabla}\vec{M}.\vec{r}_1}{r_1^{\,3}} \text{ for (i = 1,2)}$$

$$\ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 + \frac{\mu}{R^3}(\vec{r}_1 - \vec{r}_2) - \frac{3\mu\vec{R}}{R^5}\left[\vec{R}.(\vec{r}_1 - \vec{r}_2)\right]$$

$$+\lambda\left(\frac{m_1 + m_2}{m_1 \times m_2}\right)\left[\frac{|\vec{r}_1 - \vec{r}_2| - l_0}{l_0}\right]\frac{(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$$

$$+\frac{3\mu\kappa_2}{R^5}(\vec{r}_1 - \vec{r}_2) - \frac{15\mu k_2}{R^7}\left\{\vec{R}(\vec{r}_1 - \vec{r}_2)\right\}\vec{R}$$

$$+\rho_a \dot{R}\dot{\vec{R}}(C_1 - C_2) + \rho a \dot{R}\frac{\left[\dot{\vec{R}}\left(\dot{\vec{R}}.\dot{\vec{\rho}}_1\right)\right]}{R^2} + \vec{\rho}$$

$$\frac{c_1 m_2 + c_2 m_1}{m_2} = -\left[\frac{Q_1}{m_1}\left\{\dot{\vec{r}}_1 \times \vec{\nabla} \times \nabla\frac{\vec{M}.\vec{r}_1}{r_1^{\,2}} - \frac{Q_2}{m_2}\left\{\dot{\vec{r}}_2 \times \vec{\nabla}\left(\frac{\vec{m}\vec{r}_2}{r_2^{\,2}}\right)\right\}\right\}\right] \quad \text{……… (8)}$$

But, $$\vec{r}_1 - \vec{r}_2 = \vec{\rho}_1 - \vec{\rho}_2 = \left(\frac{m_1 + m_2}{m_2}\right)\vec{\rho} \quad \text{……… (9)}$$

Using (8) in (9) we get on dividing throughout by $\frac{m_1 + m_2}{m_2}$

$$\ddot{\vec{\rho}} + \frac{\mu\vec{\rho}_1}{R^3} - \frac{3\mu\vec{R}\left(\vec{R}.\vec{\rho}_1\right)}{R^5} + \frac{3\mu\kappa_2\vec{\rho}_1}{R^5} -$$

$$\frac{15\mu k_2\vec{R}}{R^7}\left(\vec{R}.\rho_1\right) + \lambda_\alpha\left[1 - \frac{\upsilon}{|\vec{\rho}_1|}\right]\vec{\rho}_1$$

$$+\rho_a \dot{R}\dot{\vec{R}}(c_1 - c_2)\frac{m_2}{m_1 + m_2} + \rho a \dot{R}\left[\dot{\vec{R}}\frac{\left(\dot{\vec{R}}.\dot{\vec{\rho}}_1\right)}{R^2} + \vec{\rho}_1\right]\frac{c_1 m_2 + c_2 m_1}{m_1 + m_2}$$

$$= \frac{-m_2}{m_1 + m_2}\left[\frac{Q}{m_1}\dot{\vec{r}}_1 \times \vec{\tau}\left\{\frac{\vec{M}.\vec{r}_1}{r_1^{\,3}}\right) - \frac{Q_2}{m_2}\vec{r}_2 \times \vec{\tau}\left(\frac{\vec{M}.\vec{r}_2}{r_2^{\,3}}\right)\right] \quad \text{……… (10)}$$

Where $$\lambda_\alpha = \frac{m_1 + m_2}{m_1 m_2}\frac{\lambda}{l_0} \quad \text{……… (11)}$$

$$\gamma = \frac{m_2 l_0}{m_1 + m_2}$$

Since $$\frac{1}{r_i^3}=\frac{1}{\left(r_i^2\right)^3}=\frac{1}{\left[\left(\vec{R}+\vec{\rho}_2\right)^2\right]^{3/2}};i=1,\quad 2$$

$$=\frac{1}{R^3}-\frac{3\vec{R}.\vec{\rho}_i}{R^5}$$

$$\frac{\vec{r}_i}{r_i^3}=\left(\vec{R}+\vec{\rho}_i\right)\left[\frac{1}{R^3}-\frac{3\vec{R}.\vec{\rho}_1}{R^5}\right];i=1,2$$

$$=\frac{\vec{R}+\vec{\rho}_i}{R^3}-\frac{3\vec{R}.\vec{\rho}_i}{R^5}\left(\vec{R}+\vec{\rho}_i\right)i=1,2 \qquad \text{......... (12)}$$

Hence we have

$$\frac{Q_1}{m_1}\left\{\dot{\vec{r}}_1\times\vec{\nabla}\left(\frac{\vec{M}.\vec{r}_1}{r_1^3}\right)\right\}\frac{Q_2}{m_2}\left\{\dot{\vec{r}}_2\times\vec{\nabla}\left(\frac{\vec{m}.\vec{r}_2}{r_2^3}\right)\right\}$$

$$=\vec{R}\times\vec{\nabla}\left(\frac{\vec{M}.\vec{R}}{R^3}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right) \qquad \text{......... (13)}$$

We get the linearised vector equation of motion for the particle of mass m_1 relative to the centre of mass the system in the form

$$\frac{-m_2}{m_1+m_2}\left[\vec{R}\times\vec{\nabla}\left(\frac{\vec{M}.\vec{R}}{R^3}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\right] \qquad \text{......... (14)}$$

The condition of constraint reduces to

$$\left|\vec{\rho}_1\right|^2\le\upsilon^2 \qquad \text{......... (15)}$$

The inequality sign holds then the system will be moving with loose string.

The normalize the vector $\vec{\rho}_1$ by introducing.

$$\vec{\rho}_1=\frac{\upsilon\vec{\rho}_1*}{l_0} \qquad \text{......... (16)}$$

Then the vector equation of particle of mass m_1 takes the form

$$\ddot{\vec{\rho}}_1 * + \frac{\mu}{R^3}\vec{\rho}_1 * - \frac{3\mu\vec{R}(\vec{R}.\vec{\rho}_1 *)}{R^5} + a_1\dot{\vec{R}} + a_2\left[\frac{\dot{\vec{R}}(\vec{R}.\dot{\vec{\rho}}_1)}{R^2} + \dot{\vec{\rho}}_1 *\right]$$

$$+3\mu\kappa_2\vec{\rho}_1^{\,*} - \frac{15\mu\kappa_2\vec{R}}{R^7}\left(\vec{R}.\vec{\rho}_1^{\,*}\right) +$$

$$\lambda_\alpha\left[1 - \frac{l_0}{\left|\vec{\rho}_1^{\,*}\right|}\right]\vec{\rho}_1^{\,*} = -\vec{R}\times\vec{\nabla}\left(\frac{\vec{M}.\vec{R}}{R^3}\right)\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right) \qquad \text{......... (17)}$$

Where

$$a_1 = \rho a R(c_1 - c_2)$$

$$a_2 = \rho a \vec{R}\frac{(c_1 m_2 + c_2 m_1)}{m_2} \qquad \text{......... (18)}$$

The normalized vector equation of relative motion of the particle of mass m_1 with respect to the centre of mass of the system

Conclusion:

The atmospheric drag consists of two terms, one with coefficient and other with coefficient as the term with coefficient E.g. $\vec{\rho_1}*$ as a factor which indicates that this is the part of the atmospheric drag arising out of air friction. The coefficient of which is parameter of the aerodynamic force is a small quantity which is multiplied by small quantity $\vec{\rho_1}*$ and neglect this term as we are considering preservative force of first order only.

References:

[1] Elsgotts, L. : Differential equations and the calculus of variations, PP. 211-250, Mir Publishurs Moscow, 1973.

[2] Etkin, B. : Dynamics of gravity oriented orbiting systems with applications to passive stabilization. A.I.A.A. Journal, Vol. 2, No. 6 PP. 1008-104 June, 1964.

[3] Jha, M. : The effect of earth's oblateness on the motion of a system of two satellites connected by an extensible cable Ph. D. Thesis, submitted to Bihar University, Muzaffarpur, Oct. 1985.

[4] Kumar Kamlesh: Effect of magnetic force on the motion and stability of an extensible cable connected statellite system. Ph. D. thesis submitted to the University of Bihar, Muzaffarpur, 1988.

[5] Liapunov, A.M. : Sabrania Sachimeiviya Vol. 2, AN USSR, MOSCOW, 1959 (Russian)

[6] Mecuskey, S.W. : Introduction to celestial mechanics Addison Wesley Publishing Company, Inc. Reading Massachusetts, 1962.

[7] Sarichev, V.A. : Effect of the atmospheric resistance on a gravity stabilization system. Kosmicheskiye, ISS ledovania, Vol. 2 No. 1 PP. 23-32 (Russian)

[8] Sharma, B. : The motion of a system of two cable connected satellites in the atmosphere Ph.D. Thesis submitted to the University of Bihar, Muzaffarpur, No. 1974.

[9] Singh, R.B. & Demia, V.G. : About the motion of a heavy flexible string attached to the satellites in the central field of attraction, Celestial mechanics, An International Journal of space dynamics, No. 6, PP. 268-277, 1972.

[10] Singh R.B. : Three dimensional motion of two-connected bodies in the central gravitational field of force. Russia collections problem of guided motion in mechanics PP. 210-215, 1971 (Russian).

[11] Singh R.B. : The motion of two connected bodies in an elliptical orbit, Bulletin of the Moscow State University Mathematics-Mechanics, No. 3 PP. 82-86, 1973 (Russian).

[12] Singh, U.C. : Atmospheric drag effect on the non linear motion and stability of a cable connected satellites system in orbit Ph.D. thesis submitted to B.U. Muz. May 1979.

[13] Sharma R.B. : Effect of earth's oblatenes and atmospheric drag on the motion and stability of extensible cable connected satellite system in the orbit. Ph. D. Thesis submitted to university of Bihar, Muzaffarpur 1989.

[14] Zare, K. : The possible motion of satellite about an oblate planet. 'Celestial Mechanics' An international Journal of space dynamics, Vol, 30, No. 1 P. 49 May 1983.