LUCKY AND PROPER LUCKY LABELING FOR THE EXTENDED DUPLICATE GRAPH OF TRIANGULAR SNAKE

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Abstract: In this paper, we investigate the extended duplicate graph of triangular snake graph admits lucky labeling and proper lucky labeling.

1.Introduction

In 1967, Rosa[3] introduced the concept of graph labeling. The concept of lucky labeling was introduced by A.Ahai et al.,[5]. Lucky labeling is applied in real life situations such as transportation network, where pairwise connections are given some numerical values and each weight could represent the stations or city with certain expenses or costs etc. For a dynamic survey of various graph labeling we refer to J.A.Gallian [4]. The concept of proper lucky labeling was introduced by Kins Yenoke et al.,[6]. E.Sampthkumar[1] introduced the concept of duplicate graph. Thirusangu et al.,[2] have introduced the notion of extended duplicate graph .

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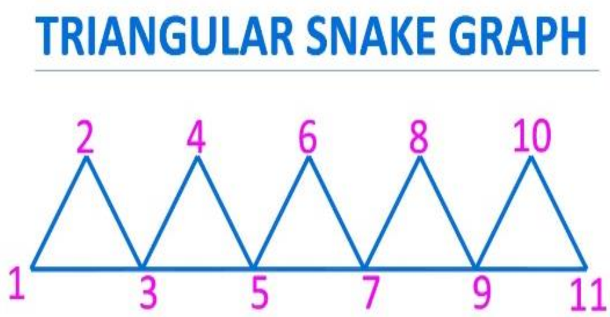
Key words : Duplicate graph, Extended duplicate graph, Triangular snake graph, Lucky labeling, Proper lucky labeling.

2. **Preliminaries**

The following definitions are useful for understanding its results of this research paper.

Definition :2.1 Triangular snake

A triangular snake TS_m is obtained from a path $u_1, u_2, \ldots, u_{m+1}$ by connecting u_i and u_{i+1} to a new vertex v_i, for $1 \leq i \leq m$, where 'm' is the number of edges of the path.



Definition:2.3 Duplicate graph

A Simple graph G with vertex set V and edge set E.The duplicate graph of G is DG = (V_1, E_1) where the vertex set V_1=VUV′ and V∩V′ = ϕ and f:V →V′ is bijective.The edge set E_1 of DG is defined as the edge ab∈E iff both edges ab' and $a'b$ are in E_1.

Definition:2.4 Extended duplicate graph of triangular snake

Let DG=(V_1, E_1) be a duplicate graph of the triangular snake graph G(V, E). Extended duplicate graph of triangular snake is attained by accumulating the edge v_2v_2' to the duplicate graph.This is indicated by EDG(TS_m).And it has 4m+2 vertices and 6m+1 edges,where 'm' is the number of edges of the path.

Definition : 2.5 Lucky labeling

Let f:V(G) →N be a labeling of the vertices of a graph by positive intergers. Let S(v) denote the sum of labels of the neighbours of the vertex v in G.If v is an isolated vertex of G.We put S(v)=0.A labeling is lucky if S(u)≠S(v) whenever u and v are adjacent. The least integer k for which a graph G has a lucky labeling from the set {1,2,3,...,k} is the lucky number of G denoted by η(G).

Definition:2.6 Proper lucky labeling

A lucky labeling is proper lucky labeling if the labeling f is proper as well as lucky, that is if u and v are adjacent in G then f(u)≠f(v) and S(u)≠S(v).The proper lucky number of G is denoted by η_p(G) is the least positive integer K such that G has a proper lucky labeling with {1,2,3,...,k} as the set of labels.

3. MAIN RESULTS

3.1: LUCKY LABELING FOR EDG(TS_m), m≥ 1

Here ,we present an algorithm and prove the existence of lucky labeling for EDG(TS_m). **Algorithm-1**

Procedure - [Lucky-labeling for EDG(TS_m) , m ≥ 1]

V ← {v_1,v_2, v_3… v_{2m},v_{2m+1}, v'_1,v'_2,v'_3…v'_{2m} ,v'_{2m+1}}

E ← {e_1, e_2, e_3 …e_{3m} , e_{3m+1},e'_1,e'_2,… e'_{3m} }

if m ≥ 1

v_1 ← 1, v'_1 ← 2, v_2 ← 1, v'_2 ←2, v_3← 2, v'_3← 1

end if

if m ≥ 2

v_4 ← 1, v'_4 ← 2, v_5 ← 2, v'_5 ← 1

end if

if m ≥ 3

for i = 0 to (m-3)/2 do

for j = 0 to 1

$v_{6+4i+j} \leftarrow 1$; $v'_{6+4i+j} \leftarrow 2$

end for

end for

end if

if $m \geq 4$

for i = 0 to (m-4)/2 do

$v_{8+4i} \leftarrow 2$; $v_{9+4i} \leftarrow 1$; $v'_{8+4i} \leftarrow 1$; $v'_{9+4i} \leftarrow 2$

end for

end if

end procedure

Theorem 1: The extended duplicate graph of triangular snake graph admits lucky labeling and its lucky number is 2.

Proof: Let TS_m be the triangular snake graph and $EDG(TS_m)$ be the extended duplicate graph of triangular snake graph.

Define the set of vertices and edges are

$V(G) = \{v_1, v_2, v_3 \ldots v_{2m}, v_{2m+1}, v'_1, v'_2, v'_3 \ldots v'_{2m}, v'_{2m+1}\}$

$E(G) = \{e_1, e_2, e_3 \ldots e_{3m}, e_{3m+1}, e'_1, e'_2, \ldots e'_{3m}\}$

Define the mapping $f: V(G) \rightarrow N$ such that the labeling is a lucky labeling if $S(u) \neq S(v)$, whenever u and v are adjacent in G and S(v) denote the sum of labels of the neighbors of the vertex v in G.

Using the algorithm 1,

if $m \geq 1$, the vertices v_1, v_2, v_3, v'_1, v'_2 and v'_3 receive the labels 1 , 1 , 2 , 2 , 2 and 1 respectively ;

if $m \geq 2$, the vertices v_4, v_5, v'_4 and v'_5 receive the labels 1 , 2 , 2 and 1 respectively ;

if $m \geq 3$, the vertices v_{6+4i+j} receive the labels 1 and the vertices v'_{6+4i+j} receive the labels 2 for $0 \leq i \leq (m-3)/2$ and $0 \leq j \leq 1$;

if $m \geq 4$, the vertices v_{8+4i} receive the labels 2 ; the vertices v_{9+4i} receive the labels 1 ; the vertices v'_{8+4i} receive the labels 1 and the vertices v'_{9+4i} receive the labels 2 for $0 \leq i \leq (m-4)/2$.

Thus all 4m+2 vertices are labeled.

Now to prove that $EDG(TS_m)$ is lucky labeling.

that is to prove $S(u) \neq S(v)$.

The sum neighborhood of the vertices are as follows:

If $m \geq 1$,

$S(v_1) \leftarrow 3$, $S(v'_1) \leftarrow 3$, $S(v_2) \leftarrow 5$, $S(v'_2) \leftarrow 4$, $S(v_3) \leftarrow 7$, $S(v'_3) \leftarrow 5$

If $m \geq 2$,

$S(v_4) \leftarrow 2$, $S(v'_4) \leftarrow 4$, $S(v_5) \leftarrow 7$, $S(v'_5) \leftarrow 5$

If $m \geq 3$,

$S(v_6) \leftarrow 3$, $S(v'_6) \leftarrow 3$, $S(v_7) \leftarrow 6$, $S(v'_7) \leftarrow 6$

If $m \geq 4$,

$S(v_{8+2i}) \leftarrow 4$, $S(v'_{8+2i}) \leftarrow 2$, $S(v_{9+2i}) \leftarrow 7$, $S(v'_{9+2i}) \leftarrow 5$ for $0 \leq i \leq (m-4)$

Clearly, we observe that $S(u) \neq S(v)$.

Hence, the extended duplicate graph of triangular snake graph admits lucky labeling and lucky number of $EDG(TS_m)$ is 2. i.e $\eta(G) = 2$.

Example 1: Lucky labeling diagram in $EDG(TS_m)$ is shown in figure (1) & (2)

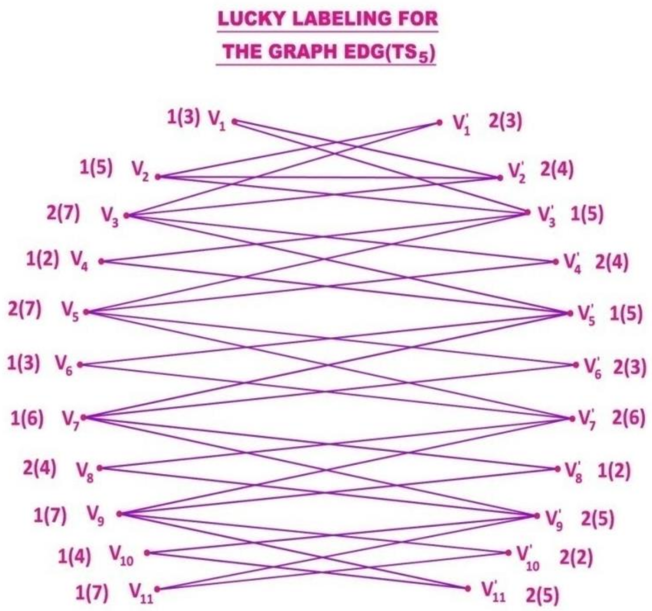


Fig 1: $EDG(TS_5)$

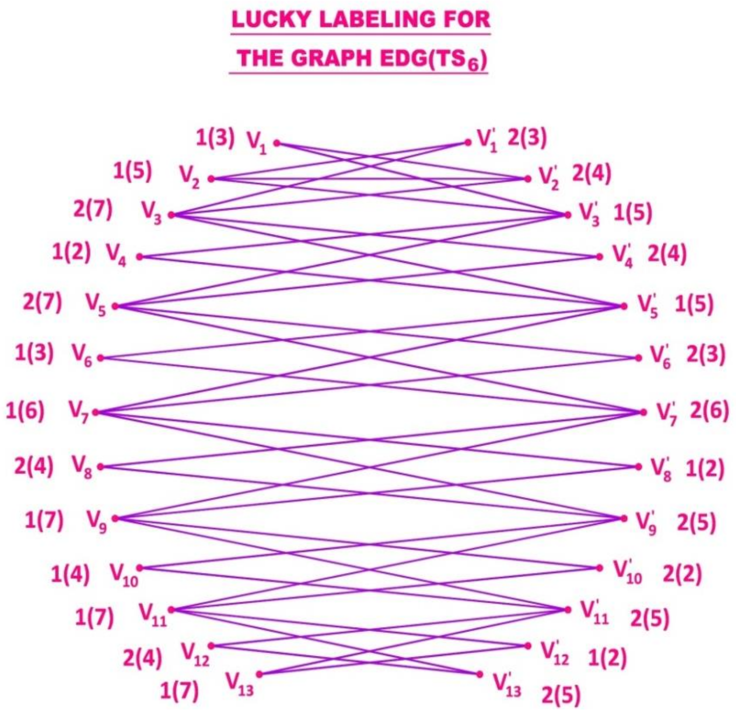


Fig 2: $EDG(TS_6)$

3.2: PROPER LUCKY LABELING FOR EDG(TS_m) , m≥ 1

Here, we present an algorithm and prove the existence of proper lucky labeling for EDG(TS_m).

Algorithm-2

Procedure - [Proper lucky labeling for EDG(TS_m) , m ≥ 1]

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    V ← {v1,v2, v3… v2m,v2m+1, v'1,v'2,v'3…v'2m ,v'2m+1}
    E ← {e1, e2, e3 …e3m , e3m+1,e'1,e'2,… e'3m }
if  m ≥ 1
      v'1 ← 2 , v'2 ← 2
      for i=1 to (2m+1)
            vi ←1
      end for
      for i = 1 to (m+1)/2 do
          v'4i-1 ←3
      end for
end if
if m ≥ 2
      for i = 0 to (m-2)/2 do
            for j = 0 to 1 do
                  v'4+4i+j ← 2
            end for
      end for
end if
end procedure
```

Theorem 2 : The extended duplicate graph of triangular snake graph admit proper lucky labeling and the proper lucky number is

$$\eta(G) = EDG(TSm) = \begin{cases} 3, & if\ m\ is\ odd \\ 2, & if\ m\ is\ even \end{cases}..$$

Proof: Let TSm be the triangular snake graph and EDG(TS_m) be the extended duplicate graph of triangular snake graph.

Define the set of vertices and edges are

$$V(G) = \{v_1,v_2, v_3\ldots v_{2m},v_{2m+1}, v'_1,v'_2,v'_3\ldots v'_{2m} ,v'_{2m+1}\}$$

$$E(G) = \{e_1, e_2, e_3 \ldots e_{3m} , e_{3m+1},e'_1,e'_2,\ldots e'_{3m} \}$$

Define a mapping f: V(G) → N such that the labeling is proper lucky if f(u) ≠ f(v) & S(u)≠S(v) whenever u and v are adjacent in G and S(v) denote the sum of labels of the neighbors of the vertex v in G.

Using the algorithm 2,

if $m \geq 1$, the vertices v'_1 and v'_2 receive the label '2' ; the vertices v_i receive the label '1' for $1 \leq i \leq (2m+1)$ and the vertices and v'_{4i-1} receive the label '3' for $1 \leq i \leq (m+1)/2$.

if $m \geq 2$, the vertices v'_{4i+j+4} receive the label '2' for $1 \leq i \leq (m-2)/2$ and $0 \leq j \leq 1(m-2)$. Clearly,We observe that $f(u) \neq f(v)$.

Thus all the 4m+2 vertices are labeled.

Now to prove that EDG(TSm) is proper lucky labeling.

i.e) $S(u) \neq S(v)$.

The sum neighbourood of the vertices are as follows:

if $m \geq 1$,

$S(v_1) \longleftarrow 5$, $S(v_2) \longleftarrow 7$, $S(v_3) \longleftarrow 8$, $S(v'_1) \longleftarrow 2$, $S(v'_2) \longleftarrow 3$, $S(v'_3) \longleftarrow 4$.

if $m \geq 2$,

Case (1):_ If m is odd,

For $0 \leq i \leq (m-3)/2$,

$S(v_{6+4i}) \longleftarrow 5$,$S(v_{7+4i}) \longleftarrow 8$,$S(v'_{6+4i}) \longleftarrow 2$,$S(v'_{7+4i}) \longleftarrow 4$

Case (2): If m is even,

For $0 \leq i \leq (m-2)/2$,

$S(v_{4+4i}) \longleftarrow 5$, $S(v_{5+4i}) \longleftarrow 10$.

For $0 \leq i \leq (m-2)/2$, & $0 \leq j \leq 1$

$S(v'_{4+4i+j}) \longleftarrow 2$

From the above cases, ,we get $S(u) \neq S(v)$

Hence, the extended duplicate graph of triangular snake graph is proper lucky labeling and the proper lucky number is

$$\eta(G) = EDG(TS_m) = \begin{cases} 3, & if\ m\ is\ odd \\ 2, & if\ m\ is\ even \end{cases}..$$

Example 2:: Proper lucky labeling diagram in EDG(TS_m) is shown in figure (3) & (4)

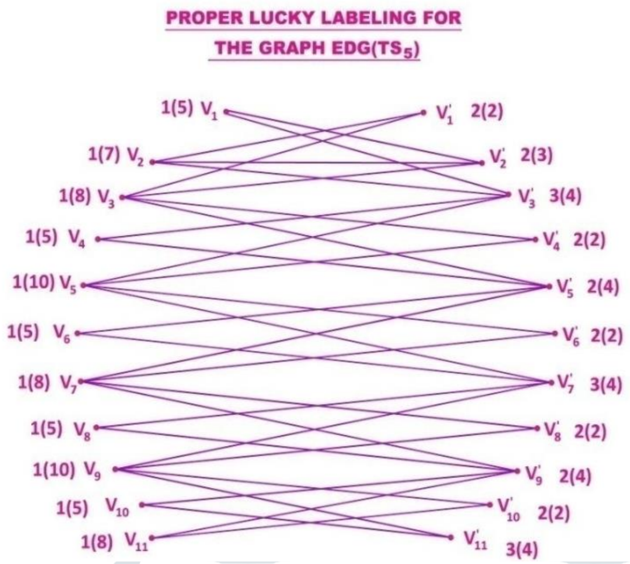

FIG 3: EDG(TS_5)

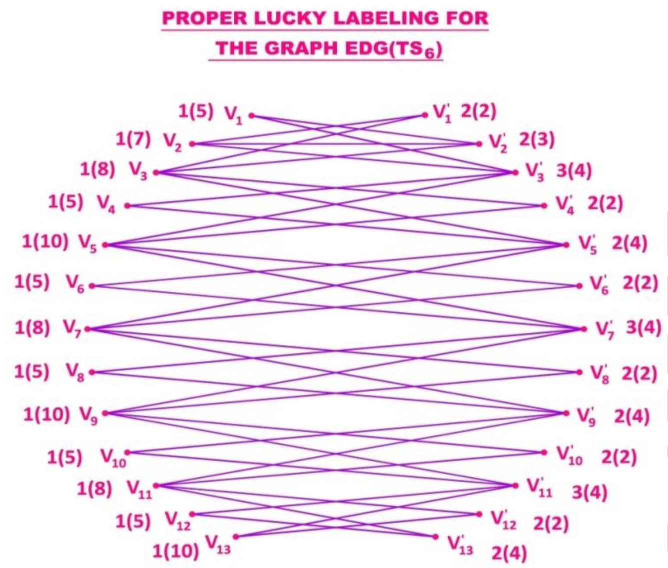

FIG 4: EDG(TS_6)

4. Conclusion: In this research paper,we have presented and investigated algorithms and their results in Lucky Labeling and Proper Lucky labeling for the extended duplicate graph of triangular snake graph EDG(TS_m), $m \geq 1$.

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