Graph Energies of Anthraquinone: Energy, Inverse Sum Connectivity energy, Sum Eccentricity energy, Randic Colour energy, Minimum Degree energy, Degree Sum energy, Degree Square Sum energy

Dr. M. Gayathri

Assistant Professor, Department of Mathematics,
Government First Grade College, Varthur, Bangalore, Karnataka, India.

Abstract:
In this paper I compute Energy, Inverse sum connectivity energy, Sum Eccentricity energy, Randic Colour energy, Minimum Degree energy, Degree Sum energy, Degree Square Sum energy of Anthraquinone.

Key words:
Anthraquinone, Eigen values, Characteristic equation, Energy, Inverse sum connectivity energy, Sum Eccentricity energy, Randic Colour energy, Minimum Degree energy, Degree Sum energy, Degree Square Sum energy.

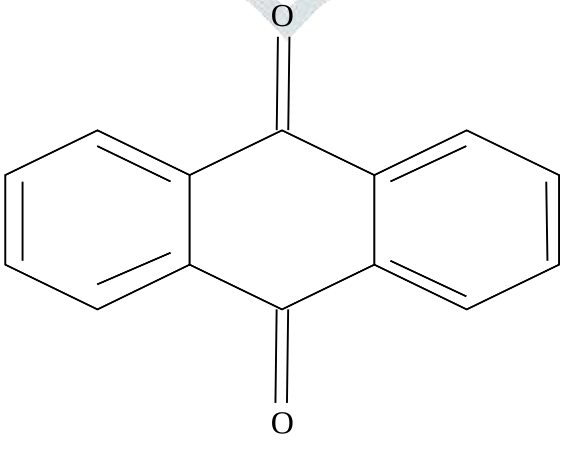
1. Introduction

Anthraquinones are active components of many plant blends which are used as medicines and exhibit laxative, diuretic, estrogenic and immunomodulatory effects. Anthraquinones are structurally related to anthracene and possess the 9,10-dioxoanthracene. Anthraquinones typically occur in their glycosidic forms. These compounds impart colour to plants and have been widely utilized as natural dyes. Anthraquinone is an important and widely used raw material for the manufacture of vat dyes, which are a class of water-insoluble dyes that can easily be reduced to a water soluble and usually colourless leuco form that readily impregnates fibres and textiles. In addition, they are also used as laxatives and possess antifungal and antiviral activities. It is also used as a seed dressing or in seed treatments. Other major uses are as a pesticide, as a bird repellent (especially for geese), and as an additive in chemical alkaline pulp processes in the paper and pulp industry.

So far, 79 naturally occurring anthraquinones have been identified which include emodin, physcion, cascarin, catenarin and rhein. A large body of literature has demonstrated that the naturally occurring anthraquinones possess a broad spectrum of bioactivities, such as cathartic, anticancer, anti-inflammatory, antimicrobial, diuretic, Vaso relaxing and phytoestrogen activities, suggesting their possible clinical application in many diseases.

In this paper I compute Energy, Energy, Inverse sum connectivity energy, Sum Eccentricity energy, Randic Colour energy, Minimum Degree energy, Degree Sum energy, Degree Square Sum energy of Anthraquinone.

2. Structural and Molecular formulae

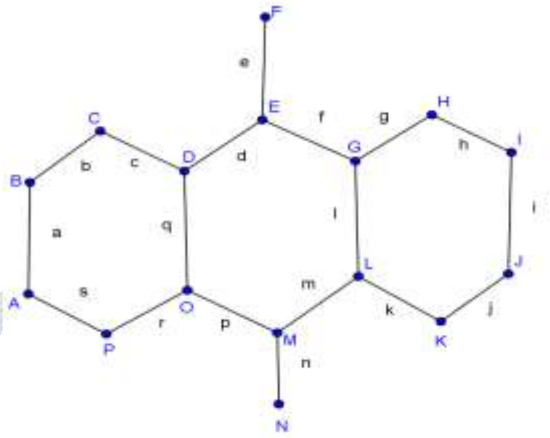


Molecular formulae: $C_{14}H_8O_2$

3. Energy of a Graph

The energy of a graph is one of the emerging concepts within graph theory. This concept serves as a frontier between chemistry and mathematics and is defined in1978 by I. Gutman [1] and originating from theoretical chemistry. In this paper we consider all graphs are simple without loops and multiple edges, finite and undirected. For standard terminology and notations related to graph theory, we follow Balakrishnan and Ranganathan [2]. The energy of a graph is zero if and only if it is trivial. The energy of a graph is one of the emerging concepts within graph theory. This concept serves as a frontier between chemistry and mathematics [3].

Let us consider the graph of Anthraquinone (i.e., Graph G) as shown in the following fig.



Graph G: Graph of Anthraquinone

Here the vertices A, B, C, D, ...M, N, O, P are treated as the vertices $v_1, v_2, \ldots\ldots v_{13}, v_{14}, v_{15}, v_{16}$.

In general, G be a graph possessing n vertices and m edges. Let $v_1, v_2, \ldots\ldots v_n$ be the vertices of G. Then the adjacency matrix $A(G)$ of the graph G is the square matrix of order n whose (i, j) entry is defined as

$$a_{ij} = \begin{cases} 1 & \text{if} \quad i \neq j \quad \text{and } v_i \text{ and } v_j \text{ are adjacent} \\ 0 & \text{if} \quad i \neq j \quad \text{and } v_i \text{ and} v_j \text{ are not adjacent} \\ 0 & \text{if} \quad i = j \end{cases}$$

The eigenvalues $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots\ldots \lambda_n$ of the graph G are the eigen values of its adjacency matrix. Since $A(G)$ is real symmetric, the eigen values of G are real numbers whose sum equal to zero.

The energy of a graph G is the sum of absolute values of the eigen values of a graph G and denoted it by $E(G)$. Hence

$$E(G) = \sum_{i=1}^{n} |\lambda_i|$$

$E(G)$ will be referred as the ordinary energy of the graph G.

Energy of the Anthraquinone:

Adjacency matrix of Anthraquinone is shown in matrix 1.

The Characteristic equation is

$$x^{16} - 18x^{14} + 127x^{12} - 456x^{10} + 903x^8 - 998x^6 + 593x^4 - 168x^2 + 16 = 0$$

The Eigen values of above characteristic equation are

$\lambda_1 = -2.49889, \lambda_2 = 2.49889, \lambda_3 = -2, \lambda_4 = 2, \lambda_5 = -1.6624, \lambda_6 = 1.6624, \lambda_7 = -1.49592, \lambda_8 = 1.49592$ $\lambda_9 = -1,$ $\lambda_{10} = 1, \lambda_{11} = -1, \lambda_{12} = 1,$ $\lambda_{13} = -0.757366, \lambda_{14} = 0.757366, \lambda_{15} = -0.424945,$ $\lambda_{16} = 0.424945.$

The Energy of Anthraquinone is

$$\varepsilon(C_{14}H_8O_2)= |-2.49889| + |2.49889| + |-2| + |2| + |-1.6624| + |1.6624| + |-1.49592| + |1.49592| + |-1| + |1| + |-1| + |1| + |-0.757366| + |0.757366| + |-0.424945| + |0.424945|$$

$$= 21.679042$$

4. Inverse sum connectivity graph:

The inverse sum connectivity of a graph is introduced by K. N. Prakasha [4]. The inverse sum connectivity matrix $ISC(G) = (I_{ij})_{n\times n}$ is defined as

$$I_{ij} = \begin{cases} \sqrt{d_i + d_j} & if \quad v_i \sim v_j \\ 0 & otherwise \end{cases}$$

Let $ISC(G)$ be the inverse sum connectivity matrix. The characteristic polynomial of $ISC(G)$ will be denoted by $\phi_{ISCD}(G,\lambda)$ and defined as

$$\phi_{ISCD}(G,\lambda) = \det(\lambda I - ISC(G))$$

Since the inverse sum connectivity matrix is real and symmetric its eigen values are real numbers and we label them in non-increasing order $\lambda_1 > \lambda_2 > \lambda_3 > \dots\dots > \lambda_n$. The inverse sum connectivity energy of G is similarly defined by

$$ISCE(G) = \sum_{i=1}^{n} |\lambda_i|$$

Inverse sum connectivity Energy of the Anthraquinone:

The inverse sum connectivity matrix is as shown in matrix 2.

The characteristic equation is

$$x^{16} - 86.0607x^{14} + 2815.79x^{12} - 45694.9x^{10} + 398751x^{8} - 1.8919\times10^{6}\times x^{6} + 4.68239\times10^{6}\times x^{4} - 5.30716\times10^{6}\times x^{2} + 1.91398\times10^{6} = 0$$

The eigen values are

$\lambda_1 = -5.79622$, $\lambda_2 = -4.27534$, $\lambda_3 = -3.69792$, $\lambda_4 = -3.00531$, $\lambda_5 = -2.18695$, $\lambda_6 = -1.99918$, $\lambda_7 = -1.43487$, $\lambda_8 = -0.800763$, $\lambda_9 = 0.800763$, $\lambda_{10} = 1.43487$, $\lambda_{11} = 1.99918$, $\lambda_{12} = 2.18695$, $\lambda_{13} = 3.00531$, $\lambda_{14} = 3.69792$, $\lambda_{15} = 4.27534$, $\lambda_{16} = 5.79622$

The inverse sum connectivity energy of a graph = 46.393106

5. Sum Eccentricity Energy:

The sum-eccentricity matrix of a graph G is denoted by $S_e(G)$ and defined as $S_e(G) = (s_{ij})$ where

$$s_{ij} = \begin{cases} e(v_i) + e(v_j) & if \quad v_i v_j \in E \\ 0 & otherwise \end{cases}$$

If $\lambda_1, \lambda_2, \lambda_3, \dots\dots\dots, \lambda_n$ are the sum-eccentricity eigen values, then the sum-eccentricity energy is $ES_e(G) = \sum_{i=1}^{n} |\lambda_i|$

Sum Eccentricity Energy of the Anthraquinone:

The sum eccentricity matrix is as shown in matrix 3.

The characteristic equation is

$$x^{16} - 2295x^{14} + 2081474x^{12} - 963204372x^{10} + 24633929896x^{8} - 35191961521344x^{6}$$
$$+2686809369722112x^{4} - 95623215979631616x^{2} + 106220340611481600 = 0$$

The eigen values are

$\lambda_1 = -26.7548$, $\lambda_2 = -24.4807$, $\lambda_3 = -18.3478$, $\lambda_4 = -16.5539$, $\lambda_5 = -12.0000$, $\lambda_6 = -11.5193$
$\lambda_7 = -8.5651$, $\lambda_8 = -4.37577$, $\lambda_9 = 4.37577$, $\lambda_{10} = 8.5651$, $\lambda_{11} = 11.5193$, $\lambda_{12} = 12.0000$, $\lambda_{13} = 16.5539$,
$\lambda_{14} = 18.3478$, $\lambda_{15} = 24.4807$, $\lambda_{16} = 26.7548$.

The sum-eccentricity energy is $ES_e(G) = \sum_{i=1}^{n} |\lambda_i|$ = 245.19474

6. Randic colour energy:

The Randic colour matrix $A_{RC}(G) = (r_{ij})$ is a square $n \times n$ matrix defined by

$$r_{ij} = \begin{cases} \frac{1}{\sqrt{d_i d_j}} & if\ v_i\ and\ v_j\ are\ adjacent\ with\ c(v_i) \neq c(v_j) \\ \frac{-1}{\sqrt{d_i d_j}} & if\ v_i\ and\ v_j\ are\ non-adjacent\ with\ c(v_i) = c(v_j) \\ 0 & otherwise \end{cases}$$

The characteristic polynomial of $A_{RC}(G)$ is $|\lambda I - A_{RC}(G)|$. Let $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n$ be eigenvalues of Randic colour matrix $A_{RC}(G)$ is real and symmetric matrix. So, its eigenvalues are real numbers and that their sum is zero. The Randic colour energy $E_{RC}(G)$ of a coloured graph G is defined as

$$E_{RC}(G) = \sum_{i=1}^{n} |\lambda_i|$$

Randic colour energy of the Anthraquinone:

The Randic colour matrix is as shown in matrix 4.

The characteristic polynomial is

$$x^{16} - 17.0988x^{14} + 29.8727x^{13} - 42.6985x^{12} + 178.625x^{11} - 195.603x^{10} + 58.8808x^{9} - 43.1904x^{8}$$
$$+32.9254x^{7} - 1.37518x^{6} + 3.48433x^{5} - 0.464133x^{4} + 0.107067x^{3} - 0.0158473x^{2}$$
$$+5.26923 \times 10^{-7} x = 0$$

The eigen values are

$\lambda_1 = -4.3556$, $\lambda_2 = -2.54976$, $\lambda_3 = -0.442072$, $\lambda_4 = -0.260501$, $\lambda_5 = -0.247549$, $\lambda_6 = 0$,
$\lambda_7 = 0.0000332575$, $\lambda_8 = 0.147204$, $\lambda_9 = 0.261741$, $\lambda_{10} = 0.782336$, $\lambda_{11} = 0.837539$, $\lambda_{12} = 0.898423$,
$\lambda_{13} = 0.942072$, $\lambda_{14} = 1.27698$, $\lambda_{15} = 1.32552$, $\lambda_{16} = 1.38363$.

The Randic colour energy is $E_{RC}(G)=$

$|-4.3556| + |-2.54976| + |-0.442072| + |-0.260501| + |-0.247549| + 0 + |0.0000332575| + |0.147204| + |0.261741| + |0.782336| + |0.837539| + |0.898423| + |0.942072| + |1.27698| + |1.32552| + |1.38363| = 15.71096$

7. Minimum degree energy:

The minimum degree matrix of a graph G of order n is an $n \times n$ symmetric matrix $MD(G)=\left[md_{ij}\right]$, whose elements are defined as

$$md_{ij} = \begin{cases} \min\left(d_i, d_j\right) & if \quad i \neq j \\ 0 & otherwise \end{cases}$$

Let I be the identity matrix. The minimum degree polynomial of a graph G is defined as

$$P_{MD}(G:\lambda) = |\lambda I - MD(G)|$$

The eigen values of the matrix $MD(G)$, denoted by $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots\ldots, \lambda_n$ are called the minimum degree eigen values of G and their collection is called the minimum degree spectra of G.

Let G be a graph of order n with minimum degree eigen values $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots\ldots, \lambda_n$,then the minimum degree energy [5] is $E_{MD}(G) = \sum_{i=1}^{n} |\lambda_i|$

Minimum Degree energy of the Anthraquinone:

The minimum degree matrix of Anthraquinone is as shown in matrix 5.

The characteristic polynomial is

$$x^{16} - 96x^{14} + 3346x^{12} - 55868x^{10} + 483153x^8 - 22143480x^6 + 4438064x^4 - 3248000x^2 + 160000 = 0$$

The eigen values are

$\lambda_1 = -6.53065$, $\lambda_2 = -4.43366$, $\lambda_3 = -3.66288$, $\lambda_4 = -3.0808$, $\lambda_5 = -2.42586$, $\lambda_6 = -1.85952$, $\lambda_7 = -1.1788$, $\lambda_8 = -0.230223$, $\lambda_9 = 0.230223$, $\lambda_{10} = 1.1788$, $\lambda_{11} = 1.85952$, $\lambda_{12} = 2.42586$, $\lambda_{13} = 3.0808$, $\lambda_{14} = 3.66288$, $\lambda_{15} = 4.43366$, $\lambda_{16} = 6.53065$.

The Minimum degree energy $E_{MD}(G) = \sum_{i=1}^{n} |\lambda_i|$

$= |-6.53065| + |-4.43366| + |-3.66288| + |-3.0808| + |-2.42586| + |-1.85952| + |-1.1788| + |-0.230223| + |0.230223| + |1.1788| + |1.85952| + |2.42586| + |3.0808| + |3.66288| + |4.43366| + |6.53065| = 46.804786$

8. Degree sum energy:

Let G be a simple graph with n vertices $v_1, v_2, \ldots\ldots\ldots v_n$ and let d_i be the degree of v_i, $i = 1, 2, \ldots\ldots, n$ then $DS(G) = \left[d_{ij}\right]$ is called the degree sum matrix of a graph G, where

$$d_{ij} = \begin{cases} d_i + d_j & if \quad i \neq j \\ 0 & Otherwise \end{cases}$$

The characteristic polynomial of $DS(G)$ denoted by $f_n(G,\lambda)=|\lambda I - DS(G)|$.Since $DS(G)$ is real and symmetric, its eigen values are real numbers and we label them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \lambda_3 .... \geq \lambda_n$. The maximum degree energy of G is then defined as

$$E_{DS}(G)=\sum_{i=1}^{n}|\lambda_i|.$$

Degree sum energy of the Anthraquinone:

The degree matrix of Anthraquinone is as shown in matrix 6.

The characteristic polynomial is

$$x^{16}-2528x^{14}-107392x^{13}-2369120x^{12}-34102528x^{11}-349395968x^{10}-2660884480x^{9}$$
$$-15413523200x^{8}-68638248960x^{7}-235274141696x^{6}-616097710080x^{5}$$
$$-1210718748672x^{4}-1728391938048x^{3}-1691898347520x^{2}-1015139008512x$$
$$-281303580672=0$$

The eigen values are

$\lambda_1 = -6.1847$, $\lambda_2 = -6.15749$, $\lambda_3 = -6.11356$, $\lambda_4 = -5.93151$, $\lambda_5 = -5.87839$, $\lambda_6 = -5.80151$, $\lambda_7 = -4.23987$, $\lambda_8 =$ -4.19138, $\lambda_9 =$ -4.06022, $\lambda_{10} = -4.01338$, $\lambda_{11} = -3.89182$, $\lambda_{12} = -3.82407$, $\lambda_{13} =$ -3.79264, $\lambda_{14} = -2.67913$, $\lambda_{15} = -2$, $\lambda_{16} = 68.6791$.

The degree sum energy of G = $E_{DS}(G)=\sum_{i=1}^{n}|\lambda_i|$

= $|-6.1847|+|-6.15749|+|-6.11356|+|-5.93151|+|-5.87839|+|-5.80151|+|-4.23987|+|-4.19138|+|-4.06022|$ $|-4.01338|+|-3.89182|+|-3.82407|+|-3.79264|+|-2.67913|+|-2|+|68.6791|=137.43877$

Degree Square Sum energy:

The degree square sum matrix of a graph G is an $n \times n$ matrix denoted by $DSS(G)=[dss_{ij}]$ and whose elements are defined as

$$dss_{ij}=\begin{cases} d_i^{\,2}+d_j^{\,2} & if\ i \neq j \\ 0 & Otherwise \end{cases}$$

Let I be an identity matrix then the degree square sum polynomial of a graph G is defined as

$$P_{DSS(G)}(\mu)=\det(\lambda I - DSS(G))$$

The eigen values of DSS(G) are denoted by $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n$ are called the degree square sum eigen values of G and their collection is called degree square sum spectra of G.

9. Degree square sum energy of the Anthraquinone:

The degree square sum matrix of Anthraquinone is as shown in matrix 7.

The characteristic polynomial is

$$x^{16} - 16368x^{14} - 1733376x^{13} - 93948192x^{12} - 3272394240x^{11} - 79853722368x^{10} - 1424763445248x^{9}$$
$$-19006488020736x^{8} - 191448106688512x^{7} - 1456528601382912x^{6} - 8295831553179648x^{5}$$
$$-34683929317343232x^{4} - 102768751279079424x^{3} - 202899038045995008x^{2} - 237170913731149824x$$
$$-122495332025106432 = 0$$

The eigen values are

$\lambda_1 = -21.6028$, $\lambda_2 = -18.129$, $\lambda_3 = -18.0111$, $\lambda_4 = -18.0071$, $\lambda_5 = -17.9661$, $\lambda_6 = -17.8908$,

$\lambda_7 = -8.00265$, $\lambda_8 = -8$, $\lambda_9 = -8$, $\lambda_{10} = -8$, $\lambda_{11} = -8$, $\lambda_{12} = -7.99867$, $\lambda_{13} = -7.99867$, $\lambda_{14} = -4.1176$, $\lambda_{15} = -2$, $\lambda_{16} = 173.724$.

The degree sum energy of G = $E_{DSS}(G) = \sum_{i=1}^{n} |\lambda_i|$

$= |-21.6028| + |-18.129| + |-18.0111| + |-18.0071| + |-17.9661| + |-17.8908| + |-8.00265| + |-8| + |-8| + |-8| + |-8| +$

$|-7.99867| + |-4.1176| + |-2| + |173.724| = 339.44982$

$$A(C_{14}H_8O_2) = \left[\begin{array}{c|cccccccccccccccc}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\
v_1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
v_2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_3 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_4 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
v_5 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_6 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_7 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
v_8 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
v_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
v_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
v_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
v_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
v_{15} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
v_{16} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]$$

Matrix 1: Adjacency Matrix of Anthraquinone

$$ISC(G)=\begin{bmatrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\ v_1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ v_2 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_3 & 0 & 2 & 0 & \sqrt{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_4 & 0 & 0 & \sqrt{5} & 0 & \sqrt{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_5 & 0 & 0 & 0 & \sqrt{6} & 0 & 2 & \sqrt{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_6 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_7 & 0 & 0 & 0 & 0 & \sqrt{6} & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_8 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ v_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & \sqrt{5} & 0 & 0 & 0 & 0 \\ v_{12} & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{6} & 0 & 0 & 0 & \sqrt{5} & 0 & \sqrt{6} & 0 & 0 & 0 \\ v_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{6} & 0 & 2 & \sqrt{6} & 0 \\ v_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ v_{15} & 0 & 0 & 0 & \sqrt{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{6} & 0 & 0 & \sqrt{5} \\ v_{16} & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{5} & 0 \end{bmatrix}$$

Matrix 2: Inverse Sum Connectivity Matrix of Anthraquinone

$$S_e\left(C_{14}H_8O_2\right)=\left[\begin{array}{ccccccccccccccccc}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\
v_1 & 0 & 14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 \\
v_2 & 14 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_3 & 0 & 13 & 0 & 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_4 & 0 & 0 & 11 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 \\
v_5 & 0 & 0 & 0 & 10 & 0 & 10 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_6 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_7 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 11 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 \\
v_8 & 0 & 0 & 0 & 0 & 0 & 0 & 11 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 14 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 14 & 0 & 13 & 0 & 0 & 0 & 0 & 0 \\
v_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 11 & 0 & 0 & 0 & 0 \\
v_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 11 & 0 & 9 & 0 & 0 & 0 \\
v_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 9 & 9 & 0 \\
v_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 \\
v_{15} & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 11 \\
v_{16} & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 11 & 0
\end{array}\right]$$

Matrix 3: Sum Eccentricity Matrix of Anthraquinone

$$RC\left(C_{14}H_8O_2\right)=\begin{bmatrix}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\
v_1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & 0 & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{2} \\
v_2 & \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & 0 & 0 & -\frac{1}{2} \\
v_3 & -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & 0 \\
v_4 & 0 & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 & \frac{1}{3} & -\frac{1}{\sqrt{3}} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{3} & 0 & \frac{1}{3} & -\frac{1}{\sqrt{6}} \\
v_5 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{6}} & \frac{1}{3} & 0 & \frac{1}{\sqrt{3}} & \frac{1}{3} & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & -\frac{1}{3} & 0 \\
v_6 & 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{3}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\
v_7 & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{6}} & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & -\frac{1}{\sqrt{6}} \\
v_8 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & 0 \\
v_9 & 0 & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & 0 & 0 & -\frac{1}{2} \\
v_{10} & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & 0 \\
v_{11} & 0 & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & 0 & -\frac{1}{2} \\
v_{12} & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{3} & 0 & \frac{1}{3} & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 & \frac{1}{3} & -\frac{1}{\sqrt{3}} & -\frac{1}{3} & 0 \\
v_{13} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{6}} & \frac{1}{3} & 0 & \frac{1}{\sqrt{3}} & \frac{1}{3} & -\frac{1}{\sqrt{6}} \\
v_{14} & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{3}} & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} & 0 \\
v_{15} & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{6}} & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{6}} \\
v_{16} & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} & 0
\end{bmatrix}$$

Matrix 4: Randic colour Matrix of Anthraquinone

$$MD\left(C_{14}H_8O_2\right)=\left[\begin{array}{ccccccccccccccccc}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\
v_1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
v_2 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_3 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_4 & 0 & 0 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
v_5 & 0 & 0 & 0 & 3 & 0 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_6 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_7 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 2 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
v_8 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
v_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 \\
v_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 2 & 0 & 3 & 0 & 0 & 0 \\
v_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 1 & 3 & 0 \\
v_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
v_{15} & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 2 \\
v_{16} & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0
\end{array}\right]$$

Matrix 5: Minimum Degree Matrix of Anthraquinone

$$DS\left(C_{14}H_8O_2\right)=\begin{bmatrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\ v_1 & 0 & 4 & 4 & 5 & 5 & 3 & 5 & 4 & 4 & 4 & 4 & 5 & 5 & 3 & 5 & 4 \\ v_2 & 4 & 0 & 4 & 5 & 5 & 3 & 5 & 4 & 4 & 4 & 4 & 5 & 5 & 3 & 5 & 4 \\ v_3 & 4 & 4 & 0 & 5 & 5 & 3 & 5 & 4 & 4 & 4 & 4 & 5 & 5 & 3 & 5 & 4 \\ v_4 & 5 & 5 & 5 & 0 & 6 & 4 & 6 & 5 & 5 & 5 & 5 & 6 & 6 & 4 & 6 & 5 \\ v_5 & 5 & 5 & 5 & 6 & 0 & 4 & 6 & 5 & 5 & 5 & 5 & 6 & 6 & 4 & 6 & 5 \\ v_6 & 3 & 3 & 3 & 4 & 4 & 0 & 4 & 3 & 3 & 3 & 3 & 4 & 4 & 2 & 4 & 3 \\ v_7 & 5 & 5 & 5 & 6 & 6 & 4 & 0 & 5 & 5 & 5 & 5 & 6 & 6 & 4 & 6 & 5 \\ v_8 & 4 & 4 & 4 & 5 & 5 & 3 & 5 & 0 & 4 & 4 & 4 & 5 & 5 & 3 & 5 & 4 \\ v_9 & 4 & 4 & 4 & 5 & 5 & 3 & 5 & 4 & 0 & 4 & 4 & 5 & 5 & 3 & 5 & 4 \\ v_{10} & 4 & 4 & 4 & 5 & 5 & 3 & 5 & 4 & 4 & 0 & 4 & 5 & 5 & 3 & 5 & 4 \\ v_{11} & 4 & 4 & 4 & 5 & 5 & 3 & 5 & 4 & 4 & 4 & 0 & 5 & 5 & 3 & 5 & 4 \\ v_{12} & 5 & 5 & 5 & 6 & 6 & 4 & 6 & 5 & 5 & 5 & 5 & 0 & 6 & 4 & 6 & 5 \\ v_{13} & 5 & 5 & 5 & 6 & 6 & 4 & 6 & 5 & 5 & 5 & 5 & 6 & 0 & 4 & 6 & 5 \\ v_{14} & 3 & 3 & 3 & 4 & 4 & 2 & 4 & 3 & 3 & 3 & 3 & 4 & 4 & 0 & 4 & 3 \\ v_{15} & 5 & 5 & 5 & 6 & 6 & 4 & 6 & 5 & 5 & 5 & 5 & 6 & 6 & 4 & 0 & 5 \\ v_{16} & 4 & 4 & 4 & 5 & 5 & 3 & 5 & 4 & 4 & 4 & 4 & 5 & 5 & 3 & 5 & 0 \end{bmatrix}$$

Matrix 6: Degree Sum Matrix of Anthraquinone

$$DSS\left(C_{14}H_8O_2\right)=\left[\begin{array}{c|cccccccccccccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\ v_1 & 0 & 8 & 8 & 13 & 13 & 5 & 13 & 8 & 8 & 8 & 8 & 13 & 13 & 5 & 13 & 8 \\ v_2 & 8 & 0 & 8 & 13 & 13 & 5 & 13 & 8 & 8 & 8 & 8 & 13 & 13 & 5 & 13 & 8 \\ v_3 & 8 & 8 & 0 & 13 & 13 & 5 & 13 & 8 & 8 & 8 & 8 & 13 & 13 & 5 & 13 & 8 \\ v_4 & 13 & 13 & 13 & 0 & 18 & 10 & 18 & 13 & 13 & 13 & 13 & 18 & 18 & 10 & 18 & 13 \\ v_5 & 13 & 13 & 13 & 18 & 0 & 10 & 18 & 13 & 13 & 13 & 13 & 18 & 18 & 10 & 18 & 13 \\ v_6 & 5 & 5 & 5 & 10 & 10 & 0 & 10 & 5 & 5 & 5 & 5 & 10 & 10 & 2 & 10 & 5 \\ v_7 & 13 & 13 & 13 & 18 & 18 & 10 & 0 & 13 & 13 & 13 & 13 & 18 & 18 & 10 & 18 & 13 \\ v_8 & 8 & 8 & 8 & 13 & 13 & 5 & 13 & 0 & 8 & 8 & 8 & 13 & 13 & 5 & 13 & 8 \\ v_9 & 8 & 8 & 8 & 13 & 13 & 5 & 13 & 8 & 0 & 8 & 8 & 3 & 13 & 5 & 13 & 8 \\ v_{10} & 8 & 8 & 8 & 13 & 13 & 5 & 13 & 8 & 8 & 0 & 8 & 13 & 13 & 5 & 13 & 8 \\ v_{11} & 8 & 8 & 8 & 13 & 13 & 5 & 13 & 8 & 8 & 8 & 0 & 13 & 13 & 5 & 13 & 8 \\ v_{12} & 13 & 13 & 13 & 18 & 18 & 10 & 18 & 13 & 13 & 13 & 13 & 0 & 18 & 10 & 18 & 13 \\ v_{13} & 13 & 13 & 13 & 18 & 18 & 10 & 18 & 13 & 13 & 13 & 13 & 18 & 0 & 10 & 18 & 13 \\ v_{14} & 5 & 5 & 5 & 10 & 10 & 2 & 10 & 5 & 5 & 5 & 5 & 10 & 10 & 0 & 10 & 5 \\ v_{15} & 13 & 13 & 13 & 18 & 18 & 10 & 18 & 13 & 13 & 13 & 13 & 18 & 18 & 10 & 0 & 13 \\ v_{16} & 8 & 8 & 8 & 13 & 13 & 5 & 13 & 8 & 8 & 8 & 8 & 13 & 13 & 5 & 13 & 0 \end{array}\right]$$

Matrix 7: Degree Square Sum Matrix of Anthraquinone

10. Conclusion:

In this article, I compute Energy, Inverse sum connectivity energy, Sum Eccentricity energy, Randic Colour energy, Minimum Degree energy, Degree Sum energy, Degree Square Sum energy of Anthraquinone.

References:

1. I. Gutman. The energy of a graph, Ber. Math. Statist. S ekt. forschungsz. Graz 103(1978) 1-22
2. R. Balakrishnan, The energy of a graph, Lin. Algebra Appl.387(2004) 287-295.
3. I. Gutman, Chemical graph theory- The mathematical connection, in: J. R. Sabin, E. J. Brandas (Eds.), Advances in Quantum chemistry 51, Elsevier, Amsterdam,2006, pp.125-138.
4. K. N. Prakasha, The inverse sum connectivity index of graphs.
5. B. Basavana Goud and Praveen Jakkannavar, Minimum Degree energy of graphs, Electronic Journal of Mathematical Analysis and Applications, Vol.7(2) July 2019, PP. 230-243.
6. S.R. Jog, S.P. Hande, D.S. Revankar, Degree sum polynomial of graph valued functions on regular graphs, Int. J. Graph Theory,1(2013),108-115
7. B. Basavana goud and E. Chitra, Degree square sum energy of graphs, International Journal of Mathematics and its Applications,6(2-B) (2018),193-205.