Analytical Study of Dependence of Directivity on the Geometries of the Microstripline Coupler

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Abstract:

When two microstriplines are placed in close proximity with edges parallel to each other microstripline coupler is formed and natural coupling exists between them. The energy flowing in the first microstripline is coupled to the second microstripline both in forward and reverse directions. The forward coupling is called even-mode and reverse coupling is called odd-mode. The directivity is an important parameter of the coupler and is related with the characteristic impedances and guide wavelengths for even and odd-modes of propagation. This is the function of width of the conductor strip, spacing between them and relative permittivity of the substrate. The present study is related with the dependence of directivity on the geometrical parameters of the coupled microstriplines.

Keywords: Microstripline, directivity, relative permittivity, coupling coefficient.

1. Introduction

The advent of microwave integrated circuits (MICs) has developed several planar transmission structures suitable for microwave frequencies especially in giga hertz range. The planar transmission structures are stripline, microstripline, slot line, coplanar strips, coplanar waveguide and their variants. Special feature of such stripline structure are reduction in size and weight, lower cost, increased reliability and easily replaceable. The dominant modes are TEM-mode for stripline, Quasi-TEM-modes for microstripline, Non-TEM-mode for slotline and other variants.

Among various planar transmission structures microstripline is simplest one. It is open structure and simple in realization. It consists of narrow conductor strip on one side of dielectric substrate with other side metalized to serve as a ground plane shown in figure 1 a. The electric and magnetic field configurations are shown in figure 1 b. These field lines are confined in the vicinity of the strip conductor with larger concentration inside the substrate material and smaller in air region. Due to inhomogeneous medium effective permittivity in place of relative permittivity of the dielectric substrate is used for the purpose of the study of characteristic parameters of the single and coupled microstripline structure. The knowledge of the characteristic parameters and reflection coefficient for even and odd-modes provide the calculation of directivity of the coupled structure which is formed by placing two similar microstriplines in close proximity with edge parallel to each other as shown in fig.2. Natural coupling exits between them due to linking of...
energy flux one line to other. Thus the directivity is the functions of the geometrical parameters of the directional coupler.

Fig. 1a: Microstripline structure  
Fig. 1 b: Microstrip field configuration

Fig. 2: Coupled microstripline  
Fig. 3: The Even-mode forward coupling  
Fig. 4: The odd-mode reverse coupling

2. Formulation of characteristics parameter of the coupled structure

The characteristics impedance, phase velocity and guide wavelength are important characteristic parameters of microstripline coupler. These parameters are the functions of geometry of the conductor strips, height and permittivity of the dielectric substrate. As from conformal transformation, microstripline is supposed to be parallel plate capacitor. The characteristic impedance is related with the capacitance and phase velocity of the wave traveling through the structure and is given by

$$Z_o = \frac{1}{V_p C_p}$$  \hspace{1cm} (1)

where $V_p$ = phase velocity of the wave travelling along the microstripline

$C_p$ = capacitance per unit length of the line.

The study of microstripline coupler involves the analysis of even and odd-modes of propagation. In the even-mode, energy travelling down, one microstrip line is coupled into a parallel line and travels in the same direction, whereas in the odd-mode energy travels in the reverse direction after coupling. The derivation of the characteristic impedances, phase velocity and guide wavelength for even and odd-modes is based on the equation 1. In this coupled structure the field configurations get altered as shown figure 3 and figure 4 for even and odd-mode respectively. The capacitance $C_p$ is replaced by $C_{pe}$ and $C_{po}$, phase velocity $V_p$ is replaced by $V_{pe}$ and $V_{po}$ and effective permittivity $\varepsilon_{eff}$ is replaced by $(\varepsilon_{eff})_e$ and $(\varepsilon_{eff})_o$ respectively for even and odd-modes. Similarly $Z_{oe}$ and $Z_{oo}$ represent the respective characteristic impedances for even and odd-modes.
Even mode characteristics impedance ($Z_{oe}$)

The total capacitance is constituted by the following components:

- $C_{PPE}$ = parallel plate capacitance for even mode
- $C_{PPU}$ = capacitance between upper surface of the conductor and ground plane
- $C'_{PPU}$ = capacitance between strip conductor and ground plane enclosed between two microstriplines

$$
= \left(2\varepsilon_{\text{reff}}/3c \eta\right)(w/h)(1/[w/s]+1)
$$

(2)

$C_F$ = Fringe capacitance at the edge of the striplines

$$
= (\varepsilon_c/c \eta) \left(2.7/\log(4h/t) \left(1/[w/s]+1\right)\right)
$$

(3)

Thus the total capacitance for even-mode coupled lines is expressed as

$$
C_{PE} = C_{PPE} + (1/2)C_{PPU} + (1/2)C_F + (1/2)C'_{PPU} + (1/2)C'_{F}
$$

(4)

Now we can write the characteristic impedance for even-mode as

$$
Z_{oe} = \left(\eta/\sqrt{\varepsilon_{\text{reff}}}\right)\left[1/[w/h]+(w/3h)\left(1.35/\log(4h/t) + (w/3h)\right)(1/[w/s]+1)\right]
$$

(5)

and for $t = 0$

$$
Z_{oe} = \left(\eta/\sqrt{\varepsilon_{\text{reff}}}\right)\left[1/[w/h]+(1/3\sqrt{\varepsilon_{\text{reff}}}\right)\left(1.35/\log(4h/t) + (w/3h)\right)(1/[w/s]+1)\right]
$$

(6)

Odd-mode characteristic impedance ($Z_{oo}$)

In the case of odd-mode coupled lines, the total capacitance ($C_{PO}$) is determined in terms of the following components:

- $C''_{PPU}$ = capacitance between strip conductor and the ground plane spaces enclosed between the two micristrip lines

$$
= (8/3)\left[\sqrt{\varepsilon_{\text{reff}}} / c \eta\right]
$$

(7)

$C''_F$ = Fringe capacitance between edges of the microstrip lines and is given as

$$
= (\varepsilon_{\text{reff}} / c \eta) \left[2.7/\log(4s \tan(4h/s))/nt\right]
$$

(8)

The total capacitance of the odd-mode coupled lines is thus expressed as

$$
C_{PO} = C_{PP} + (1/2)C_{PPU} + (1/2)C_{PPU}^* + (1/2)C_F + (1/2)C_F^*
$$

(9)
and the odd-mode characteristic impedance \( Z_{oo} \) is given as

\[
Z_{oo} = \left[ \frac{\eta}{\sqrt{\varepsilon_{reff}}} \right] \left[ \left( \frac{w}{3h} \sqrt{\varepsilon_{reff}} \right) + \left( \frac{4}{3} \sqrt{\varepsilon_{reff}} \right) \right] \left( \frac{1}{s/w} + 1 \right) + \left( \frac{1.35}{\log \left( 4s \tan \left( \frac{4h}{s} \right) / \pi \right)} \right) \]

(10)

and for \( t = 0 \)

\[
Z_{oo} = \left[ \frac{\eta}{\sqrt{\varepsilon_{reff}}} \right] \left( \frac{w}{h} + \left( \frac{1}{3} \sqrt{\varepsilon_{reff}} \right) + \left( \frac{4}{3} \sqrt{\varepsilon_{reff}} \right) \right) \left( \frac{1}{s/w} + 1 \right) \]

(11)

The phase velocity for even and odd-mode are given by

For even mode

\[
\left( V_p \right)_e = c/\left( \sqrt{\varepsilon_{reff}} \right)_e
\]

(12)

and for odd mode

\[
\left( V_p \right)_o = c/\left( \sqrt{\varepsilon_{reff}} \right)_o
\]

(13)

and similarly guide wavelength for even and odd-modes are given by

For even mode propagation

\[
\left( \lambda_{g} \right)_e = \frac{V_p}{f} = \frac{\lambda_o}{\left( \sqrt{\varepsilon_{reff}} \right)_e}
\]

(14)

For odd-mode propagation

\[
\left( \lambda_{g} \right)_o = \frac{V_p}{f} = \frac{\lambda_o}{\left( \sqrt{\varepsilon_{reff}} \right)_o}
\]

(15)

3. Study of dependence of characteristic impedance of coupled microstripline for even and odd-mode with width and spacing between them

The study of variation of characteristic impedance has been performed for a given dielectric substrate by varying the width of metal strips and spacing between them. In case of even and odd-modes and results are obtained which shows that with increase of stripwidth characteristic impedance decreases both for even and odd-modes but rate of decrease is faster for even-mode than that for odd-mode. Further the results reveal that even-mode characteristic impedance decrease and that for odd-mode increases with increase of spacing.

4. Directional coupler

Directional coupler is used to sample the part of energy passing through the main waveguide. It is four port devices. There are no reflections at the junctions of these four ports. When the power is incident on port(1) then it is passed to port (2) and port (4) but it is not passing to port (3), thus the port (3) is uncoupled or isolated. Similarly when the power is incident from port (3) this is coupled to port (2) and port (4) but not coupled to port (1) as shown in Figure-5

Fig.5: Directional Coupler
Directivity is defined as the ratio of power coupled in the forward to power coupled in the backward direction in the auxiliary arm. It is the measure of discrimination of a directional coupler between forward and backward waves and is also defined as the ratio of the voltage coupled to the desired port and of the voltage coupled to the undesired port. It is denoted by ‘D’. It is given in decibels.

\[
D = 10 \log_{10} \frac{P_{\text{aux(forward)}}}{P_{\text{aux(backward)}}}
\]

where,

\[
P_{\text{aux(forward)}} = \text{Power coupled in the auxiliary arm due to power in forward direction}
\]

\[
P_{\text{aux(backward)}} = \text{Power coupled in the auxiliary arm due to power in backward direction}
\]

Also \[D = \frac{V_4}{V_3}\]

\[
D(dB) = -20 \log \left( \frac{V_4}{V_3} \right)
\] (16)

Thus if the value of directivity is more it indicates the leakage of power in the auxiliary arm is less. Generally D is in between 30 to 35 dB.

For an ideal forward directional coupler directivity is infinity, i.e. voltage at port (3) should be ideally zero. The signal is coupled only to port (4), port (2) and (4) being perfectly matched.

With microstrip the differing field patterns associated with the odd and even modes, give rise to different phase velocities. This results in some coupling to the unwanted port as well. The greater differences in the phase velocities of the even and odd modes makes the coupling tighter. This parallel microstrip directional coupler may not give a wide band width performance for tight coupling. Further the directivity depends on microstrip geometry and substrate property \(\varepsilon_r\). An approximate but simpler mathematical expression for the directivity of the coupled microstrip coupler is given as

\[
D = \left[4|\xi|/\Delta \pi (1-|\xi|^2)\right]^{-2}
\]

\[
D = \left[\Delta \pi (1-|\xi|^2)/4|\xi|^2\right]^{-2}
\] (17)

Where,

\[
\Delta = [\lambda_{go}/\lambda_{ge}] - 1
\] (18)

\(\lambda_{go}\) and \(\lambda_{ge}\) are the guide wavelengths of the coupled lines for even and odd modes respectively and \(\xi\) is expressed by equation

\[
\xi = [\rho_e/1-\rho_e^2] - [\rho_o/1-\rho_o^2]
\] (19)

where,

\(\rho_e\) = Reflection coefficient for even mode

\(\rho_e = \frac{Z_{oe} - Z_o}{Z_{oe} + Z_o}\)

and \(\rho_o\) = Reflection coefficient for odd mode

\(\rho_o = \frac{Z_{oo} - Z_o}{Z_{oo} + Z_o}\)
5. Study of directivity and its dependence on strip width and spacing between them

The directivity can be calculated manually using calculator by measuring the values of characteristic impedances and guide wavelengths for even and odd-modes. By varying the width of the microstripline for different values of spacing between these lines directivity has been obtained at a given frequency and for a given dielectric substrate. The graphs with stripwidth and spacing on x-axis and directivity on y-axis have been plotted separately as shown in graph 1 and graph 2.

![Graph 1: Variation of Directivity with stripwidth.](image1)

![Graph 2: Variation of Directivity with spacing.](image2)

The results shows that as stripwidth increases directivity decreases at moderate rate showing the greater amount of power coupled to the neighboring microstripline in forward direction which can result in greater coupling coefficient. Also the results show that the directivity decreases with increase of spacing between two strips with relatively larger rate than that of variation of directivity with strip width. This shows that spacing between two strip lines affects the flow of power.

6. Conclusion

From the above discussion of the results in different sections it can be concluded that directivity of the microstripline coupler is the function of geometry of the structure along with spacing between two strips. These parameters are also the functions of guide wavelength, effective relative permittivity and frequency. Further these parameters are the functions of different attenuations occurring within the structure due to propagation of waves in even and odd-modes both. This is very useful for design of microstripline coupler and isolators.
References: