COMPARATIVE ANALYSIS AND DESIGN OF FLAT AND GRID SLAB SYSTEM WITH CONVENTIONAL SLAB SYSTEM

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Abstract- Timoshenko’s theory of plates is used to evaluate design moments and shears occurring in ribs of a Grid floor accurately. However, it involves assuming magnitudes of parameters like spacing of ribs, thickness of slab and width of rib which have considerable influence on the overall economy of the Grid floor slab. The aim of this study is to analytically find the effect of these assumed parameters on the overall economy of the structure, Grid floors of sizes 12 m X 16 m, 14 m X 16 m and 16 m X 16 m were designed for various dimensions of slab, rib and different spacing. The cost of each slab is estimated and interaction curves are developed. From this study, it can be concluded that the cost of the grid floor would be minimum if minimum thickness of slab, minimum width of ribs and maximum spacing of ribs is adopted. Further, for the typical case considered, the approximate method of Ranking - Grashoff theory underestimates the moments by around 20 %.

Keywords: grid floor, economic design, width of ribs, spacing of ribs, thickness of slab, computer aided analysis and design.

1. INTRODUCTION

Slabs may be viewed as moderately thick plates that transmit loads to the supporting walls or beams and sometimes directly to the columns by flexure, shear and torsion. It is because of this complex behavior that it is difficult to decide whether the slab is a structural element, component or a system in itself. Purushothaman (1984) visualises Slabs as intersecting, closely spaced grid-beams and hence they are highly indeterminate. This is indirectly helpful to designers, since multiple load-flow paths are available. However, rigorous elastic solutions are not available for many practically important boundary conditions. Iyengar (2004) opines that Finite difference and finite element methods and more methods have been developed to find the collapse loads of various types of slabs through the yield-line theory and strip-methods. The study and detailing of slabs is a challenge as precise technical information is not readily available. An intuitive feel is still the basis for the design of slab as many parameters that affect its performance are assumed. As the greatest volume of concrete that goes into a structure is in the form of slabs, even a slightest change in these parameters will affect the economy. The basic idea behind intuitive or indirect design in engineering is the memory of past experiences, subconscious motives, incomplete logical processes, random selections or sometimes mere superstition. This will not lead to the best design. The shortcomings of the indirect design can be overcome by adopting a direct or optimal design procedure. The feature of the optimal design is that it consists of only logical decisions by setting out the constraints and then minimizing or maximizing the objective function (which could be either cost, weight or merit function).

1. INTRODUCTION

Most civil engineering structures are even to-day designed on the basis of permissible stress criterion. However, some of the recent methods use a specified factor of safety against ultimate failure of the structure. Presently, the approach is based on the design constraints expressing the maximum probability of various types of events such as local or ultimate failure. During the early fifties there have been considerable advances in ‘art’ and economy of the structural design through the use of better structural materials and refined knowledge of structural design processes. Thus, the aim was to put structural design on a scientific basis. The need for innovation and optimization arose in the challenging problems faced by the aerospace industry, which gave a Philip to research activities in this area.

2. GRID FLOORS

2.1 General

The cast-in-situ reinforced concrete roof and floor slab is the simplest form of slab construction, but it is rather wasteful in materials, particularly cement. Substantial savings can be affected by modifying the composition of the slab so that its weight is reduced without impairing its strength or behavior. Ribbed and waffle slabs are examples. Amit A. Sathawane and R.S. Deotale (2011) compared Flat Slab and Ribbed floor and found that the former is more economical. The formwork for such a system is complex and the extra initial cast may not be justified where a small-sized domestic construction is involved. Some weight reduction can be effected by the use of hollow clay blocks which eliminates the need for special form-work by acting as a part of the formwork in the construction of the in-situ ribbed slabs. Ribbed, hollow block or voided slab construction has been covered in
the IS:456 (1978) code for the first time. Ibrahim. S. Vepari and H.S.Patel (2011), observed that Grid or coffered floor systems consisting of beams spaced at regular intervals in perpendicular directions, monolithic with a slab are generally employed for architectural reasons for large rooms such as auditoriums, vestibules, theatre halls, show rooms of shops where column free space is often the main requirement. The rectangular or square voids formed in the ceiling are advantageously utilized for concealed architectural lighting. The size of the beams running in perpendicular directions is generally kept the same. Instead of rectangular beam grid, a diagonal grid can also be used with the beams inclined at 45° to the sides.

2.2 Methods of analysis
According to the Indian Standard Code of practice for Plain and Reinforced Concrete i.e. IS: 456 (2000), the ribbed slab system can be analyzed as a solid slab if the spacing of the ribs is not be greater than 1.5 m and 12 times the flange thickness. In situ ribs should not be less than 65 mm wide. The ribs should be formed along each edge parallel to the span, having a width equal to that of the bearing. The moments and shears per unit width of grid are determined from Table 22 of IS: 456 (2000) code and the reinforcements are designed in the ribs. In slabs, reinforcement generally consists of a mesh or fabric. A second approximate method which is applicable to the grid floor system is the Ranking Grashoff (1857) Theory of equating deflections at the junctions of ribs. However this method does not yield the twisting moments in the beams. For small span grids with spacing of ribs not exceeding 1.5 m, it can be used. But for grids of larger spans with spacing of ribs exceeding 1.5 m, a rigorous analysis based on orthotropic plate theory is generally used. A reinforced concrete grid floor with ribs at close intervals in two mutually perpendicular directions connected by slab in between in the ribs can be considered as an orthotropic plate freely supported on four sides. Timoshenko and S. Woinowsky-Krieger (1963) evaluated the moments and shears in the grid which depend upon the deflection surface. Maximum Bending moments develop at center of span while maximum torsional moments are generated at the corners of the grid and maximum shear forces develop at mid points of longer side supports. Al-Ansari (2006) adopted MATHCAD for two way ribbed slab analysis and design.

2.3 Minimum cost design of grid floor
The most common form of reinforced concrete construction of private and public buildings is T-beam and grid floor. The design of these structures is generally based on either stress design or strength design. It has been well established that the strength design is more logical and also economical. For the design slabs of various shapes and edge conditions limit design procedures have also been well established. These methods result in considerable economy in the design of reinforced concrete structures. However, one can further improve the design if one chooses the dimensions optimally.

The cost of the structure is often a nonlinear function of the dimensions of the structure. It is necessary that the structure in addition to being low cost must meet the safety and functional requirements. These are also generally nonlinear. Adidam (1978) investigated the optimal design of T-beam and grid floors using Nonlinear Mathematical Programming Technique in which the objective function represents the cost of one beam and slab assembly per unit length along the beam span per unit spacing. This is also expressed as a ratio of cost per unit area of floor to the cost of one unit of concrete. An existing square grid of 18.83 meter span was optimized. He found that the optimal design turns out to be 1.2 meter square grid instead of existing one meter square. This indirectly results in saving of form work and material.

From study of literature, it can be understood that the economy of a Grid slab is not only affected by the Geometry, but also the Design parameters. The following are some of the parameters that affect the overall cost of a grid floor.

1. Size of Grid Floor and Spacing of Ribs (in X and Y directions)
2. Grade of Concrete and Grade of Steel
3. Live load on the slab
4. Thickness of slab, Width of Rib and Depth of Rib
5. However, the structural design engineer has control over Thickness of slab, Width of Rib and Depth of Rib only and hence the study of their effect on the cost of the Grid floor is important.

3. COMPUTERISED ANALYSIS AND DESIGN OF GRID FLOOR — NECESSITY

Grid Slabs, being highly indeterminate, are difficult to analyse by elastic theories. Since slabs are sensitive to support restraints and fixities, rigorous elastic solutions are not available for many practically important boundary conditions. Since large volumes of reinforced concrete go into slabs, the slightest change in different parameters will lead to considerable economy in the final analysis. Economy is the basis of all good designs, the slab being no exception to this golden rule. Repeated analysis and design leads to economical design but is tedious. Hence it is necessary that the analysis and design of Grid Floors be computerized to arrive at a more rational design.

The approximate method underestimates the bending moments developed in X and Y directions, to the extent of 17 % and 21 % respectively. The moments are very much under estimated in the long span direction.
4. OBJECTIVE

The following are the objective of the present study:

1. To design various form of slab system for example conventional slab, flat slab and grid floor slab for the given plan area and their comparative study.
2. To study various parameters related to Grid Floor slab.
3. Grid floor slab will be tried to analyzing using Timoshenko’s Plate Theory.
4. Grid floor slab will be tried to analyzing using Rankine Grashoff Theory
5. To study comparative costing of various types of slab system.

5. BACKGROUND AND MOTIVATION

Building construction is the engineering deals with the construction of building such as residential houses. In a simple building can be define as an enclose space by walls with roof, food, cloth and the basic needs of human beings. In the early ancient times humans lived in caves, over trees or under trees, to protect themselves from wild animals, rain, sun, etc. as the times passed as humans being started living in huts made of timber branches. The shelters of those old have been developed nowadays into beautiful houses. Rich people live in sophisticated condition houses.

2. LITERATURE REVIEW

CH.RAJKUMAR et al.( 2017) In the early ancient times humans lived in caves, over trees or under trees, to protect themselves from wild animals, rain, sun, etc. as the times passed as humans being started living in huts made of timber branches. The shelters of those old have been developed nowadays into beautiful houses. Rich people live in sophisticated condition houses. They are generally employed for architectural reasons for large rooms such as auditoriums, vestibules, theatre halls, show rooms of shops where column free space is often the main requirement. The rectangular or square void formed into ceiling is advantageously utilized for concealed architectural lighting. The sizes of the beams running in perpendicular directions are generally kept the same. Instead of rectangular beam grid a diagonal. In the present problem G+ Building is consider and analysis and design is done for both Gravity and lateral (earth quake and wind) loads. And this is compared with the flat slab.

Anitha.K et al. (2017) They are generally employed for architectural reasons for large rooms such as auditoriums, theatre halls e.c.t; where column and free space is often the main requirement A waffle slab is a type of building material that has two directional reinforcement on the outside of the material, giving it the shape of the pockets on a waffle. The parameters considered in this study are span to depth ratio, spacing of transverse beams, thickness of web and thickness of flange. The magnitude of span to depth ratio considered is 16 to 60. The spacing of transverse beams is varied from 0.5m to 2.0 m. Thickness of slab and the rib is made constant and is equal to 0.1m and 0.15m respectively. The bending moment, the shear force and the mid span deflection developed in grid floor beams have been predicted by conventional and numerical methods and the results are compared. The parametric study is carried out using the model proposed by ANSYS 12.0 software. The results of the study give an insight to the range for the magnitude of the various parameters to be considered for the optimum performance of grid floors.

Syed Abdul Qavi et al.(2016) Floor systems consisting of flat slabs are very famous in countries where cast-in place building is prime form of construction because of numerous advantages in terms of architectural flexibility, use of space, easier formwork, and shorter creation time. Flat slabs are being used chiefly in office buildings due to reduced formwork cost, fast excavation, and easy establishment. That's why it's crucial to think what you're getting into (or under) so you can maximize the comeback on your investment. Grid floor systems comprising of beams move apart at regular intervals in perpendicular directions, monolithic with slab. GRID SLAB interconnected grid systems are being commonly used or supporting building floors bridge decks and overhead water tanks slabs. In this study, slab system design and analysis for Low rise, medium and High rise building for different seismic zones and having medium soil condition by using ETABS 9.7.4 The analysis and design of slab system is done as per IS456-2000 and IS 1983-2002. Design of the slab system is done for different spacing/ grid size of column to find out which grid size of the column or plan area which slab is economical.

BharathNishan et al. (2017) In this paper a parametric study is conducted on different methods of analysis of grid slabs. A general purpose program has been developed in Visual Basic environment by combining the methodologies. The program was validated with results of standard software available for the intended purpose. The results obtained from the program are plotted for comparison and the conclusions are drawn. These methods are elaborate and time consuming to obtain the solution .The task of solving the problem manually is often tedious, unrewarding and the brain often gets tired and distracted. The probability of making mistakes increases. There was a need for developing a general purpose program which reduces the time required for solving the problem and produce results with high degree of accuracy Grid floors are types of floors with a flat surface, resting on series of interconnected beams called grids. It can be of solid concrete, wood, steel or composite. A grid floor essentially gives a beautiful houses. Rich people live in sophisticated condition houses.

AnithuDev et al. (2017) slabs are used to when a large column free area is the main requirement. There are various methods available for analyzing the grid slab system. In present study some of these methods are method, Timoshenko’s plate theory and stiffness method and compared with each other. The comparison is done on the basis of flexural parameters such as bending moments, shear forces and deflection obtained from various methods. The parameters considered for study include span to depth ratio, spacing of transverse beams, thickness of web and thickness of flange. The magnitude of span to depth ratio considered is 16 to 60. The spacing of transverse beams is varied from 0.75m to 1.75 m. Thickness of slab and the ribs are made constant and are equal to 0.1m and 0.15m respectively. The bending moment, the shear force and the mid span deflection developed in grid floor beams have been predicted by conventional and numerical methods and the results are compared and software validated. The parametric study is carried out using the model proposed by ETABS.15 software. More over the cost analysis is carried out with the help of MS.EXCEL program for finding economical design. The spacing of transverse beams is varied from 0.5m to 2.0 m. The bending moment, the shear force and the mid span deflection developed in grid floor beams have been predicted by...
conventional and numerical methods and the results are compared. The parametric study is carried out using the model proposed by ANSYS software. They found out that Timoshenko’s plate theory is in good agreement with ANSYS results.

**AVINASH PATEL et al. (2015)** Flat Slabs are highly versatile elements widely used in construction, providing minimum depth, fast construction and allowing flexible column grids. In flat slabs, the beams used in conventional slabs are done away and the slab is made to rest directly over the columns. In case of higher loads, a drop panel or a column head is provided to reduce the intensity of loads. Flat slabs are particularly appropriate for areas where tops of partitions need to be sealed to the slab soffit for acoustic or fire reasons. Grid floor systems consist of beams spaced at regular intervals in perpendicular directions, monolithic with slab. The rectangular or square void formed in the ceiling is advantageously utilized for concealed architectural lighting. They are generally employed for architectural reasons for large rooms such as auditoriums, vestibules, theatre halls, show rooms of shops where column free space is often the main requirement. Earthquake is the bane of such tall structures. As the earthquake forces are haphazard in nature & unpredictable, we need to accumulate engineering tools for analyzing structures under the action of these forces. This paper focuses on studying the behavior of conventional slab, flat slab and grid slab separately. A comparative study was done to identify the best slab system.

**S. A. Halkude et al. (2014)** The maintenance cost of these floors is less. However, construction of the grid slabs is cost prohibitive. By investigating various parameters the cost effective solution can be found for the grid slabs, for which proper method of analysis need to be used. There are various approaches available for analyzing the grid slab system. In present study some of these approaches are studied and compared with each other. The comparison is done on the basis of flexural parameters such as bending moments and shear forces obtained from various methods. For carrying out study, halls having constant width 10.00m and varying ratio of hall dimensions (L/B) from 1 to 1.5 are considered. In these types of slab, a mesh or grid of beams running in both the directions is the main structure, and the slab is of nominal thickness. It is used to cover a large area without obstruction of internal columns. They are generally employed for architectural reasons for large rooms such as auditoriums, vestibules, theatre halls, show rooms of shops where column free space is often the main requirement.

**J. Prasad et al. (2005)** In this paper elaborates the results obtained from the analytical study carried out on waffle slab medium size floor system with a view to achieve the optimum dimensions of rib spacing, its depth and width. The waffle slab has been considered as monolithically connected to band beams. In addition to these strength requirements, they are required to satisfy the deformation criteria also, in terms of low deflection and crack width. This entails into the study of the role of various structural elements specially provided to meet one or more of the requirements in the domain of strength or deformation.

**MuhammedYoosaf.K.T et al. (2016)** This paper deals with the influence of various parameters on economical spacing of the transverse beams in grid floor. The parameters considered in this study are span to depth ratio, spacing of transverse beams, thickness of web and thickness of flange. The magnitude of span to depth ratio considered is 16 to 60. The spacing of transverse beams is varied from 0.5m to 2.0 m. Thickness of slab and the rib is made constant and is equal to 0.1m and 0.15m respectively. The bending moment, the shear force and the mid span deflection developed in grid floor beams have been predicted by conventional and numerical methods and the results are compared. The parametric study is carried out using the model proposed by ANSYS 12.0 software. The results of the study give an insight to the range for the magnitude of the various parameters to be considered for the optimum performance of grid floors. Grid floor system consisting of beams spaced at regular intervals in perpendicular directions and monolithic with slab are used for large rooms such as auditoriums, vestibules, theatre hall, show room shops etc., Different patterns of grid are possible namely, rectangular, square, diagonal, continuous etc. Grids are found to be very efficient in load transferring. It is generally adopted when large column free space is required and reduces the span to depth ratio of rectangular grids.

**S. A. Halkude et al. (2014)** The Grid structure is monolithic in nature and is stiffer. It provides pleasing appearance and also has less maintenance cost. However, construction of the grid floor is cost prohibitive. By investigating various parameters involved, a cost effective solution can be found for the grid floor. The present work includes the parametric investigation in terms of flexural actions such as bending moments, torsion and shear force. Spacing of Grid beam is one of the parameters considered for investigation. The other parameter considered for investigation is the depth of Grid beams along with the design feasibility. Mesh or grid of beams running in both the directions is the main structure, and the slab is of nominal thickness. These floors are used to cover a large column free area and they are generally employed for architectural reasons for large rooms such as auditoriums, vestibules, theatre halls, show rooms of shops where column free space is often the main requirement.

**Ulfat Saboree et al. (2018)** Grid slabs are those structures in which beams are provided at square or rectangular intervals, in perpendicular direction with slabs, in recent days grid slabs are more used by the construction companies and designers, they are using grid slab for making auditoriums, theater, vestibules, show rooms, where column free space are required. In this Paper analysis and design of Fat, Grid and their different combinations for several parameters have been carried out; such as for seismic behavior, summation of total moment, dead loads, base shear, storey drift, displacement Vs. height of the structure, design specification and their comparisons.

**Hayder M.K. Al-Mutairee et al. (2019)** In this paper concerns to use the isotropic perpendicular RC straight joists to resist the external load. The yield-line theory was adapted to analysis the circular waffle slabs. The steps of design were according to the ACI Code provisions. Fixed and simply supported circular slabs were presented. Closed form equations have been driven by author for the purposes of analysis and design this type of slabs by the present procedure. Uniformly distributed load was considered, that represent almost practical cases. Useful illustration example is presented in this study according to the available materials in Iraq to facilitate the job of designers. The good performance of RC circular slab which design by the present procedure proved clearly the efficiency of this technique. They may be floor and/or roof of building, halls, water tanks, silos, bunkers, etc., hence, many researchers devoted their studies for this field. The simply supported and fixed edge circular floors under uniformly distributed loads represent the most popular practical cases, thus, they are considered here.

3. LOADS AND METHODS OF ANALYSIS LOADS
The types of loads acting on structures for buildings and other structures can be broadly classified as vertical loads, horizontal loads and longitudinal loads. The vertical loads consist of dead loads, live load and impact load. The horizontal loads comprises of wind load and earthquake load. The longitudinal loads i.e. tractive and braking forces are considered in special case of design of bridges, gantry girders etc.

Types of loads acting on the structure are:
- Dead loads
- Imposed loads
- Wind loads
- Snow loads
- Earthquake loads
- Special loads

Other loads and effects acting on structures
As per the clause 19.6 of IS 19.6 of IS 456 – 2000, in addition to above load discussed, account shall be taken of the following forces and effects if they are liable to affect the safety and serviceability of the structure.
  a) Foundation movement (see IS 1904)
  b) Elastic axial shortening
  c) Soil and fluid pressure (see IS 875, part 5)
  d) Vibration
  e) Fatigue
  f) Impact (see IS 875, part 5)
  g) Impact (see IS 875, part 5)
  h) Erection loads (see IS 875, part 2) and
  i) Stress concentration effect due to point load and the like.

3.1 STIFFNESS METHOD OF STRUCTURAL ANALYSIS
Stiffness method or displacement method is an important approach to the analysis of structures. This is used in its basic form for the analysis of structures that are linear and elastic although it can be adapted to nonlinear analysis. It is generally used for the analysis of statically determine cases. This method in its basic form considers the nodal displacements of the structures as unknown.

The Direct Stiffness Method is a highly organized, conceptually simple approach for the analysis of all types of structures that is easily implemented in the form of a computer aided analysis procedure using a matrix formation.

The most far-reaching developments in structural engineering has been the ability to analyze automatically almost all types of structures with a high degree of accuracy and at reasonable cost. The availability of digital computer has made development possible Methods of analysis that could easily be computerized were quickly developed.

Merits:
One basic form of the stiffness method could be applied to a wide range of structures, with only minor adjustments to cope with
each variant. The advantages of the method can be summarized as:
  1. A general purpose program is easy to write.
  2. It requires a minimum of input data.
  3. It can be made automatic. Its use requires no understanding of structural mechanics.
Demerits:
The method has a major disadvantage in that no account is taken of the degree of indeterminacy and therefore there is little opportunity to benefit from the structural expertise of the operator. Equally this will be seen as an essential concomitant of the advantage listed in above. The time required performing an analysis and the amount of computer storage depends almost entirely on the number of degree of freedom involved. Structures having many degrees of freedom but new degree of static indeterminacy should be much more economically analyzed by the flexibility rather than the stiffness method.

3.2 STRUCTURAL COMPONENTS IN BUILDING
Generally, a building can be defined as an enclosed structure intended for human occupancy. However, a building includes the structure itself and nonstructural components (e.g., cladding, roofing, interior walls and ceilings, HVAC systems, electrical systems) permanently attached to and supported by the structure. The scope of the Provisions provides recommended seismic design criteria for all buildings except detached one- and two-family dwellings located in zones of relatively low seismic activity and agricultural structures (e.g., barns and storage sheds) that are only intended to have incidental human occupancy. The Provisions also specifies seismic design criteria for nonstructural components in buildings that can be subjected to intense levels of ground shaking.

4. METHODOLOGY AND PROBLEM FORMULATION

4.1 METHODS OF ANALYSIS
Grid is highly redundant structural system and therefore statically indeterminate. Various approaches available for the analysis of grid floor frame are as listed below.

2. Analysis by plate theory.

4.2 RANKINE - GRASHOFF METHOD
This is an approximate method. It is based on equating deflections in either direction at the junctions of ribs. This method is suitable for small span grids with the spacing of ribs not exceeding 1.50 m. In this method the slab is considered as simply supported on edges. (Refer Figure. No.2) This method computes moments and shear force per unit width of slab strip.

4.3 PLATE ANALOGY METHOD
This is a rigorous method of analysis. This is based on Timoshenko’s analysis of orthotropic plate theory considering plane stress analysis. As in Rankine-Grashoff method, in this method also the analysis is done by considering the grid simply supported on edges (Refer Fig. No.2). Bending & torsion moments and shears are obtained per unit width of slab strip.

Figure No. 7 Typical grid considered in Rankine–Grashoff and Plate theory (Below grid)

4.4 STIFFNESS METHOD
This method is based on matrix formulation of the stiffness of the structure and gives closed form solution. By using this method the analysis can be done by considering rigid supports as well. Various application software’s are available to carry out analysis by
this method. In the present work while analyzing grid floor frame by stiffness method, the simple supports are considered at closer distance so as to simulate the support conditions similar to Rankine-Grashoff method and Plate theory. (Refer Fig. No.3).

Figure No. 8 Typical Grid floor considered in stiffness method

4.5. THEORETICAL FORMULATION

4.5.1 Typical Geometrical Data for L/B=1.0

Width of Hall \(a\) = 10.00 m, Length of Hall \(b\) = 10.00 m Spacing of grids in \(x\)-direction \(a1\) and \(y\)-direction \(b1\) = 1.00 m Thickness of slab \(Df\) = 0.1 m Width of ribs \(bw\) = 0.23 m

4.5.2 Load Calculations

The loads on floor slab are calculated on the basis of Density of reinforced concrete and floor finish considered as 25kN/m². Live load intensity=5 kN/m² Total dead load of floor area \((10.00 \times 10.00) = 730.180\) kN Total live load on floor area \((10.00 \times 10.00) = 500\) kN

\[1.5(DL + LL) = 1.5(730.180 + 500) = 1845.27\] kN

\[load\ per\ unit\ area\ (q) = \frac{1845.27}{10 \times 10} = 18.4527\] kN/m²

4.5.3 Rankine-Grashoff Method

Using load intensity given in (1), the design bending moments and shears are calculated as follows:

Load intensity in \(x\) direction \(q_x = q \times \frac{b^4}{a^4 + b^4} = 9.2263\) kN/m

Load intensity in \(y\) direction \(q_y = q \times \frac{a^4}{a^4 + b^4} = 9.2263\) kN/m

4.5.3.1 Moment calculations

Moment in beams running in \(x\) direction \(M_x = \frac{q_x b x a^2}{8} = 115.329\) kN.m

Moment in beams running in \(y\) direction \(M_y = \frac{q_y a x b^2}{8} = 115.329\) kN.m

4.5.3.2 Shear Force calculations

Shear force in beams running in \(x\) direction \(Q_x = \frac{q_x a x b x 1}{2} = 46.132\) kN
Shear force in beams running in y direction \((Q_y) = \frac{q_1 b x a_s}{2} = 46.132 \text{ kN}\)

### 4.5.4 Plate Theory

Using the load intensity as mentioned in (1) design bending and torsion moments, and shear forces are calculated and presented below:

#### 4.5.4.1 Section Properties of Ribs

\[
\frac{D_f}{D} = \frac{0.1}{0.6} = 0.167 \quad \text{And} \quad \frac{b_w}{b_f} = \frac{0.25}{1.00} = 0.23
\]

Second moment of inertia of beam in \(x\)-direction \((I_1)\) and in \(y\)-direction \((I_2)\) = \(k \cdot b_w \cdot D^3\)

Where \(k = \) constant taken from Reynolds designers handbook = 0.1455

\[I_1 = I_2 = 0.1455 \times 0.23 \times 0.6^2 = 7.22844 \times 10^6 \text{mm}^4\]

Flexural rigidity per unit length of plate along \(x\)-direction = \(D_x = \frac{E I_1}{b_1} = 0.00722 \text{E}\)

Flexural rigidity per unit length of plate along \(y\)-direction = \(D_y = \frac{E I_2}{b_2} = 0.00722 \text{E}\)

Torsional rigidity per unit length of plate along \(x\) direction = \(C_t = k_1 G (2a^2)2b\)

Torsional rigidity per unit length of plate along \(y\) direction = \(C_2 = k_1 G (2b^2)2a\)

Where,

\[G = \frac{E}{2(1+\nu)} = \frac{E}{2(1+0.18)} = 0.43478 \text{E}\]

\[k_1 = \text{Constant of Torsional rigidity given by Timoshenko}\]

Here in this case,

\[C_1 = C_2 = 0.252 \times 0.43478 \text{E} \times (2 \times 10)^3 \times (2b) = 0.007998 \text{E}\]

\[E = 5700 \sqrt{h/c} = 5700 \sqrt{20} = 24.4911 \times 10^6 \text{ kN/m}^2\]

#### 4.5.4.2 Deflection at center of span

\[2H = \frac{C_1}{b_1} + \frac{C_2}{a_2} = 0.0016 \text{E}\]

\[\frac{D_x}{a_2^4} = 18.426 \text{ and} \quad \frac{D_y}{b_y^4} = 18.426\]

\[\text{deflection} = \frac{16q}{E} \left[ \sin \left( \frac{2\pi x}{a_2} \right) \sin \left( \frac{2\pi y}{b_y} \right) \right] = 0.07526 \text{ m}\]

### 5. RESULT AND DISCUSSION

**Part 1** = results of the analysis carried by Rankine-Grashoff method, Plate theory, and Stiffness method

The results of the analysis carried by Rankine-Grashoff method, Plate theory, and Stiffness method are presented below. To carry out analysis of different hall sizes automation in analysis procedure is required which is done by using excel worksheets. These worksheets are used for Rankine-Grashoff method and Plate theory. The analysis by Stiffness method is carried out using STAAD.pro, application software. After analyzing such grids by above discussed three methods, the results are presented here.
Figure no. 11 comparison of maximum bending moments for beams in x-direction (Mx)

The Figure No.4 shows the variations in maximum bending moment (Mx) for the beams which are running in x-direction for various (L/B) ratios by various methods of analysis.

The graph shows that the bending moment (Mx) is increasing with increasing L/B ratio. The nature of bending moment variation is non-linear for Rankine-Grashoff method and Plate theory approach. However, using stiffness method bending moment (Mx) is increasing almost linearly with increasing L/B ratio.

Up to L/B = 1.3, the bending moment (Mx) is in close proximity for Plate Theory and Stiffness method. With increase in L/B beyond 1.3, the bending moment (Mx) given by Plate theory is lower than those given by the Stiffness method in the range of 5% to 17%. Rankine-Grashoff method estimates lowest values of Mx, amongst all above three methods.

Figure no.12 comparison of maximum bending moments for beams in y-direction (My)

The Figure No.5 shows the variations in maximum bending moment (My) for the beams which are running in y-direction for various (L/B) ratios by various methods of analysis. The bending moment (My) is decreasing as L/B goes on increasing for all three methods. As the L/B ratio increases, the variation is observed to be non-linear for Rankine-Grashoff method and Stiffness method. However, for Plate theory approach the variation is almost linear. The graph also shows that, the bending moment (My) is in close proximity for Plate theory and Stiffness method up to L/B= 1.1. With increase in L/B beyond 1.1, the bending moments (My) given by Plate theory is lower than that given by the Stiffness method in the range of 7% to 48%. With increasing L/B ratio these values become lower for plate theory.
Figure no. 13 comparison of maximum shear force for beams in x-direction (Qx)

The Figure No.6 shows the variations in maximum Shear Force (Qx) for the beams which are running in x-direction for various (L/B) ratios by various methods of analysis.

The variation of shear force (Qx) is observed to be nonlinear for Rankine-Grashoff method and Plate theory.

However, for stiffness method the variation of shear force (Qx) is almost linear. Rankine-Grashoff method estimates lowest values of shear force (Qx) amongst above three methods. For a given L/B ratio, the Stiffness method shows highest value of shear force (Qx), than that is shown by Plate theory and Rankine-Grashoff method. Plate theory shows less value of shear force (Qx) by 30% to 37% than the Stiffness method for L/B = 1 to 1.5.

Figure no. 14 comparison of maximum shear force for beams in y-direction (Qy)

The Figure No.7 shows the variations in maximum Shear Force (Qy) for the beams which are running in y-direction for various (L/B) ratios by various methods of analysis.

The graph shows that shear force (Qy) is decreasing almost linearly with increasing L/B ratio, for all the three methods.

The stiffness method shows highest value of shear force (Qy) for the given L/B ratio. Rankine-Grashoff method shows lower values in the range of 32% to 60% for L/B= 1 to 1.5 than that of stiffness method. Plate theory shows values lower by 30% to 68% for L/B =1 to 1.5.

Part 2- Using the Excel worksheet, the quantity of steel and concrete and the total cost are determined for different Grid floor sizes, thicknesses of slab (Table 3), width of Ribs (Table 4) and spacing of Ribs (Table 5) and the results are tabulated for the three Cases defined in the scope.
CASE 1

Table No 4: Grid floor Material costs for different Grid size’s for varying thicknesses of slab

<table>
<thead>
<tr>
<th>Size of Grid Floor (m²)</th>
<th>Thickness of slab (mm)</th>
<th>Quantity of steel (m³)</th>
<th>Quantity of Steel (kg)</th>
<th>Quantity of concrete (m³)</th>
<th>Total Cost (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 x 16</td>
<td>90</td>
<td>0.210</td>
<td>1648.50</td>
<td>34.01</td>
<td>277,192.50</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.226</td>
<td>1774.10</td>
<td>35.60</td>
<td>293,316.50</td>
</tr>
<tr>
<td></td>
<td>110</td>
<td>0.242</td>
<td>1899.70</td>
<td>37.19</td>
<td>309,440.50</td>
</tr>
<tr>
<td>14 x 16</td>
<td>90</td>
<td>0.262</td>
<td>2056.70</td>
<td>43.83</td>
<td>352,825.50</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.282</td>
<td>2213.70</td>
<td>45.68</td>
<td>372,290.50</td>
</tr>
<tr>
<td></td>
<td>110</td>
<td>0.302</td>
<td>2370.70</td>
<td>47.53</td>
<td>391,755.50</td>
</tr>
<tr>
<td>16 x 16</td>
<td>90</td>
<td>0.293</td>
<td>2300.05</td>
<td>54.85</td>
<td>423,743.25</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.298</td>
<td>2339.30</td>
<td>56.96</td>
<td>436,854.50</td>
</tr>
<tr>
<td></td>
<td>110</td>
<td>0.366</td>
<td>2873.10</td>
<td>59.07</td>
<td>482,111.50</td>
</tr>
</tbody>
</table>

Figure No. 15 Variation of thickness of slab with total material cost for different sizes of Grid floor

From Fig. 15, it can be observed that
1. The cost of material requirement of slab increases with increase in thickness of slab.
2. For an increase in thickness of slab by 10 mm, the increase in cost of steel and concrete requirement for a given grid size is in the range of 3 to 10%.
3. The % increase in cost for increase in thickness of slab decreases with increase in grid size.
4. The graphs plotted for thickness of slab against cost of material requirement appear to diverge which means that at a greater thickness of slab, the cost of the material requirement of the slab is going to vary considerably with the size of grid.
5. The slope of these graphs decreases with increase in thickness of slab which means that the cost of the slab is not going to increase considerably beyond certain thickness of slab.

CASE 2

Table No 5: Grid floor Material costs for different Grid size’s for varying width of Ribs

<table>
<thead>
<tr>
<th>Size of Grid Floor (m²)</th>
<th>Width of Rib (mm)</th>
<th>Quantity of steel (m³)</th>
<th>Quantity of Steel (kg)</th>
<th>Quantity of concrete (m³)</th>
<th>Total Cost (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 x 16</td>
<td>200</td>
<td>0.24</td>
<td>1899.70</td>
<td>37.19</td>
<td>309,440.50</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>0.23</td>
<td>1766.25</td>
<td>45.23</td>
<td>340,946.25</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>0.21</td>
<td>1632.80</td>
<td>53.26</td>
<td>372,452.00</td>
</tr>
<tr>
<td>14 x 16</td>
<td>200</td>
<td>0.30</td>
<td>2370.70</td>
<td>47.53</td>
<td>391,755.50</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>0.28</td>
<td>2231.55</td>
<td>58.98</td>
<td>439,290.75</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>0.26</td>
<td>2048.85</td>
<td>70.42</td>
<td>485,295.25</td>
</tr>
<tr>
<td>16 x 16</td>
<td>200</td>
<td>0.34</td>
<td>2653.30</td>
<td>59.07</td>
<td>467,824.50</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>0.32</td>
<td>2504.15</td>
<td>74.53</td>
<td>535,409.75</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>0.30</td>
<td>2323.60</td>
<td>89.98</td>
<td>600,954.00</td>
</tr>
</tbody>
</table>
From Fig. 16, it can be observed that
1. The cost of the material requirement of the slab increases with increase in width of rib.
2. For an increase in width of rib by 50 mm, the decrease in the cost of the steel and concrete for a given grid size is in the range of 9 to 15%.
3. The % increase in cost for increase in width of rib increases with increase in grid size.
4. The graphs plotted for width of ribs against cost of material requirement appear to diverge which means that at a greater the width of rib, the cost of the material requirement of the slab is going to vary considerably with the size of grid.
5. The slope of these graphs increases with increase in width of rib which means that the cost of the slab is going to increase considerably beyond certain width of rib.

**CASE 3**

**Table No 6: Grid floor Material costs for different Grid size's for varying spacing of Ribs**

<table>
<thead>
<tr>
<th>Size of Grid Floor (m2)</th>
<th>Width of Rib (mm)</th>
<th>Quantity of steel (m3)</th>
<th>Quantity of Steel (kg)</th>
<th>Quantity of concrete (m3)</th>
<th>Total Cost (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 x 16</td>
<td>1.5</td>
<td>0.280</td>
<td>2198</td>
<td>43.464</td>
<td>360190.0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.242</td>
<td>1899.7</td>
<td>37.192</td>
<td>309440.5</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>0.207</td>
<td>1624.95</td>
<td>33.428</td>
<td>272761.75</td>
</tr>
<tr>
<td>14 x 16</td>
<td>1.5</td>
<td>0.348</td>
<td>2731.8</td>
<td>56.342</td>
<td>459,277.00</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.302</td>
<td>2370.7</td>
<td>47.532</td>
<td>391,755.50</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>0.274</td>
<td>2150.9</td>
<td>42.245</td>
<td>351,033.50</td>
</tr>
<tr>
<td>16 x 16</td>
<td>1.5</td>
<td>0.419</td>
<td>3289.15</td>
<td>70.848</td>
<td>568,034.75</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.366</td>
<td>2873.1</td>
<td>59.072</td>
<td>482,111.50</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>0.325</td>
<td>2551.25</td>
<td>52.006</td>
<td>425,861.25</td>
</tr>
</tbody>
</table>
From Fig. 5, it can be observed that
1. The cost of the material requirement of the slab decreases with increase in spacing of ribs.
2. For an increase in spacing of 0.5 m, the decrease in the cost of the steel and concrete for a given grid size is in the range of 10 to 16%.
3. The % decrease in cost for increase in spacing increases with increase in grid size.
4. The graphs plotted for spacing of ribs against cost of material requirement appear to converge which means that at a greater spacing, the cost of the material requirement of the slab is not going to vary much with the size of grid.
5. The slope of these graphs decreases with increase in spacing which means that the cost of the slab is not going to reduce much beyond certain spacing.

**Part 3- Methodology of finding the probability of failure of grid slab**

a) For random variations in different grades of concrete, characteristic strength of steel, dimensions and live load, corresponding Moment, Shear & Deflection values are obtained from ETABS software and is denoted as action (S).
b) Using MATLAB software simulate moment of resistance, shear resistance and deflection using above said equation (1 to 7) and denote it as resistance (R).
c) Compute safety margin i.e., \( M = R - S \)
d) Compute Reliability Index (\( \beta \)) = of safety margin \( \sigma_{\mu} \)
e) Compute probability of failure by equation \( P.F = (1 - \beta \phi) \)

**Table No 7: Reliability Index and Probability of failure for square grid in flexure by Monte Carlo simulation**

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>fck N/mm²</th>
<th>fy N/mm²</th>
<th>Spacing mm</th>
<th>Rib mm</th>
<th>width mm</th>
<th>Reliability index</th>
<th>Probability of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>415</td>
<td>1200</td>
<td>150</td>
<td>3.7380</td>
<td>0.0927 x10⁻³</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>415</td>
<td>1200</td>
<td>170</td>
<td>3.4780</td>
<td>0.2526 x10⁻³</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>415</td>
<td>1200</td>
<td>190</td>
<td>3.2330</td>
<td>0.6125 x10⁻³</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>500</td>
<td>1200</td>
<td>210</td>
<td>4.4667</td>
<td>0.0039 x10⁻³</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>500</td>
<td>1200</td>
<td>230</td>
<td>3.6432</td>
<td>0.1346 x10⁻³</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>500</td>
<td>1200</td>
<td>250</td>
<td>3.8758</td>
<td>0.0531 x10⁻³</td>
<td></td>
</tr>
</tbody>
</table>

**CONCLUSION**

Part 1-

1) Rankine-Grashoff method is an approximate method. Rankine-Grashoff method does not give the values of torsion moments. Rankine-Grashoff method underestimates critical bending moment (Mx) and shear force (Qx).
2) Plate theory and Rankine-Grashoff method are used for simple support conditions. On the contrary the stiffness method can be used for rigid supports as well.
3) In Plate theory and Rankine-Grashoff method, design moments and shear force in Peripheral beams cannot be obtained. In fact in monolithic framed construction, design moments and shears in peripheral beams will be the maximum.
4) Initially, up to L/B=1.2, Plate theory shows higher value of bending moment (Mx) with respect to stiffness method. With increasing L/B, beyond L/B=1.3 Plate theory shows lower value of bending moment (Mx) as compared to stiffness method.
5) Stiffness method shows higher value of shear force (Qx) as compared to other methods discussed. Plate theory shows less values of Qx than that of stiffness method.
6) Stiffness method is accurate & more suitable to arrive at design moments and shear force. Also Stiffness method takes less time for analysis.

Part 2-
From this study, it can be concluded that the increase in cost with increase in thickness of slab decreases with increase in Grid size. Further, Cost of slab increases with increase in width of Rib and size of grid while increase in spacing of ribs decreases the cost of Grid floor. For the scope defined for this study, the cost of material is minimum for minimum width of Rib (200 mm), minimum thickness (90 mm) and maximum spacing (2.5 m) for all sizes of Grid floors. As most of grid floors adopted in practice are of the sizes discussed in this study, the guidelines are expected to be of use to the design engineers before actually designing it. Also, from the results of the typical grid floor analysis, it can be concluded that the approximate method underestimates the moments. However, further study and more results are required to give a quantitative dimension to this aspect.

Even though some of the resulting conclusions are easily foreseeable, (ie., cost increases with slab thickness and decreases with rib spacing), the study provides a quantitative insight into the effect of dimensions on economy of Gird Floor.
1. For an increase in thickness of slab by 10 mm, the increase in cost of steel and concrete requirement for a given grid size is in the range of 3 to 10 %.
2. For an increase in width of rib by 50 mm, the decrease in the cost of the steel and concrete for a given grid size is in the range of 9 to 15 %.
3. For an increase in spacing of 0.5 m, the decrease in the cost of the steel and concrete for a given grid size is in the range of 10 to 16 %.

Part 3-
a) Explicit evaluation of safety in terms of probability of failure of a Grid slab when the design variables are random in nature and do follow given probability distributions is done using digital simulation, by Monte Carlo technique and AFOSM.

b) It is observed that the reliability index varies from 3.233 to 5.162 for square grid slab in flexure, in shear the reliability index varies from 4.895 to 5.830 and in deflection the reliability index varies from 6.248 to 7.711.

c) The reliability index varies from 3.686 to 4.861 for rectangular grid slab in flexure, in shear the reliability index varies from 4.834 to 5.841 and in deflection the reliability index varies from 6.663 to 8.166.
d) Thus there is almost a consistent level of reliability in the design methodology adopted by IS 456-2000. However the present limit state method of design does not consider the importance of a structure, exposure conditions, effect of quality control etc.

REFERENCES