C-Set in a Banach Algebra

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Abstract

I use the notation of C-Set in a linear space which is a generalization of a convex set. I have used C-Set in Banach Algebra which is also an Algebra with identity 1 and multiplicative structure is used.

Introduction

In this paper I have defined C-Set in a Banach algebra algorithm which identify 1 and in which multiplication structure is related to the norm.

Definition:

A Banach algebra is a complex Banach space which is also an algebra with identity 1, and in which the multiplicative structure is related to the norm by the following requirements:

(1) $\|xy\| \leq \|x\|.\|y\|$.

(2) $\|1\| = 1$.

It follows that $x_n \to x, y_n \to y \Rightarrow x_n y_n \to xy$ .12

Theorem (I) : Let A be a Banach algebra and G be a proper left C-set of A, then $\overline{G}$ is also a proper left C-Set of A.

Proof : Since G is a C-Set of linear space A, by theorem (I), $\overline{G}$ is a C-Set of linear space A. Let g be an element of $\overline{G}$, then there exists a sequence $\{g_n\}$ in G Such that

$$g_n \to g.$$

Let x belongs to A then

$$xg_n \to xg.$$

But $\{xg_n\} \subseteq G$, hence xg belongs to $\overline{G}$.

Therefore $\overline{G}$ is a left C-Set of A.

Since G is a proper left C-Set by theorem , it can not contain a regular element. Let S denote the set of singular elements of A, then

$$G \subseteq S.$$

Now S is a closed set.

Thus $G \subseteq \overline{G} \subseteq S.$

Since $1 \notin S, 1 \notin \overline{G}$. Therefore $\overline{G}$ is a proper left C-Set of A.

Similarly, if G is proper right C-Set of A, then $\overline{G}$ is also a proper right C-Set of A.

Finally, if G is a proper C-Set of A, then $\overline{G}$ is a proper C-Set of A.

REFERENCES

1. Simmon, G. F.; Introduction to Topology and Modern Analysis, (1963), p. 302.

2. Simmons, G.F.; Introduction to Topology and Modern Analysis, (1963), P. 306.