C-Set in a Banach Algebra

Dr. Kumar Swetank,
Jai Prakash University, Chapra(Saran), Bihar, India.

Abstract
I use the notation of C-Set in a linear space which is a generalization of a convex set. I have used C-Set in Banach Algebra which is also an Algebra with identity 1 and multiplicative structure is used.

Introduction
In this paper I have defined C-Set in a Banach algebra algorithm which identify 1 and in which multiplication structure is related to the norm.

Definition:
A Banach algebra is a complex Banach space which is also an algebra with identity 1, and in which the multiplicative structure is related to the norm by the following requirements:

(1) \[ \|x y\| \leq \|x\| \cdot \|y\|. \]
(2) \[ \|1\| = 1. \]

It follows that \[ x_n \rightarrow x, y_n \rightarrow y \Rightarrow x_n y_n \rightarrow xy. \]

Theorem (I): Let A be a Banach algebra and G be a proper left C-set of A, then \( \overline{G} \) is also a proper left C-Set of A.

Proof: Since G is a C-Set of linear space A, by theorem (I), \( \overline{G} \) is a C-Set of linear space A. Let g be an element of \( \overline{G} \), then there exists a sequence \( \{g_n\} \) in G such that \( g_n \rightarrow g \).

Let x belongs to A then \( x g_n \rightarrow x g \).

But \( \{x g_n\} \subseteq G \), hence xg belongs to \( \overline{G} \).

Therefore \( \overline{G} \) is a left C-Set of A.

Since G is a proper left C-Set by theorem, it can not contain a regular element. Let S denote the set of singular elements of A, then
\[ G \subseteq S. \]

Now \( S \) is a closed set.

Thus \( G \subseteq \overline{G} \subseteq S. \)

Since \( 1 \not\in S, 1 \not\in \overline{G} \). Therefore \( \overline{G} \) is a proper left C-Set of \( A \).

Similarly, if \( G \) is proper right C-Set of \( A \), then \( \overline{G} \) is also a proper right C-Set of \( A \).

Finally, if \( G \) is a proper C-Set of \( A \), then \( \overline{G} \) is a proper C-Set of \( A \).

REFERENCES