

Differential Equation in the Frequency-Domain - An Overview

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Abstract

This paper studies the concept of magnetism in physics, With rare exception, this is the only form of magnetism strong enough to be felt by people. Magnetic fields are generated by rotating electric charges, according to HyperPhysics. Electrons all have a property of angular momentum, or spin. Most electrons tend to form pairs in which one of them is “spin up” and the other is “spin down,” in accordance with the Pauli Exclusion Principle, which states that two electrons cannot occupy the same energy state at the same time. In this case, their magnetic fields are in opposite directions, so they cancel each other. However, some atoms contain one or more unpaired electrons whose spin can produce a directional magnetic field. The direction of their spin determines the direction of the magnetic field, according to the Non-Destructive Testing (NDT) Resource Center. When a significant majority of unpaired electrons are aligned with their spins in the same direction, they combine to produce a magnetic field that is strong enough to be felt on a macroscopic scale. Magnetic field sources are dipolar, having a north and south magnetic pole. Opposite poles (N and S) attract, and like poles (N and N, or S and S) repel, according to Joseph Becker of San Jose State University. This creates a toroidal, or doughnut-shaped field, as the direction of the field propagates outward from the north pole and enters through the south pole.

Key words: Non-Destructive Testing, Magnetic monopoles , Gauss law, magnetic field.

Introduction

Magnetism takes many other forms, but except for ferromagnetism, they are usually too weak to be observed except by sensitive laboratory instruments or at very low temperatures. Diamagnetism was first discovered in 1778 by Anton Brugnam, who was using permanent magnets in his search for materials containing iron. According to Gerald Küstler, a widely published independent German researcher and inventor, in his paper, “Diamagnetic Levitation — Historical Milestones,” published in the Romanian Journal of Technical Sciences, Brugnam observed, “Only the dark and almost violet-colored bismuth displayed a particular phenomenon in the study; for when I laid a piece of it upon a round sheet of paper floating atop water, it was repelled by both poles of the magnet.”

Objective:

This paper intends to explore Gauss' Law for magnetism which applies to the magnetic flux through a closed surface. Also, the area vector which points out from the surface.

The Gauss's law for magnetic fields in integral form is given by:

$$\oint_S \mathbf{b} \cdot d\mathbf{a} = 0, \oint_S \mathbf{b} \cdot d\mathbf{a} = 0,$$

where:

- \mathbf{b} is the magnetic flux

The equation states that there is no net magnetic flux \mathbf{b} (which can be thought of as the number of magnetic field lines through an area) that passes through an arbitrary closed surface SS . This means the number of magnetic field lines that enter and exit through this closed surface SS is the same. This is explained by the concept of a magnet that has a north and a south pole, where the strength of the north pole is equal to the strength of the south pole (Fig. 35). This is equivalent to saying that a magnetic monopole, meaning a solitary north or south pole, does not exist because for every positive magnetic pole, there must be an equal amount of negative magnetic poles.

Differential equation

Gauss's law for magnetic fields in the differential form can be derived using the divergence theorem. The divergence theorem states:

$$\int_V (\nabla \cdot \mathbf{f}) dv = \oint_S \mathbf{f} \cdot d\mathbf{a}, \int_V (\nabla \cdot \mathbf{f}) dv = \oint_S \mathbf{f} \cdot d\mathbf{a},$$

where \mathbf{f} is a vector. The right-hand side looks very similar to Using the divergence theorem, Equation is rewritten as follows:

$$0 = \oint_S \mathbf{b} \cdot d\mathbf{a} = \int_V (\nabla \cdot \mathbf{b}) dv, 0 = \oint_S \mathbf{b} \cdot d\mathbf{a} = \int_V (\nabla \cdot \mathbf{b}) dv.$$

Because the expression is set to zero, the integrand $(\nabla \cdot \mathbf{b})(\nabla \cdot \mathbf{b})$ must be zero also. Thus the differential form of Gauss's law becomes:

$$\nabla \cdot \mathbf{b} = 0, \nabla \cdot \mathbf{b} = 0.$$

Derivation using Biot-Savart law

Gauss's law can be derived using the Biot-Savart law, which is defined as:

$$\mathbf{b}(\mathbf{r}) = \mu_0 4\pi \int_V (\mathbf{j}(\mathbf{r}') dv) \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}, \mathbf{b}(\mathbf{r}) = \mu_0 4\pi \int_V (\mathbf{j}(\mathbf{r}') dv) \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3},$$

where:

- $b(r)$ is the magnetic flux at the point r
- $j(r')$ is the current density at the point r'
- μ_0 is the magnetic permeability of free space.

Taking the divergence of both sides of Equation (51) yields:

$$\nabla \cdot b(r) = \mu_0 4\pi \int \nabla \cdot (j(r') dv) \times \frac{r - r'}{|r - r'|^2} \cdot \nabla \cdot b(r) = \mu_0 4\pi \int \nabla \cdot (j(r') dv) \times \frac{r - r'}{|r - r'|^2}$$

To carry through the divergence of the integrand in Equation (52), the following vector identity is used:

$$\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)$$

Thus, the integrand becomes:

$$[j(r') \cdot (\nabla \times \frac{r - r'}{|r - r'|^2})] - [\frac{r - r'}{|r - r'|^2} \cdot (\nabla \times j(r'))]$$

- Electric Field intensity due to Infinitely long uniformly Charged Wire
- Electric Field due to Plane Sheet
- Electric Field due to Spherical shell

Thus the net electric flux through any closed surface is equal to $1/\epsilon_0$ times the net electric charge within that closed surface (or imaginary Gaussian surface)

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